Omnidirectional Edge Detection

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Abstract*--In this paper we propose a new method for extending 1-D step edge detection filters to two dimensions via complex-valued filtering. Complex-valued filtering allows us to obtain edge magnitude and direction simultaneously. Our method can be viewed either as an extension of *n*-directional complex filtering of Paplinski to infinite directions or as a variant of Canny's gradient-based approach. In the second view, the real part of our filter computes the gradient in the *x* direction and the imaginary part computes the gradient in the *y* direction. Paplinski claimed that *n*-directional filtering is an improvement over the gradient-based method, which computes gradient only in two directions. We show that our omnidirectional and Canny's gradient-based extensions of the 1-D DoG coincide. In contrast to Paplinski's claim, this coincidence shows that both approaches suffer from being confined to the subspace of two 2-D filters, even though *n*-directional filtering hides these filters in a single complex-valued filter. Aside from these theoretical results, the omnidirectional method has practical advantages over both *n*-directional and gradient-based approaches. Our experiments on synthetic and real world images show the superiority of omnidirectional and gradient-based methods over *n*-directional approach. In comparison with the gradient-based method, the advantage of omnidirectional method lies mostly in freeing the user from specifying the smoothing window and its parameter. Since the omnidirectional and Canny's gradient-based extensions of the 1-D DoG coincide, we have based our experiments on extending the 1-D Demigny filter. This filter has been proposed by Demigny as the optimal edge detection filter in sampled images.

***Index Terms*-- Complex-valued filtering, Directional edge detection, Infinite directional, Two-dimensional edge detection**

I. INTRODUCTION

DESIGNING optimal linear filters for edge detection in images has been a popular subject of research for the past three decades and a large number of methods have been proposed for this purpose. Edge detection methods can be broadly divided into three categories:

1. Methods that only detect edge magnitude and provide no directional information. These methods are usually based on the Laplacian of Gaussian (LoG) operator [1][2] and solve the problem directly in two dimensions. More sophisticated methods based on LoG also exist which are nonlinear [3]. These methods, which are not directional, are not further considered in this paper.
2. Approaches that detect both magnitude and direction of edges by extending a 1-D optimal edge detection filter to two dimensions [4][5][6][7].
3. Methods that detect both magnitude and direction of edges and solve the problem directly in two dimensions, but do not fall within the category of linear filtering, e.g. [8].

In this paper, we study approaches for extending 1-D edge detection filters to two dimensions and propose a new method as well. The standard method for this extension was proposed by Canny[5]. He argued that to detect an edge with the tangent direction $\mathbf{d}$ and the normal direction $\mathbf{n}$, one should construct a 2-D filter by posing the edge detection filter in direction $\mathbf{n}$ and

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multiply it with a projection function in direction $\mathbf{d}$. Furthermore, direction $\mathbf{n}$ can be robustly estimated by the gradient direction of the Gaussian smoothed image. On the other hand, Canny's 1-D edge detection filter is the derivative of the Gaussian (DoG). Taking parameters of these two Gaussians equal, he proposed that edge magnitude and direction can be estimated by equation (1), where G_x and G_y are 2-D filters for detecting edges in directions x and y, respectively. Finally he showed that application of G_x and G_y to an image is equivalent to first smoothing the image by a Gaussian and then computing the gradient in directions x and y. By exploiting the separability of Gaussians, this approach simplifies convolutions from two dimensions to one dimension. Although Canny's extension has been derived by using specific properties of Gaussians, his method can be generalized for extending other 1-D filters to two dimensions. We study this general approach in section 2.1.

After Demigny[9] formulated the edge detection problem in discrete space and claimed that sampled versions of continuous operators are not optimal for sampled data , researchers favored formulations of edge detection in the discrete space. In specific, [10] proposed a discrete counterpart of the Canny's method for extending 1-D filters to two dimensions which will be discussed in section 2.2. We after [10] refer to these formulations as gradient-based approaches.

Noting that complex numbers embed both magnitude and direction in themselves, Paplinski[7] proposed another solution for this extension. He argued that since the Canny gradient operators, namely G_x and G_y, estimate edges only in directions x and y, the Canny's approach is limited to two directions. So he proposed *n*-directional filtering to simultaneously estimate edges in *n* directions. He especially advocated tri-directional filtering and claimed that: "at a close examination, it appears that three directions, e.g., $(0,120,240^\circ)$, constitute the sufficient basis for the gradient calculation." His work will be briefly reviewed in Section 2.3.

In this paper, we introduce a new solution to the complex-valued edge detection problem which shows very nice properties. In contrast to *n*-directional filtering of Paplinski[7] which is biased towards several pre-specified directions, our omnidirectional filtering is isotropic[1]. Our derivation of omnidirectional filtering is discussed in section 3. In section 4, we show that omnidirectional and the Canny's gradient-based extensions of the 1-D DoG coincide. This shows that, in contrast to Paplinski's claim, *n*-directional filtering has not escaped from calculating gradient in two directions. In fact, complex-valued filtering hides these two gradient operators in real and imaginary parts of the filter.

In section 5 we generalize the result of section 4 by proving that omnidirectional and gradient-based extensions coincide on all separable smoothing functions. This shows that, from a computational point of view, omnidirectional and gradient-based extensions are equivalent. This equivalence is strengthened by the fact that the real and imaginary parts of our omnidirectional filter correspond to G_x and G_y filters in the Canny's gradient-based method. By decomposing real and imaginary parts of our filter, we have been able to fit our method in the standard framework of Canny's edge detector.

In section 6 we experimentally compare these approaches on extending the 1-D Demigny edge detection filter. In subsection A we show the superiority of the omnidirectional and the gradient-based approaches over the *n*-directional method on both synthetic and real-world images. But these experiments show no noticeable visual difference between the quality of the

[1] By isotropic we mean that step edges with different directions are detected with the same edge strength.

omnidirectional and the gradient-based extensions. In subsection B, we support this observation by quantitatively showing that the accuracy of the gradient-based and the omnidirectional approaches are almost the same. The main advantage of the omnidirectional approach is that it frees the user from specifying the smoothing window and its parameters. In subsection C, we show that despite the high sensitivity of the smoothing parameter in the gradient-based method, the omnidirectional approach is completely successful in the automatic selection of the appropriate smoothing window. We conclude the paper in section 7.

II. APPROACHES TO 2-D DIRECTIONAL EDGE DETECTION

A. Continuous Gradient Estimation

The continuous gradient estimation method was first proposed by Canny[5] for extending the DoG filter to two dimensions. To detect edges in a given direction, one can align the edge detection filter normal to the edge direction and multiply[2] it by a projection function[3] parallel to the edge direction. This produces a 2-D filter for detecting edges in the given direction. On the other hand, the gradient of a smooth surface can be determined exactly from gradients in two directions, usually x and y directions. For a general 1-D filter, $h(x)$, and a general windowing function, $s(x)$, the gradient in the direction x can be computed by the filter $D_x(x,y)=h(x)s(y)$ and gradient in the direction y can be computed by the filter $D_y(x,y)=h(y)s(x)$. The magnitude and direction of the edge at image point $I(x_0, y_0)$ are then approximated by:

$$\begin{aligned} g &= \left\{[D_x(x,y)*I(x,y)]^2 + [D_y(x,y)*I(x,y)]^2\right\}^{\frac{1}{2}}\Big|_{(x_0,y_0)} \\ \theta &= tg^{-1}\left\{\frac{D_y(x,y)*I(x,y)}{D_x(x,y)*I(x,y)}\right\}\Big|_{(x_0,y_0)} \end{aligned} \tag{1}$$

B. Discrete Gradient Estimation

In discrete gradient-based approach, in a manner analogous to the continuous case, the 2-D filter is obtained by multiplying the smoothing sequence $s[i]\ \ i=-m,...,0,...,m$ with the 1-D discrete filter $d[j]\ \ j=-m,...,0,...,m$. Then $W=sd^T$ and $W^T=ds^T$ are used respectively for estimating the gradient in directions x and y. The magnitude and direction of the edge at the center of a windowed input matrix, A, are:

$$\begin{aligned} \hat{g} &= \left\{trace^2\left[W^T A\right] + trace^2[WA]\right\}^{1/2} \\ \hat{\theta} &= tg^{-1}\left\{\frac{trace[WA]}{trace\left[W^T A\right]}\right\} \end{aligned} \tag{2}$$

[2] Although usually the term "convolve" is used, it is a misnomer and the actual operation is multiplication.

[3] Other names are windowing and smoothing functions.

C. The n-directional filtering

In [7], Paplinski proposed complex-valued filtering for simultaneous edge magnitude and direction detection. Since a complex number, $z = re^{j\theta}$, embeds both a magnitude r and a direction θ in itself, complex valued filtering is a suitable choice for directional edge detection. Assume that we want to detect edges in some direction ϕ, $0 \le \phi \le 2\pi$, and our optimal 1-D filter is $s(r)$. Then the two-dimensional complex-valued filter is specified by[4]:

$$h_\phi\left(z = re^{j\theta}\right) = s(r)p(\theta - \phi) \tag{3}$$

where $p(\theta)$ is a real valued function which attenuates as θ deviates from direction zero. Specifically, $p(\theta) = \exp\left(-c\theta^2\right)$, where c is specified by the user. To detect edges in n pre-specified directions $\phi_1, \phi_2, ..., \phi_n$ simultaneously, the following filter has been suggested:

$$h\left(z = re^{j\theta}\right) = \sum_{k=1}^{n} h_{\phi_k}\left(re^{j\theta}\right)e^{j\phi_k} \tag{4}$$

Paplinski[7] suggested *n*-directional filtering as a generalization of gradient-based methods in which gradients are computed in n, rather than just two, directions. Specifically, he suggested the use of tri-directional filtering. One major drawback of *n*-directional filtering is that it is biased towards the pre-specified directions. We remedy this drawback by introducing omnidirectional filtering. Our method is not biased to any direction and clearly is a generalization of Paplinski's idea with $n = \infty$. In section 4 we show that omnidirectional extension of the 1-D DoG filter is equivalent to Canny's gradient-based method. This shows that in contrast to Paplinski's claim, gradient-based extension of DoG filter is omnidirectional and is not biased towards the two gradient directions. In fact, complex-valued filtering can be viewed as two real-valued filtering and so suffers from the same limitations of gradient-based methods.

III. Omnidirectional Filtering

As noted in the previous section, complex-valued filtering can be used for simultaneous edge magnitude and direction detection. In this section, we tackle the problem of extending an optimal 1-D filter to a complex-valued 2-D one for step edge detection. This 2-D filter, which we denote by $f(r, \theta)$, must satisfy the following conditions:

1. For a step input edge with a jump direction ϕ (which is normal to the edge direction), the phase of the output should be ϕ. In mathematical terms:

$$\int_{\phi-\frac{\pi}{2}}^{\phi+\frac{\pi}{2}} \int_{0}^{\infty} f(r, \theta) dr d\theta = Ae^{j\phi} \tag{5}$$

   for some fixed real A independent of direction ϕ.

2. The filter should not be biased towards any direction. So it must satisfy the following

[4] Subtraction of ϕ from θ which is not specified in [7] is necessary and equation 14 in [7] shows that there has been a typo in [7].

condition (With abuse of notation, $f(.)$ represents three different functions distinguishable from the arguments: $f(r,\theta)$, $f(r)$, $f(\theta)$):

$$f(r,\theta)=\left|f(r,\theta)\right|e^{j\arg(f(r,\theta))}=\left|f(r)\right|e^{j\arg(f(r,\theta))} \quad (6)$$

Furthermore, assuming that $\arg(f(r,\theta))$ is only a function of θ and defining $f(\theta)\equiv e^{j\arg(f(r,\theta))}$ we obtain the following conditions:

$$\begin{aligned} f(r,\theta)&=\left|f(r,\theta)\right|e^{j\arg(f(r,\theta))}=f(r)f(\theta) \\ f(r)&\geq 0 \\ \left|f(\theta)\right|&=1 \end{aligned} \quad (7)$$

3. $f(\theta)$ is periodic with period 2π.
4. $f(\theta)$ is continuous in $(0,2\pi)$. This condition guarantees that

$$F(\theta)=\int_0^{\theta} f(\phi)d\phi \quad (8)$$

is differentiable and so has a Fourier series that converges to $F(\theta)$ in $(0,2\pi)$ (see theorem 8.1 of [11]). In fact, the continuity of $F(\theta)$ is sufficient for its Fourier series to converge to $F(\theta)$; but differentiability is needed in (17).

Now we show that the above requirements uniquely, up to a scale factor, determine $f(r,\theta)$. We have:

$$\begin{aligned} \int_{\phi-\frac{\pi}{2}}^{\phi+\frac{\pi}{2}}\int_0^{\infty} f(r,\theta)drd\theta &= \int_{\phi-\frac{\pi}{2}}^{\phi+\frac{\pi}{2}}\int_0^{\infty} f(r)f(\theta)drd\theta \\ &= \int_{\phi-\frac{\pi}{2}}^{\phi+\frac{\pi}{2}} f(\theta)d\theta\int_0^{\infty} f(r)dr = Ae^{j\phi} \end{aligned} \quad (9)$$

Defining

$$B=\frac{A}{\int_0^{\infty} f(r)dr} \quad (10)$$

we have:

$$\int_{\phi-\frac{\pi}{2}}^{\phi+\frac{\pi}{2}} f(\theta)d\theta = Be^{j\phi} \Rightarrow F\left(\phi+\frac{\pi}{2}\right)-F\left(\phi-\frac{\pi}{2}\right)=Be^{j\phi} \quad (11)$$

Since $f(\theta)$ is continuous in $(0,2\pi)$, $F(\theta)$ is also continuous in $(0,2\pi)$, but may be discontinuous at 0 or 2π. We show that this is not the case and $F(\theta)$ is continuous everywhere.

Proposition 1. $F(0)=F(2\pi)$.

Proof. We show that $F(2\pi)-F(0)=0$.

$$\begin{aligned} &F(2\pi)-F(0)=\int_0^{2\pi} f(\theta)d\theta=\int_0^{\pi} f(\theta)d\theta+\int_{\pi}^{2\pi} f(\theta)d\theta \\ &=e^{j\frac{\pi}{2}}+e^{j\frac{3\pi}{2}}=0 \end{aligned} \tag{12}$$

This means that Fourier series of $F(\theta)$ converges at all points to its value and $F(\theta)$ can be represented as a Fourier series:

$$F(\theta)=\sum_{n=-\infty}^{\infty} X_n e^{jn\theta} \tag{13}$$

Now from (11):

$$\begin{aligned} &F\left(\phi+\frac{\pi}{2}\right)-F\left(\phi-\frac{\pi}{2}\right)=Be^{j\phi} \\ &\Rightarrow \sum_{n=-\infty}^{\infty} X_n e^{jn\left(\phi+\frac{\pi}{2}\right)}-\sum_{n=-\infty}^{\infty} X_n e^{jn\left(\phi-\frac{\pi}{2}\right)}=Be^{j\phi} \\ &\Rightarrow \sum_{n=-\infty}^{\infty} X_n e^{jn\phi}\left(e^{j\frac{n\pi}{2}}-e^{-j\frac{n\pi}{2}}\right)=\sum_{\substack{n=-\infty \\ n=2k+1}}^{\infty} X_n e^{jn\phi}\left(e^{j\left(k\pi+\frac{\pi}{2}\right)}-e^{-j\left(k\pi+\frac{\pi}{2}\right)}\right) \\ &=\sum_{\substack{n=-\infty \\ n=2k+1}}^{\infty} 2j(-1)^k X_n e^{jn\phi}=Be^{j\phi} \end{aligned} \tag{14}$$

From uniqueness of Fourier series expansion, it follows that:

$$X_n=\begin{cases} B/(2j) & n=1 \\ 0 & n\neq 0 \end{cases} \tag{15}$$

So $F(\theta)$ is given by:

$$F(\theta)=\frac{B}{2j}e^{j\theta} \tag{16}$$

Taking derivation with respect to θ yields:

$$f(\theta)=\frac{B}{2}e^{j\theta} \tag{17}$$

Since $|f(\theta)|=1$, so $f(\theta)=e^{j\theta}$ and the omnidirectional filter is given by:

$$f(r,\theta)=f(r)e^{j\theta} \tag{18}$$

So we have proved the following theorem.

Theorem 1. The only omnidirectional 2-D complex filter satisfying the above four conditions is $f(r,\theta)=f(r)e^{j\theta}$, where $f(r)$ is the given optimal 1-D filter.

Let us now prove another nice feature of the omnidirectional filtering which is absent in the *n*-directional filtering[5].

Proposition 2. $imag\{f(x,y)\} = real\{f(y,x)\}$.

Proof.

$$imag\{f(x,y)\} = f(r)\sin\theta = f(r)\frac{y}{r} = real\{f(y,x)\} \tag{19}$$

□

Corollary. The real and imaginary parts of any discrete realization of an omnidirectional filter are transposes of each other.

Later, we will show that the real and the imaginary parts of the omnidirectional filter act as gradient filters in directions x and y. So, Proposition 3 and its corollary are counterparts of a property in the gradient-based methods in which the horizontal and the vertical filters are transposes of each other (see [10]).

IV. Equivalence Between omnidirectional and Gradient-based Extensions of DoG

Although the gradient-based filtering and complex-valued filtering are obtained through two completely different views, both yield the same 2D extended filter for the DoG 1-D filter. Before proving this claim, it is necessary to review the Canny's edge detector in more depth. Canny initially showed that the derivative of a 2-D Gaussian in some direction is a good choice for detecting edges in that direction. For directions x and y we have:

$$\begin{gathered}\nabla_{\begin{bmatrix}1\\0\end{bmatrix}} G(x,y) = \begin{bmatrix} -\dfrac{x}{\sigma^2} e^{-\frac{r^2}{2\sigma^2}} \\ 0 \end{bmatrix} \\ \nabla_{\begin{bmatrix}0\\1\end{bmatrix}} G(x,y) = \begin{bmatrix} 0 \\ -\dfrac{y}{\sigma^2} e^{-\frac{r^2}{2\sigma^2}} \end{bmatrix}\end{gathered} \tag{20}$$

where $r^2 = x^2 + y^2$. So, $-x\exp\left(-r^2/2\sigma^2\right)/\sigma^2$ detects edges in the x direction and $-y\exp\left(-r^2/2\sigma^2\right)/\sigma^2$ detects edges in the y direction. The outputs of these filters can be used to compute the edge magnitude and direction as specified in section 2.1. The problem with this approach is that it does not benefit from the separability property of the Gaussian. Canny showed that one can first convolve the image with a Gaussian, which can be implemented very fast, and then use two simple gradient operators on the output. In this section, we consider Canny's gradient-based 2D operators without exploiting the separability property of the Gaussian. We will show in the next section that a similar fast implementation is possible for omnidirectional filtering, whenever a fast implementation exists for the Canny's gradient-based method. Now the following proposition shows that the Canny's edge detection filter is equivalent to the

[5] See equation 14 in [7].

omnidirectional extension of the DoG filter.

Proposition 4. The real and imaginary parts of the omnidirectional extension of the DoG are equal to $-x\exp\left(-r^2/2\sigma^2\right)/\sigma^2$ and $-y\exp\left(-r^2/2\sigma^2\right)/\sigma^2$, respectively. So the estimated edge magnitude and direction in the omnidirectional and the Canny edge detectors are the same.

Proof. Since 1-D filter is DoG,

$$f(r) = -\frac{r}{2\sigma^2} e^{-\frac{r^2}{2\sigma^2}} \tag{21}$$

So

$$f(r,\theta) = f(r)e^{j\theta} = f(r)\cos\theta + jf(r)\sin\theta \tag{22}$$

Rewriting $f(r,\theta)$ in the *x*-*y* Cartesian coordinate we have:

$$f(x,y) = -\frac{x}{2\sigma^2} e^{-\frac{r^2}{2\sigma^2}} - j\frac{y}{2\sigma^2} e^{-\frac{r^2}{2\sigma^2}} \tag{23}$$

V. Relation Between omnidirectional and Gradient-based Extensions

In the previous section we showed that the omnidirectional and gradient-based extensions of 1-D DoG are equivalent. In this section we generalize this observation and reveal more links between the two approaches.

Proposition 5. If $f(t)$ is a 1-D step edge detection filter with the following properties[6]:

1. $f(t) = -f(-t)$
2. $t.f(t) < 0$
3. $f(\infty) = 0$

then $F(t) = \int_{-\infty}^{t} f(t)dt$ is a windowing (smoothing) function with the following properties:

1. $F(t) = F(-t)$
2. $F(t) > 0$
3. $F(0) \geq F(t) \quad \forall t$
4. $F(\infty) = 0$

Proposition 6. The real and imaginary parts of the omnidirectional extension of the 1-D filter $f(t)$ are equal to gradients of the 2-D windowing function

[6] These properties are the natural constraints on step edge detection filters as specified by [5] *and* [6].

$$F(r,\theta)=\int_{-\infty}^{r} f(r)dr \tag{24}$$

in the x and y directions.

Proof.

$$\begin{aligned}\frac{\partial F(r,\theta)}{\partial x}&=\frac{\partial F(r,\theta)}{\partial r}\frac{\partial r}{\partial x}+\frac{\partial F(r,\theta)}{\partial \theta}\frac{\partial \theta}{\partial x}\\ &= f(r)\frac{x}{r}+0= f(r)\cos\theta = real\{f(r)e^{j\theta}\}\end{aligned} \tag{25}$$

Similarly:

$$\begin{aligned}\frac{\partial F(r,\theta)}{\partial y}&=\frac{\partial F(r,\theta)}{\partial r}\frac{\partial r}{\partial y}+\frac{\partial F(r,\theta)}{\partial \theta}\frac{\partial \theta}{\partial y}\\ &= f(r)\frac{y}{r}+0= f(r)\sin\theta = imag\{f(r)e^{j\theta}\}\end{aligned} \tag{26}$$

One may think that the above proposition indicates that omnidirectional extension is a special case of the gradient-based extension. But, as we prove below, gradient-based extension can represent $\partial F(r,\theta)/\partial x$ and $\partial F(r,\theta)/\partial y$ only if $F(r)$ is separable, i.e.: $F(r,\theta)=F(r)=F(x)F(y)$.

Proposition 7. The omnidirectional extension is equivalent to the gradient-based extension by windowing function $F(t)$ if and only if $F(r,\theta)=\int_{-\infty}^{r} f(r)dr$ is separable.

Proof. We first show that if $F(r,\theta)$ is separable then both extensions are equivalent. To obtain the x-direction gradient filter, we should multiply $f(x)$ by $F(y)$, so:

$$G_x(x,y)= f(x)F(y) \tag{27}$$

Similarly, the y -direction gradient filter is

$$G_y(x,y)= f(y)F(x) \tag{28}$$

By expanding the terms and using (25) and (26) we get:

$$\begin{aligned}G_x(x,y)&= f(x)F(y)=\frac{dF(x)}{dx}F(y)\\ &=\frac{\partial}{\partial x}[F(x)F(y)]=\frac{\partial}{\partial x}F(r)=real\{f(r)e^{j\phi}\}\end{aligned} \tag{29}$$

and

$$\begin{aligned}G_y(x,y)&= f(y)F(x)=\frac{dF(y)}{dy}F(x)\\ &=\frac{\partial}{\partial y}[F(x)F(y)]=\frac{\partial}{\partial y}F(r)=imag\{f(r)e^{j\phi}\}\end{aligned} \tag{30}$$

So, the "if" part is proved. To prove the "only if" part, assume that $\partial F(r,\theta)/\partial x=G_x(x,y)$ but $F(r)$ is not separable. Since $G_x(x,y)$ is a gradient-based filter it can be expressed as the product of some filtering function, say $f(x)$, and another windowing function, say $F(y)$. So

$$\frac{\partial F(r,\theta)}{\partial x}= f(x)F(y) \tag{31}$$

Then, a simple integration with respect to x shows that $F(r)$ is separable.

VI. Experimental Results

In [6] Demigny argued that Canny's optimal filter, which is very similar to DoG, is only optimal in the continuous space and proposed $H_{\Sigma\Lambda}$ as a 1-D discrete optimal linear filter. In this section, we compare three extensions of the Demigny filter: the gradient-based, the *n*-directional, and the omnidirectional extensions. Let us first mention some details of implementation of these approaches.

1. In all experiments the low and high thresholds for hysteresis thresholding are set to 0.4 and 0.7, respectively.
2. In the gradient-based extension of a Demigny operator with parameter W (and so width 2W+1), we use a Gaussian projection function with the standard deviation $W/2$. This choice has experimentally yielded the best results for gradient-based approach as can be verified by experiments of section C.
3. There is also an implementation trick in the directional extensions. Since the Demigny filter is discrete, but the parameter r in our approach (and in *n*-directional filtering) is continuous, the filter values for non-integer radii are obtained by linear interpolation.

The rationale behind our experiments is as follows. Visual experiments in section A illustrate that *n*-directional approach is consistently the worst approach and so we can exclude it from further experiments. At this point, we have shown superiority of gradient-based and omnidirectional approaches over *n*-directional approach. But, since our visual comparison identifies no significant difference between gradient-based and omnidirectional approaches, we conduct experiments of section B to evaluate these approaches on the task of shape from motion as proposed by [12]. Again, in these experiments the gradient-based and omnidirectional extensions produced similar results. We could continue the comparison on other benchmarks (e.g. [13], [14]), but we believe that the previous experiments are sufficient to convince one that the performances of the gradient-based and the omnidirectional methods are almost the same. But, in the above experiments the smoothing parameter of the gradient-based approach has been tuned experimentally[7]. The benefit of omnidirectional over gradient-based approach is that it frees user from choosing the smoothing function and its parameter. Experiments of section C show that even though adjusting the parameter of the smoothing window is a delicate task in the gradient-based method, the omnidirectional approach performs this task automatically and with the best quality. In addition, these experiments verify the use of a Gaussian projection function with the standard deviation $W/2$ in the gradient-based extension of a Demigny operator with parameter W (and so width $2W+1$) which is used in the experiments.

A. Visual Comparison: Ruling out the n-directional Approach

In this section, we compare the gradient-based, the *n*-directional, and the omnidirectional extensions of 1-D Demigny filter on both synthetic and real-world images. Since direction plays

[7] Note that even though, the smoothing parameter has been chosen independent of images, it has been tuned for Demigny filter.

a central role in 2-D extension of filters, we have selected Fig. 1 as our synthetic reference image. Fig. 2, the cameraman, is also selected as the real-world reference image. Figures 3, 5, and 7, respectively, show the results of the application of the *n*-directional, gradient-based, and omnidirectional extensions of the Demigny filter to Fig. 1. The parameter W of the Demigny filter is set to 4, resulting in a window width of 9. In Fig. 3, we see that tri-directional filtering is biased to its three pre-specified directions and fails to detect several spokes. As Fig. 5 shows, gradient-based extension also has erroneously not detected an oblique spoke. Fig. 7 shows that omnidirectional extension has been successful in detecting all spokes.

Figures 4, 6, and 8, respectively, show the result of the application of the *n*-directional, gradient-based, and omnidirectional extensions of the Demigny filter to Fig. 2. The parameters of this experiment are identical to that of the previous experiment on Fig. 1. Comparison of Fig. 4. with figures 6 and 8 shows the apparent inferiority of the *n*-directional filtering method. As figures 6 and 8 show, comparison of the quality of the gradient-based and the omnidirectional extensions is visually hard. The remaining experiments try to uncover any benefits or drawbacks that one of these methods may have.

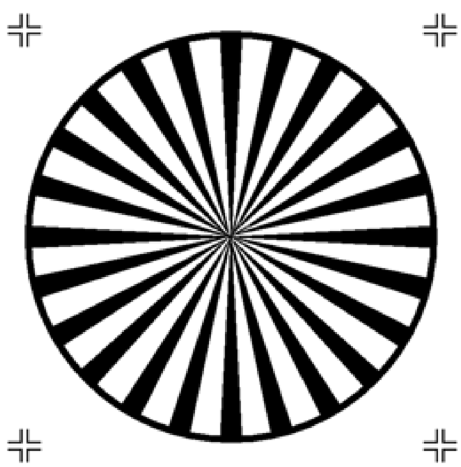


Fig. 1. The synthetic test pattern image (256 ×256).

Fig. 2. The real-world test pattern image (256 ×256).

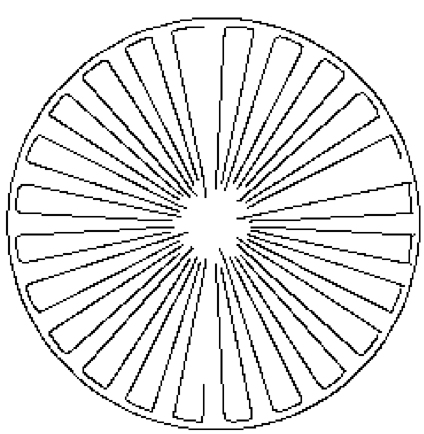


Fig. 3. Result of applying the tri-directional extension of Demigny filter with W=4 to Fig. 1. An attenuation factor of

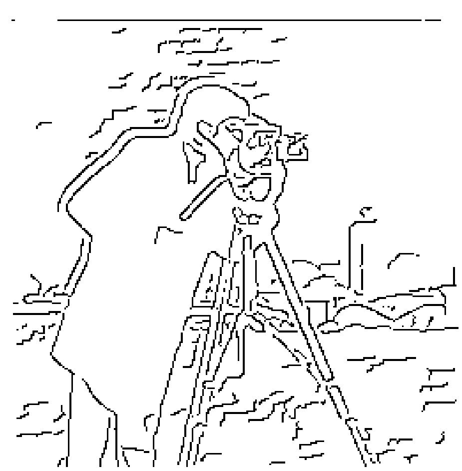


Fig. 4. Result of applying the tri-directional extension of Demigny filter with W=4 to Fig. 2. An attenuation factor of $c = 0.6321$ is

$c = 0.6321$ is obtained by setting $\delta = 0.5$.

obtained by setting $\delta = 0.5$.

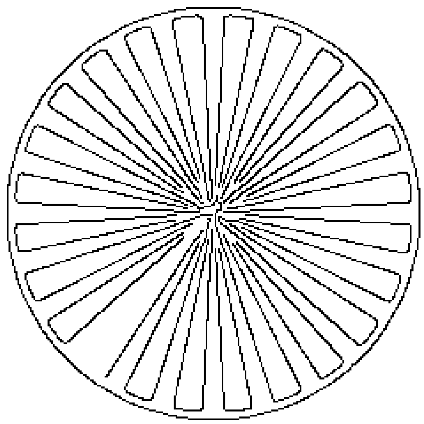


Fig. 5. Result of applying the gradient-based extension of Demigny filter with W=4 to Fig. 1 using a Gaussian projection function with $\sigma = 2$.

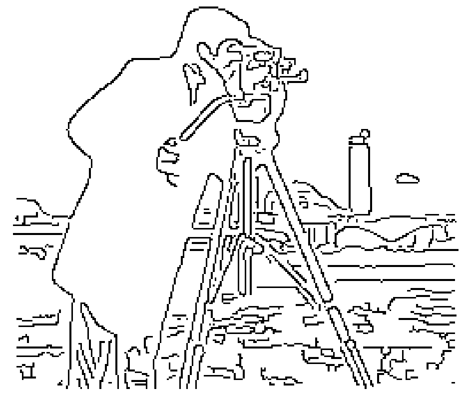


Fig. 6. Result of applying the gradient-based extension of Demigny filter with W=4 to Fig. 2 using a Gaussian projection function with $\sigma = 2$.

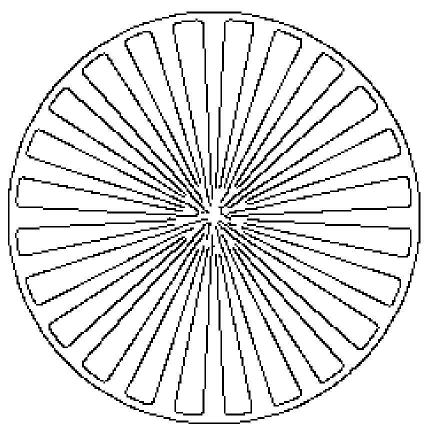


Fig. 7. Result of applying the omnidirectional extension of Demigny filter with W=4 to Fig. 1.

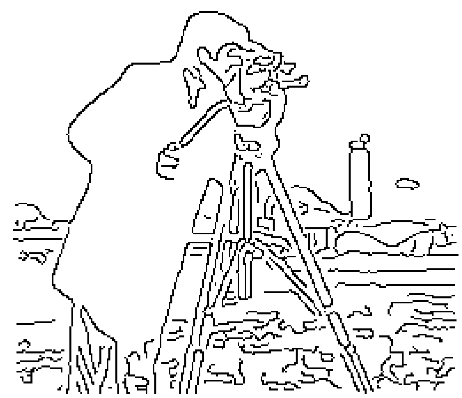


Fig. 8. Result of applying the omnidirectional extension of Demigny filter with W=4 to Fig. 2.

B. Comparison on the Structure from Motion Task

Since the tri-directional method is severely biased to its pre-specified directions, we exclude it from further experiments. In this section, we quantitatively compare our method and the gradient-based approach on the task of shape from motion as proposed by [12]. In [12] Shin et al. introduce a framework for quantitative evaluation of edge detectors on the task of reconstructing shape from motion. Eighteen sequences of images are provided, 9 from WoodBlock scene and 9 from LegoHouse scene. Accuracy of edge detectors is reported in terms of two metrics: "structure error", measured by angle difference in degrees, and "motion error", measured by distance difference in percent. Table 1 shows structure and motion error rates on WoodBlock and LegoHouse sequences for gradient-based extension of Demigny filter with parameters $W = 1,2,3,4$. Table 2 shows the same information for omnidirectional extension of Demigny filter. Both methods have obtained similar results and no method worked better than the other.

This may be due to nature of the SFM framework of [12] in which "false positive" edges do not degrade the metrics.

TABLE 1
STRUCTURE AND MOTION ERROR RATES ON SFM TASK FOR GRADIENT-BASED EXTENSION OF DEMIGNY FILTER. *W* IS THE PARAMETER OF DEMIGNY FILTER AND WINDOW WIDTH IS 2W+1

W	WB Structure	WB Motion	LH Structure	LH Motion
3	2.46	**4.24**	**3.90**	7.87
5	**1.35**	**4.71**	**2.93**	6.09
7	2.44	**4.27**	3.83	**7.65**
9	2.88	5.05	**2.63**	7.66
best	**1.35**	**4.24**	**2.63**	6.09

TABLE 2
STRUCTURE AND MOTION ERROR RATES ON SFM TASK FOR OUR EXTENSION OF DEMIGNY FILTER. *W* IS THE PARAMETER OF DEMIGNY FILTER AND WINDOW WIDTH IS 2W+1

W	WB Structure	WB Motion	LH Structure	LH Motion
3	**2.17**	4.61	5.06	**7.76**
5	2.63	4.72	3.31	**5.61**
7	**2.08**	4.49	**3.37**	8.29
9	**1.92**	**4.69**	3.12	**6.96**
best	1.92	4.49	3.12	**5.61**

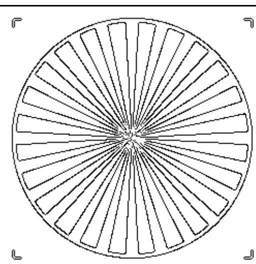
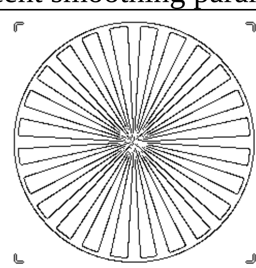
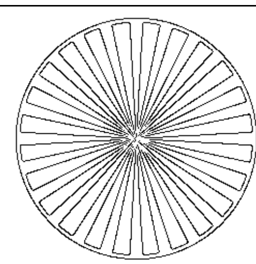
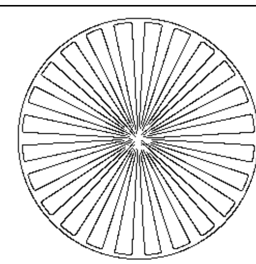
C. Automatic selection of the smoothing window

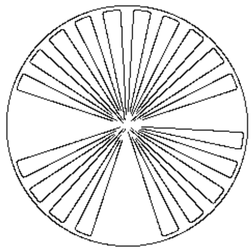
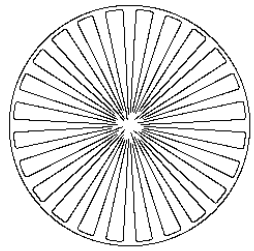
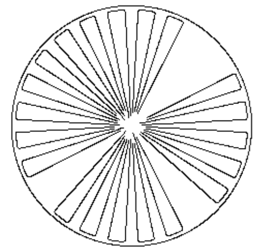
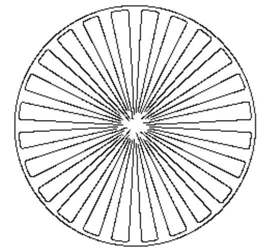
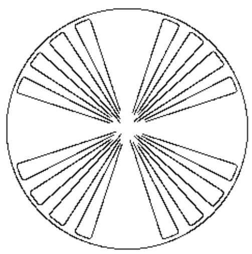
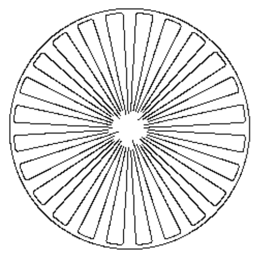
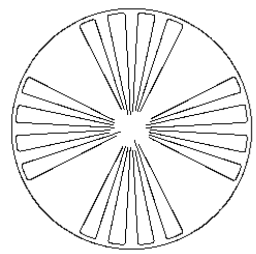
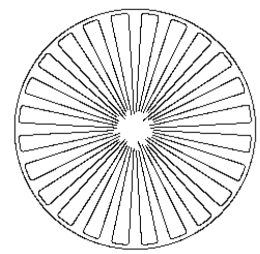
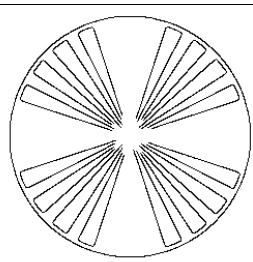
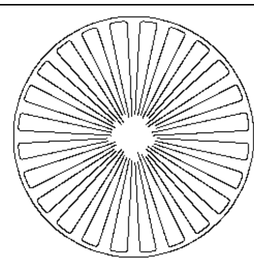
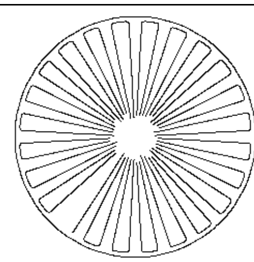
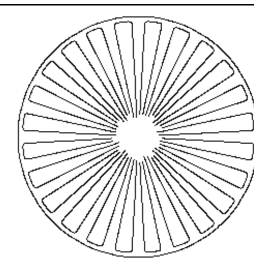
We believe that from a practical perspective, the main advantage of the omnidirectional method over the gradient-based method is the automatic selection of the smoothing window. When Canny proposed gradient-based 2-D extension of his operator, he utilized the fact that the edge detector and smoothing window were both Gaussian. Clearly, this is not the case for other edge detectors, including Demigny's, and the smoothing parameter must be chosen by experiment. In general, the smoothing window is not limited to the Gaussian form and other choices such as Hamming and Hanning windows are also possible [5].

Table 3 summarizes our first experiment. In this experiment we first applied gradient-based extensions of the Demigny filter with $W = 1,2,3,4$ and smoothing parameters 0.1, 0.3, 0.5, 0.7, 1, 1.5, 2, 2.5, and 3 to Fig. 1. Then we visually chose the best smoothing parameter for each value of W. The third column of Table 3 shows the best results obtained from the gradient-based method. We then performed the same experiment with omnidirectional extension of Demigny filter whose result is reported in the last column of Table 3. To get a feel of the sensitivity of the edge detector to the smoothing parameter, we have also reported the result for adjacent (to the best) values of the smoothing parameter in the second and forth columns of Table 3. This experiment reveals two facts. First, the quality of edge detection is very sensitive to the choice of the smoothing parameter and sometimes small deviation from the optimum choice can lead to drastic changes. Second, omnidirectional method has been successful in automatic selection of the smoothing parameter.

The same experiment was performed on the real-world image of Fig. 2 and the results are illustrated in figures 9 to 12. We only report the results for $W=2$ which leads to a 5×5 window. This is an appropriate choice for the 256×256 image of Fig. 2. Again the omnidirectional method has been successful in the automatic selection of the smoothing parameter.

Table 3: Comparison between gradient-based and omnidirectional approaches on choosing the smoothing window for edge detection on Fig. 1. Even though the choice of the smoothing parameter is a sensitive task, as the experiments on the gradient-based method reveal, the omnidirectional method has been successful in automatically making the appropriate choice.

	The best result of the Gradient-based method and results of the adjacent smoothing parameters			Omnidirectional Method
W=1				

σ	0.3	0.5	0.7	
W=2				
σ	0.7	1	1.5	
W=3				
σ	1	1.5	2	
W=4				
σ	1	1.5	2	

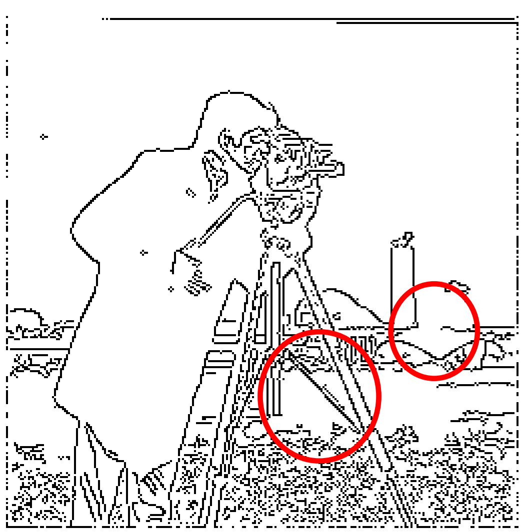


Fig. 9. Result of applying the gradient-based extension of Demigny's filter with *W=2* and $\sigma = 0.3$ to Fig. 2.

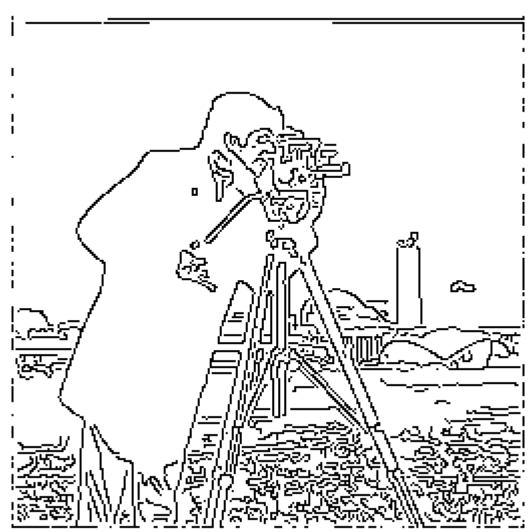


Fig. 10. Result of applying the gradient-based extension of Demigny's filter with *W=2* and $\sigma = 1$ to Fig. 2.

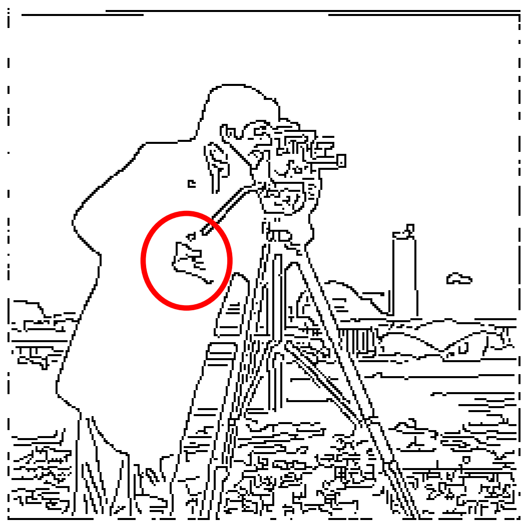


Fig. 11. Result of applying the gradient-based extension of Demigny's filter with *W=2* and $\sigma = 2$ to Fig. 2.

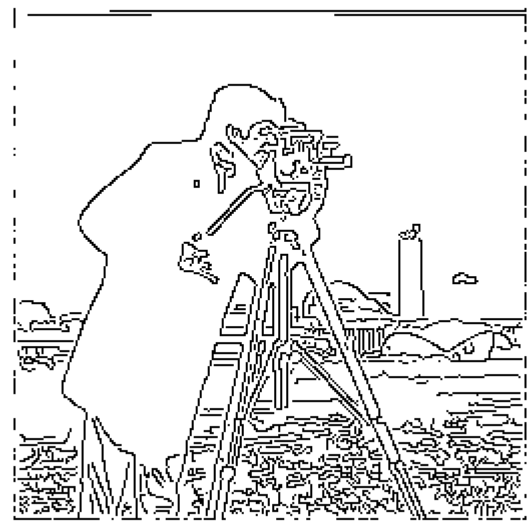


Fig. 12. Result of applying the omnidirectional extension of Demigny's filter with *W=2* to Fig. 2.

VII. CONCLUSIONS

In this paper, we studied the problem of extending 1-D edge detection filters to two dimensions and proposed an isotropic solution. Canny's gradient-based method and Paplinski's *n*-directional filtering have been the previous solutions to this problem. Canny proposed gradient-based method for extending 1-D DoG filter to two dimensions based on a firm mathematical foundation. But his extension of 1-D DoG filter exploits specific properties of the Gaussian form. Paplinski formulated the problem of directional edge detection in complex arithmetic and proposed another extension. But his *n*-directional filter is biased towards the pre-specified directions. Our omnidirectional approach is obtained as a unique solution that is not biased towards any direction. The uniqueness of the solution implies that every other solution to the problem, including Canny's gradient-based and Paplinski's *n*-directional filtering, is biased towards specific directions. So, we believe that omnidirectional filtering proposed in this paper is the true method of extending 1-D edge detection filters to two dimensions. For the specific case of 1-D DoG edge detection filter, it is proved that our omnidirectional and Canny's gradient-based methods coincide.

Experiments on synthetic and real-world images revealed the fact that omnidirectional and Canny's gradient-based methods have significantly higher quality in edge detection than Paplinski's *n*-directional approach. The main advantage of the omnidirectional extension over the gradient-based method is that the former implicitly chooses an appropriate smoothing window, while the latter requires the user to specify this parameter which, as our experiments show, is a sensitive task.

In summary, our omnidirectional method for extending 1-D edge detection filters has improved edge detection methods in two ways. First, it is a unique unbiased (to direction) method for extending 1-D edge detection filters to two dimensions. Second, it frees the user from specifying the hard-to-tune parameter of the smoothing window without sacrificing the accuracy of the edge detector. Finally, it must be mentioned that although the mathematical formulation starts with complex arithmetic, the implementation is as simple as the gradient-based method with no appearance of imaginary numbers. It is even possible to reduce the two-dimensional filtering to several one dimensional counterparts, whenever such a speedup is possible in the gradient-based method.

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