graphical nature and the physically intuitive insights it can provide, make it an important tool for practicing engineers. Applications of the describing function method to the prediction of limit cycles were detailed. Other applications, such as predicting subharmonics, jump phenomena, and responses to external sinusoidal inputs, can be found in the literature.

5.6 Notes and References

An extensive and clear presentation of the describing function method can be found in [Gelb and VanderVelde, 1968]. A more recent treatment is contained in [Hedrick, *et al.*, 1982], which also discusses specific applications to nonlinear physical systems. The describing function method was developed and successfully used well before its mathematical justification was completely formalized [Bergen and Franks, 1971]. Figures 5.14 and 5.16 are adapted from [Shinners, 1978]. The Van der Pol oscillator example is adapted from [Hsu and Meyer, 1968].

5.7 Exercises

5.1 Determine whether the system in Figure 5.28 exhibits a self-sustained oscillation (a limit cycle). If so, determine the stability, frequency, and amplitude of the oscillation.

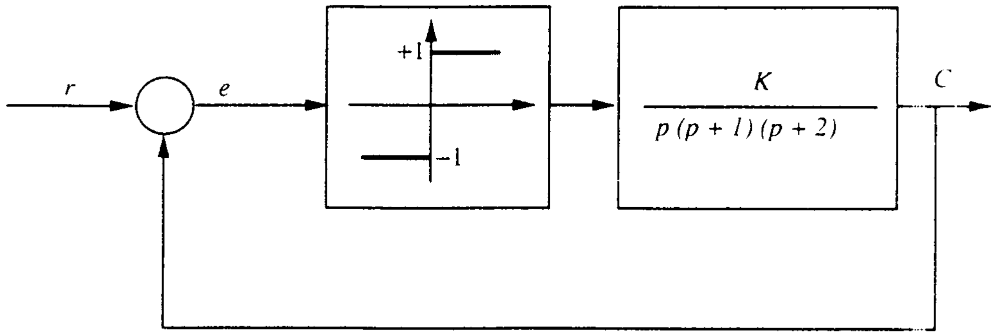

Figure 5.28 : A nonlinear system containing a relay

5.2 Determine whether the system in Figure 5.29 exhibits a self-sustained oscillation. If so, determine the stability, frequency, and amplitude of the oscillation.

5.3 Consider the nonlinear system of Figure 5.30. Determine the largest K which preserves the stability of the system. If $K = 2K_{max}$, find the amplitude and frequency of the self-sustained oscillation.

5.4 Consider the system of Figure 5.31, which is composed of a high-pass filter, a saturation function, and the inverse low-pass filter. Show that the system can be viewed as a nonlinear low-

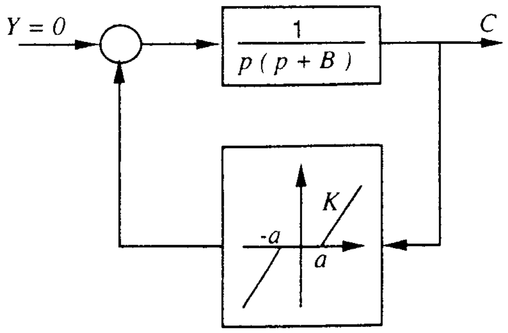

Figure 5.29 : A nonlinear system containing a dead-zone

pass filter, which attenuates high-frequency inputs *without introducing a phase lag.*

5.5 This exercise is based on a result of [Tsypkin, 1956].

Consider a nonlinear system whose output $w(t)$ is related to the input $u(t)$ by an odd function, of the form

$$w(t) = F(u(t)) = -F(-u(t)) \tag{5.18}$$

Derive the following very simple approximate formula for the describing function $N(A)$

$$N(A) \approx \frac{2}{3A}\left[\, F(A) + F(A/2) \,\right]$$

To this effect, you may want to use the fact that

$$\frac{1}{\pi}\int_{-1}^{1} \frac{f(x)}{\sqrt{1-x^2}}\, dx = \frac{1}{6}\left[\, f(1) + f(-1) + 2f(1/2) + 2f(-1/2) \,\right] + R$$

where the remainder R verifies $R = f^{6}(\xi)/(2^{5}6!)$ for some $\xi \in (-1, 1)$. Show that approximation (5.18) is quite precise (how precise?).

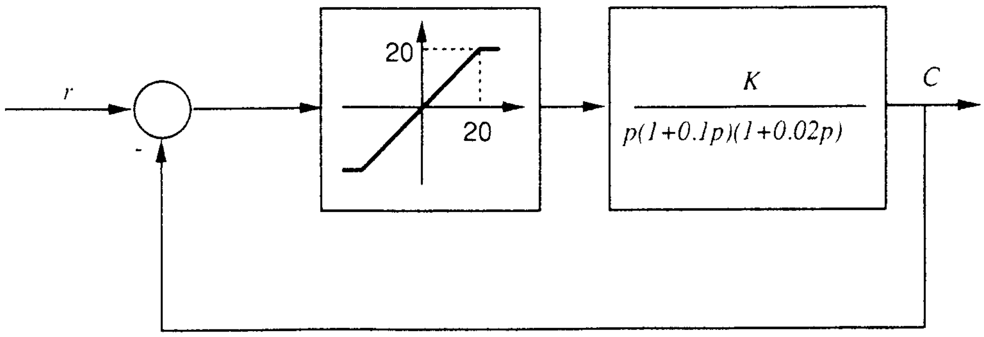

Figure 5.30 : A nonlinear system containing a saturation

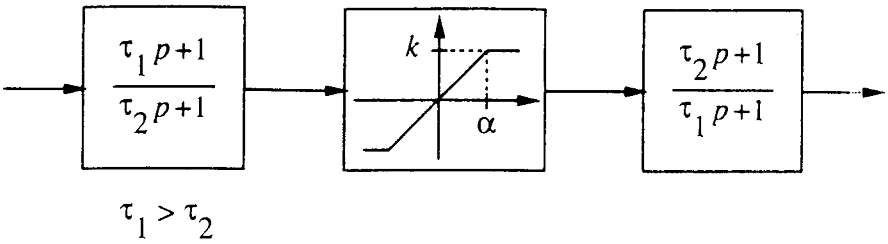

Figure 5.31 : A nonlinear low-pass filter

Invert (5.18) so as to obtain for the input-output relation a solution of the form

$$F(A) \approx \sum_{k=0}^{\infty} (-1)^k \frac{3A}{2^{k+1}} N(\frac{A}{2^k})$$

5.6 In this exercise, adapted form [Phillips and Harbor, 1988], let us consider the system of Figure 5.32, which is typical of the dynamics of electronic oscillators used in laboratories, with

$$G(p) = \frac{-5p}{p^2 + p + 25}$$

Use describing function analysis to predict whether the system exhibits a limit cycle, depending on the value of the saturation level k. In such cases, determine the limit cycle's frequency and amplitude.

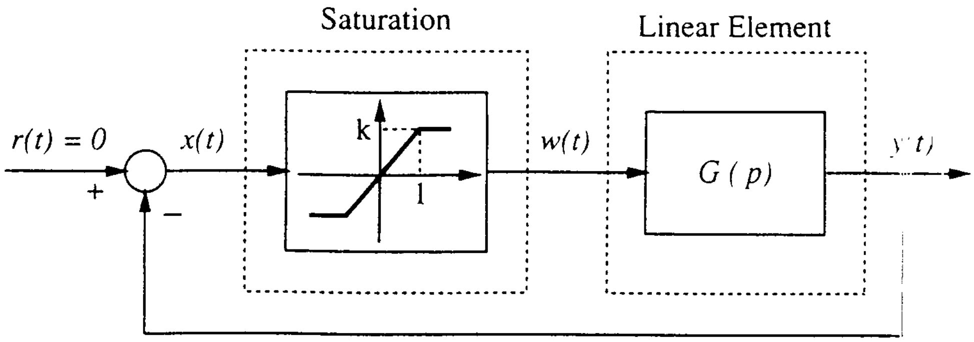

Figure 5.32 : Dynamics of an electronic oscillator

Interpret intuitively, by assuming that the system is started at some small initial state, and noticing that $y(t)$ can stay neither at small values (because of instability) nor at saturation values (by applying the final value theorem of linear control).