LINEAR CONTROL SYSTEMS

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Lecture 12

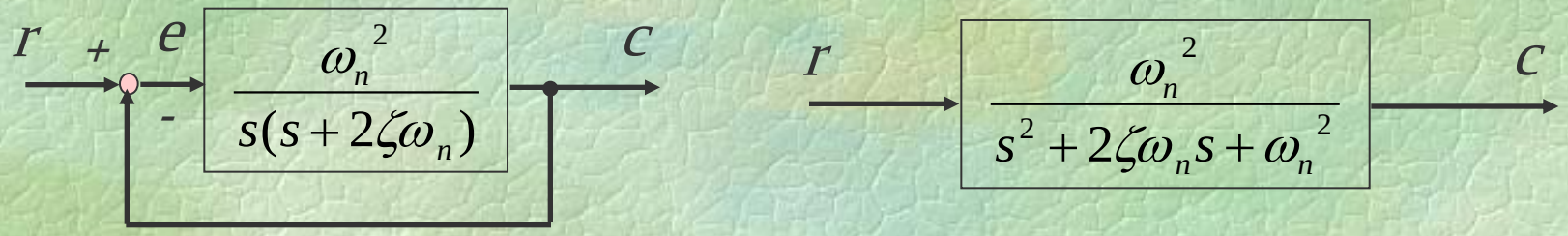
Time domain analysis of control systems

Topics to be covered include:

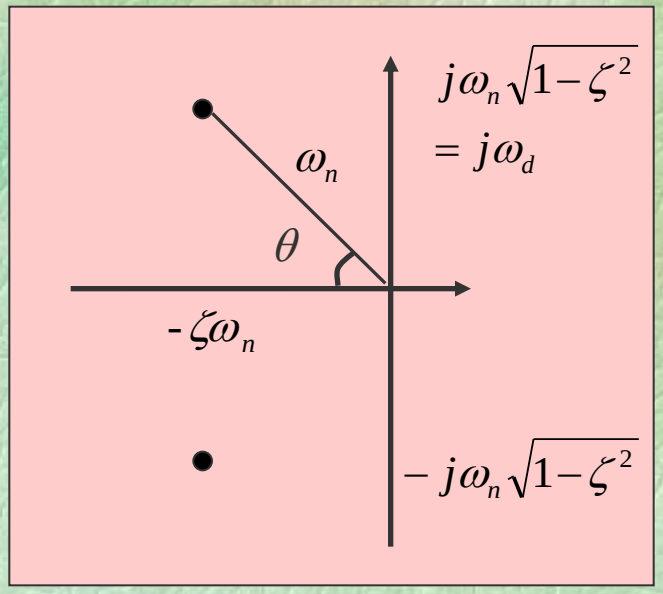
- Different region of S plane.
  - Constant natural frequency loci.
  - Constant natural damped frequency loci.
  - Constant damping factor loci.
  - Constant damping ratio loci.
- Transient response of a position control system.

Introducing a prototype second order system.

معرفی یک سیستم نمونه درجه ۲



Poles are : $-\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2} = -\alpha \pm j\omega_d$ if $0 \le \zeta \le 1$



ω_n Natural frequency — فرکانس طبیعی

$\alpha = \zeta\omega_n$ Damping factor — ضریب میرائی

ζ Damping ratio — نسبت میرائی

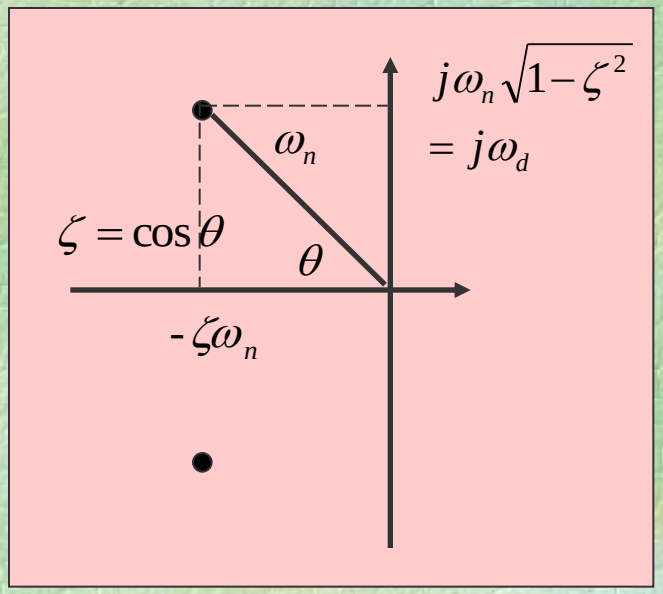
$\omega_d = \omega_n\sqrt{1-\zeta^2}$ Natural damped frequency

فرکانس طبیعی میرا شده

Different region of S plane.

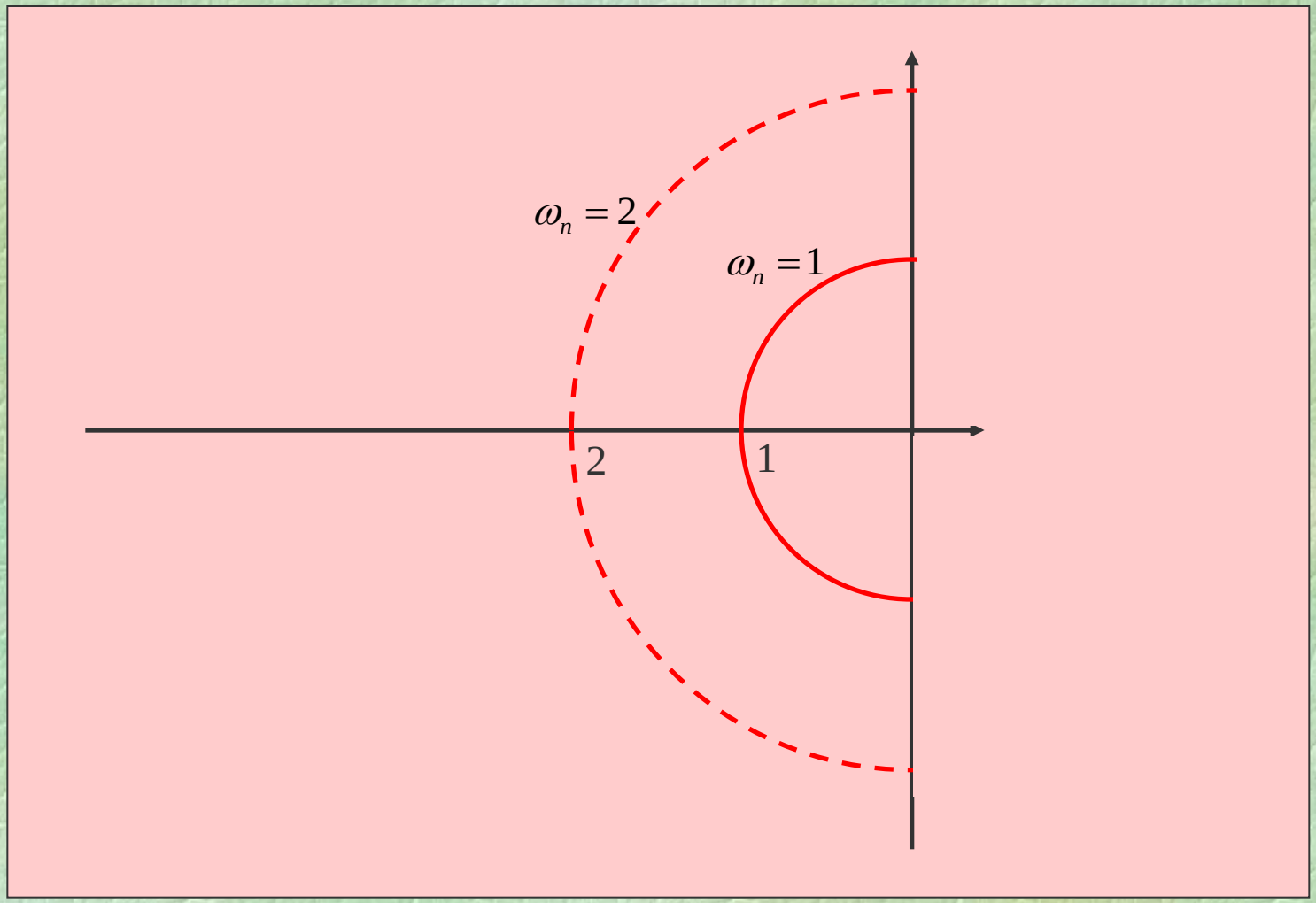
نواحی مختلف در صفحه S

- Constant natural frequency loci.
- Constant natural damped frequency loci.
- Constant damping factor loci.
- Constant damping ratio loci.



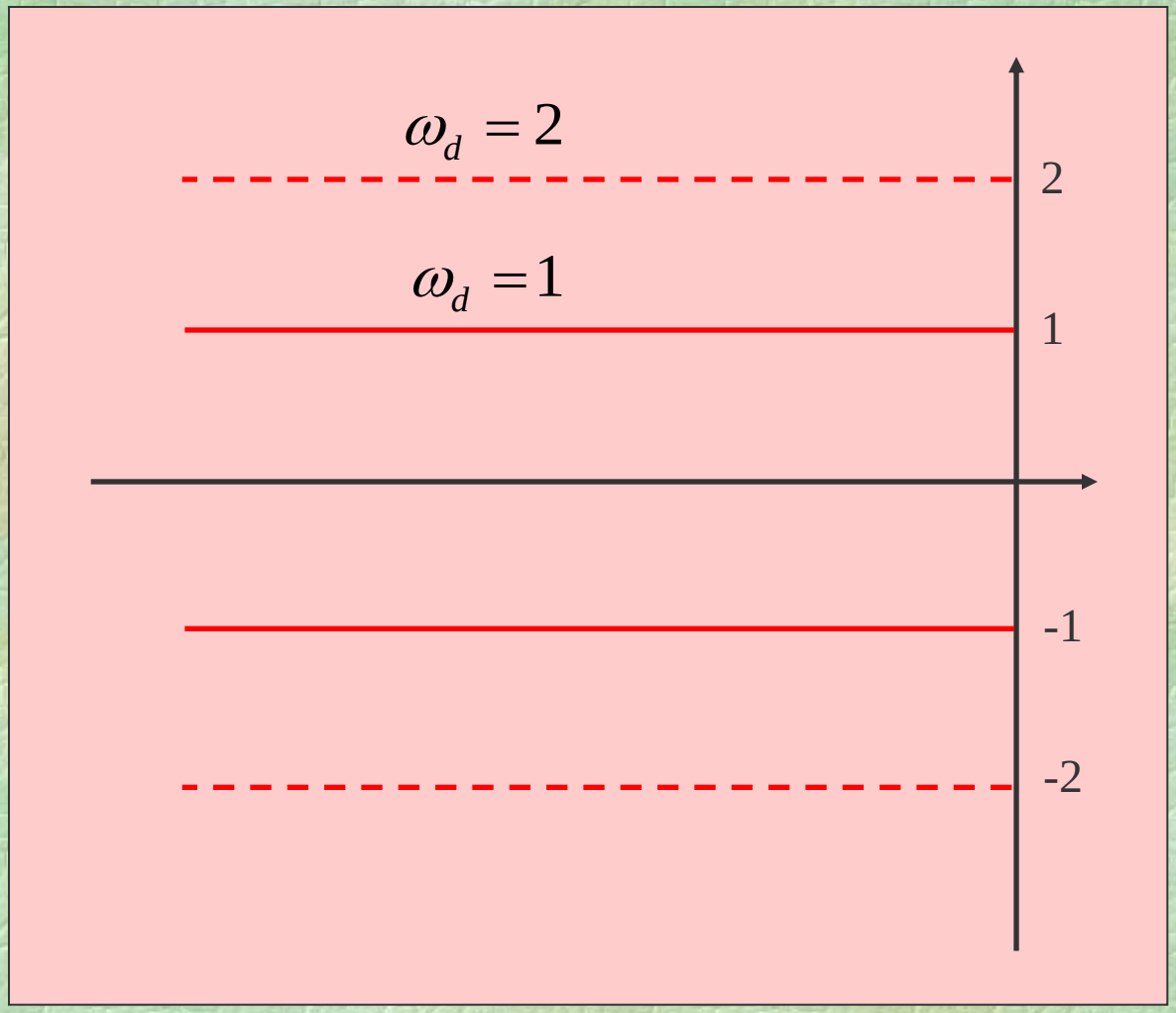
Constant natural frequency loci.

مکان فرکانس طبیعی ثابت



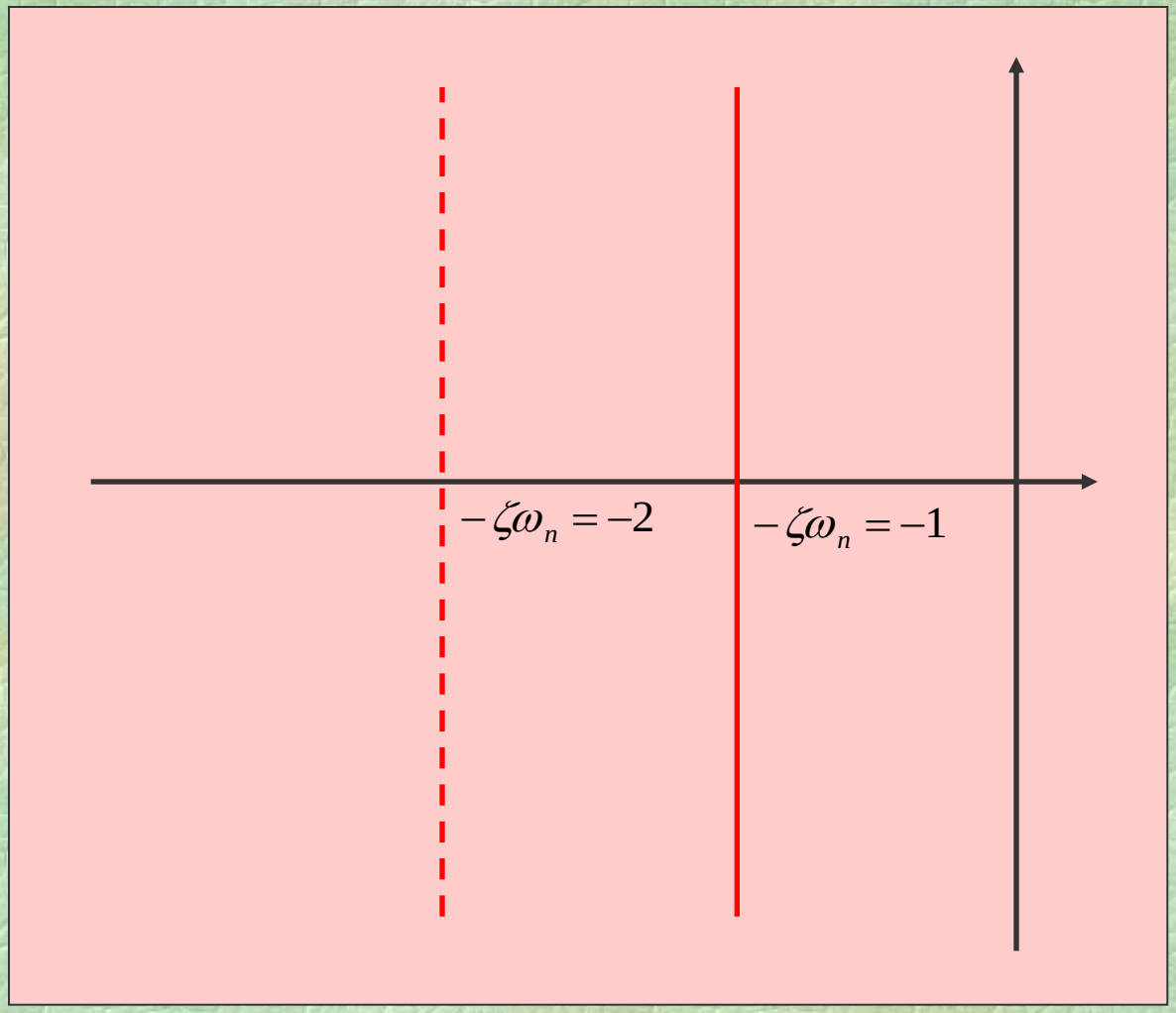
Constant natural damped frequency loci.

مکان فرکانس طبیعی میرا شده ثابت



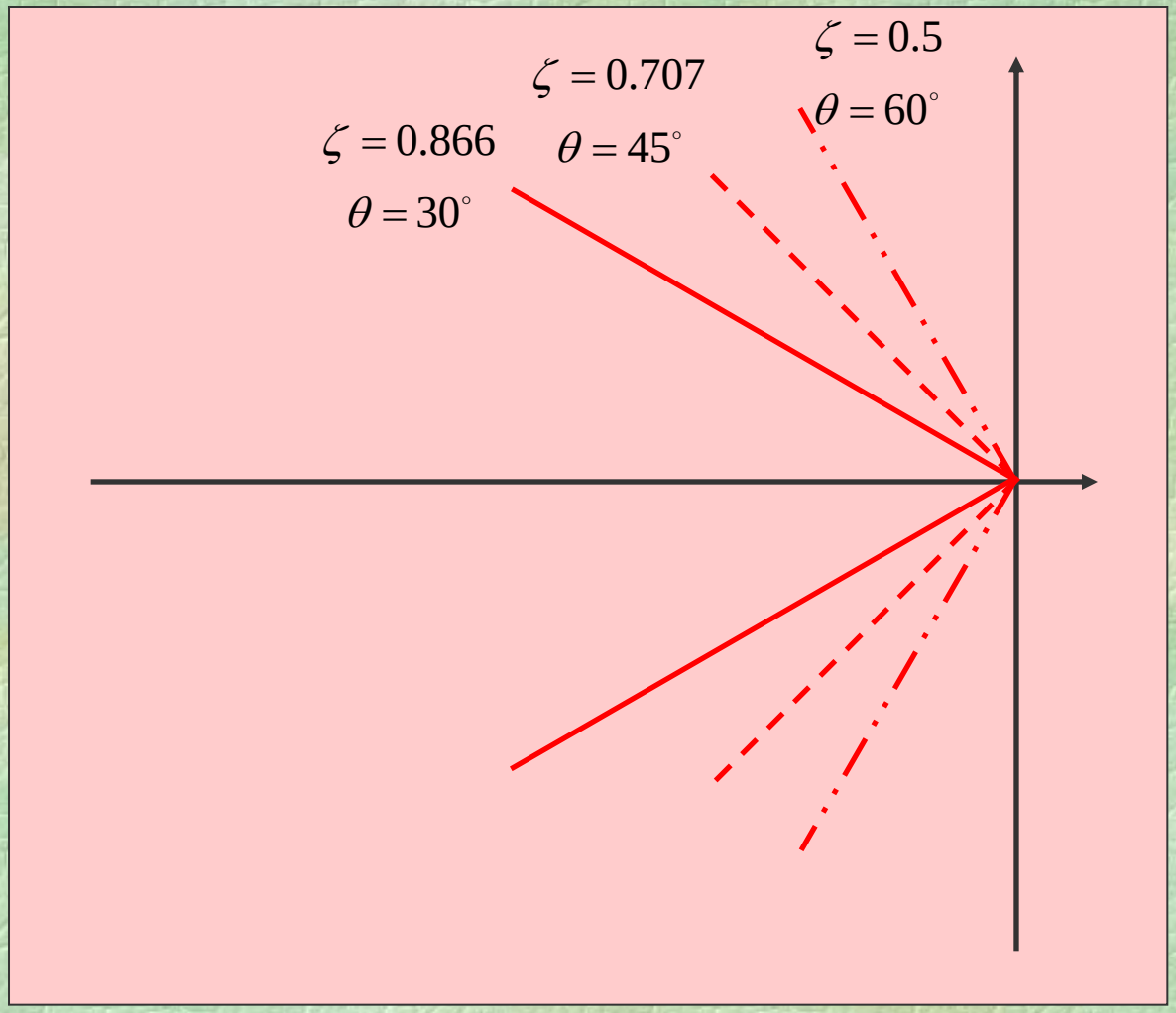
Constant damping factor loci.

مکان ضریب میرائی ثابت

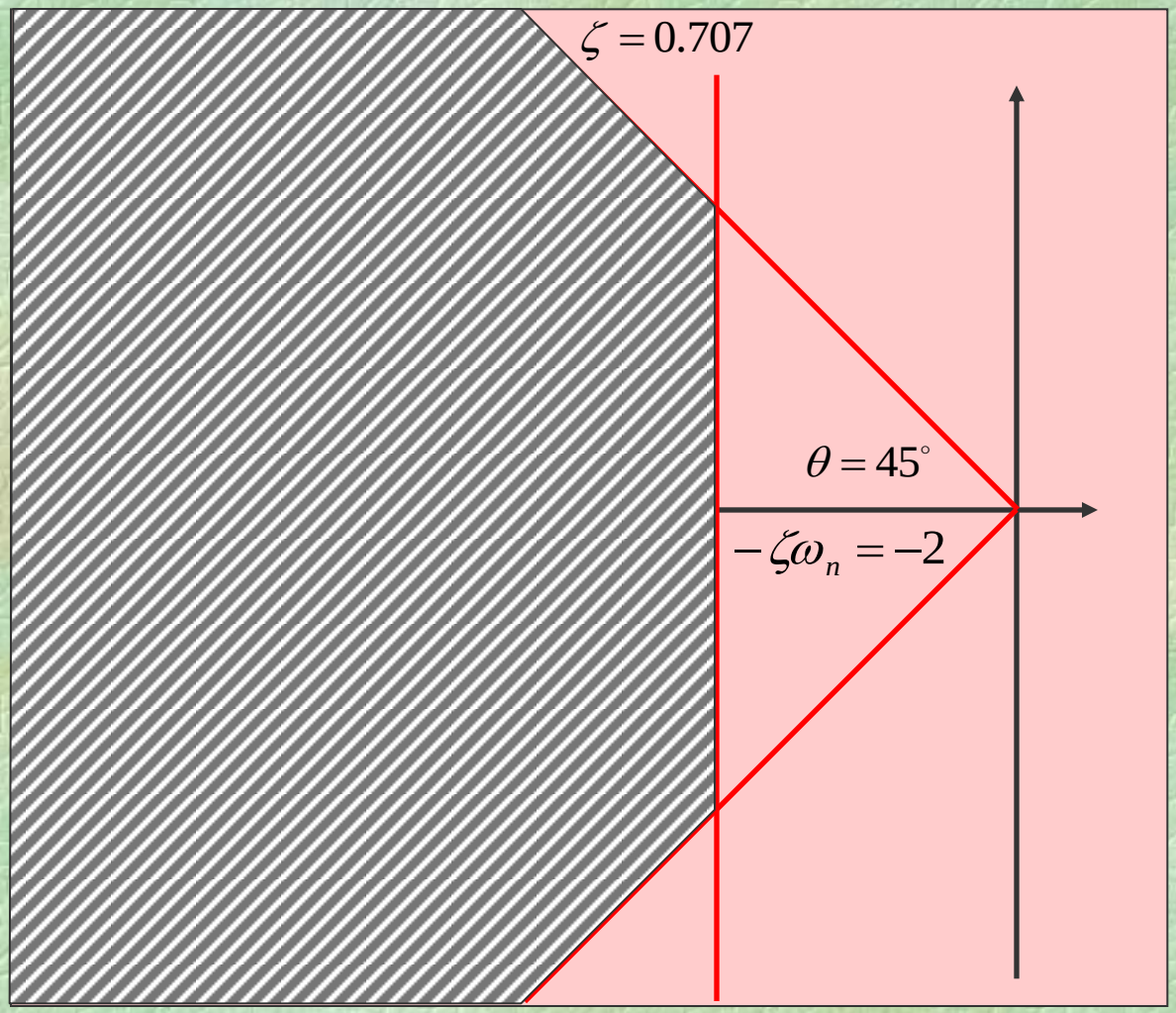


Constant damping ratio loci.

مکان نسبت میرائی ثابت



Example 1: Find the loci of a system where damping ratio is greater then 0.707 and damping factor greater than 2



Example 2: Find the loci that the percent overshoot is less than 16% and settling time is less than 1 sec (according to 5% bound).

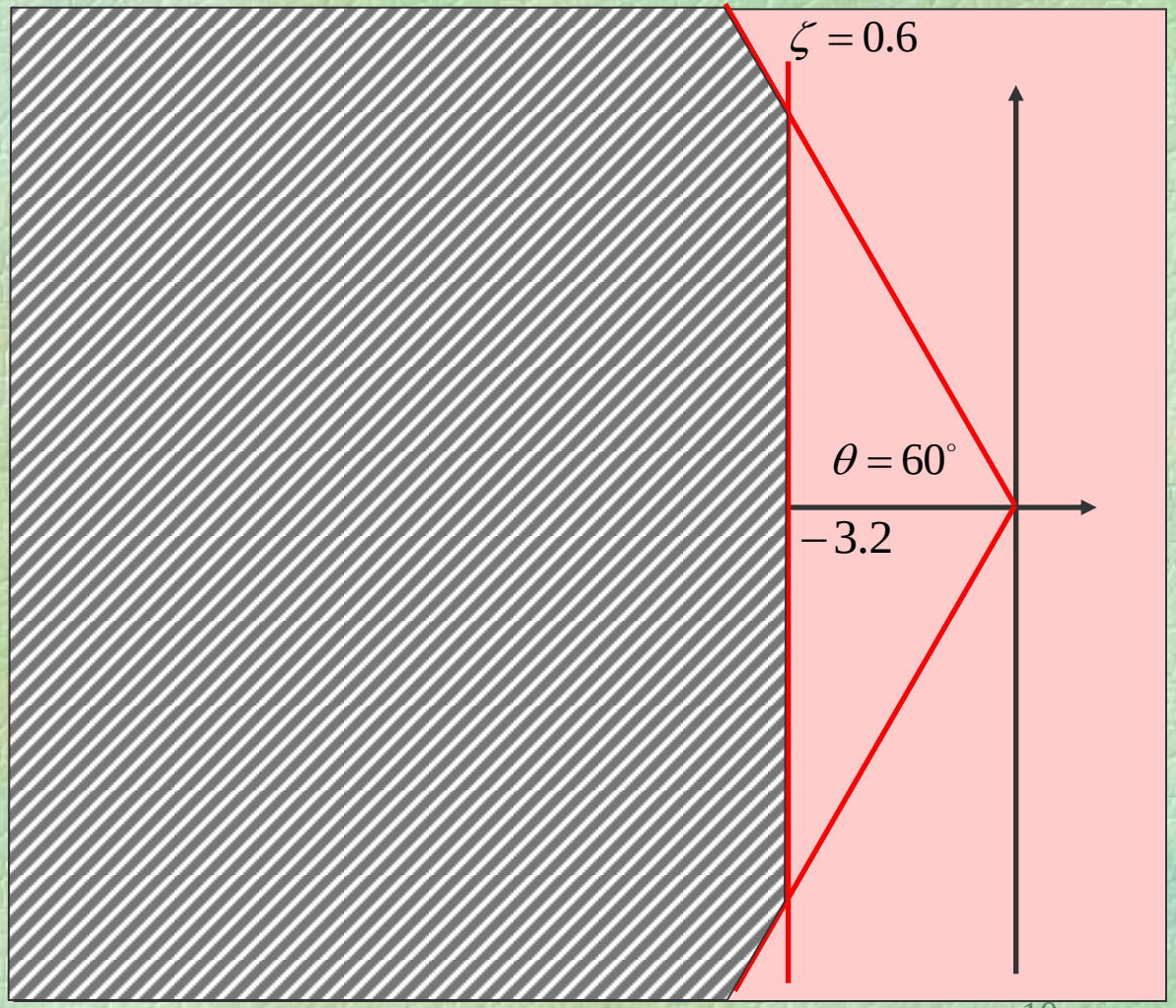
$$P.O. \le 16\% \Rightarrow \zeta \ge 0.5$$

$$\zeta \ge 0.5 \quad \Rightarrow \quad \theta \le 60^\circ$$

$$t_s = \frac{3.2}{\zeta \omega_n}$$

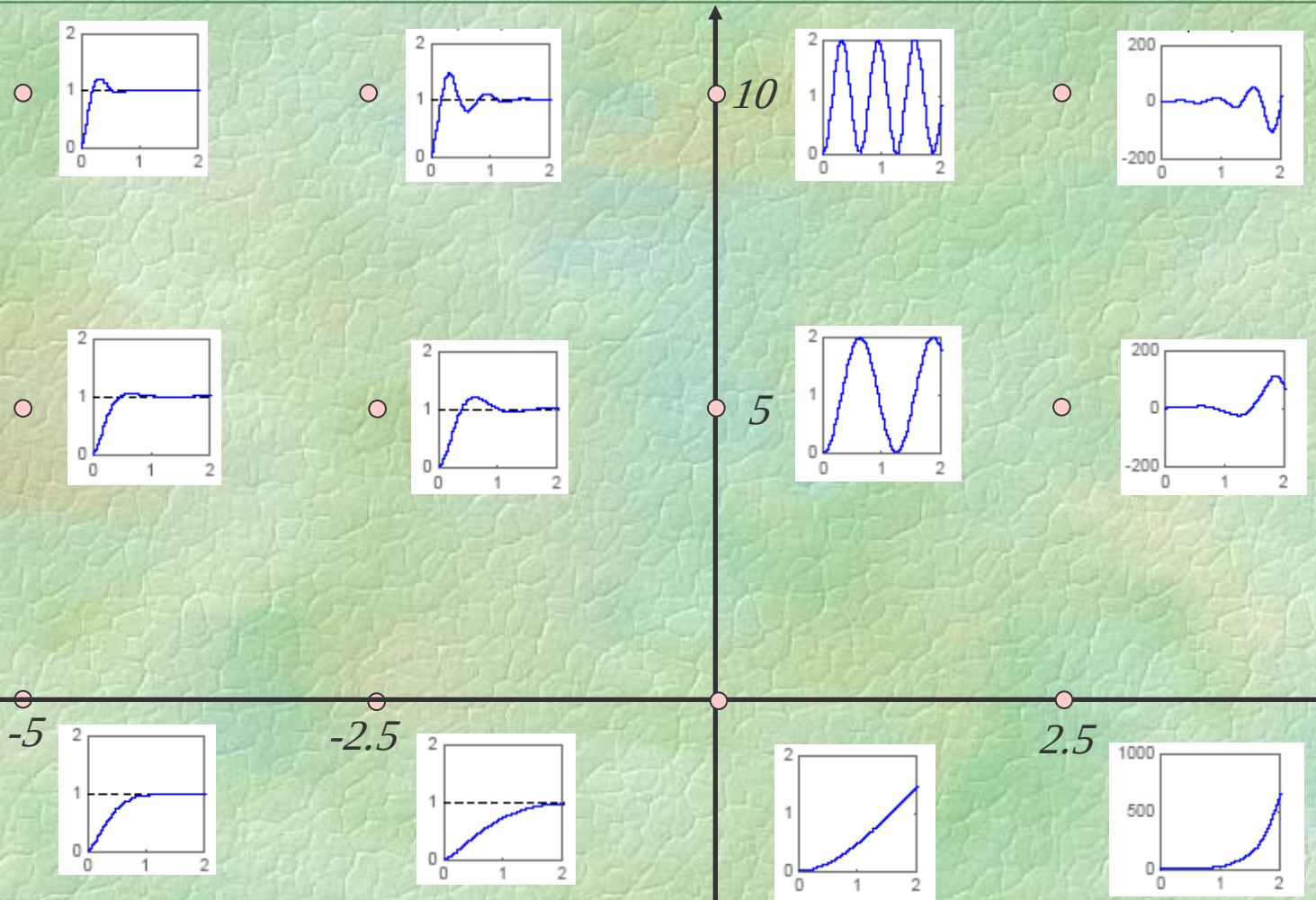
$$t_s \le 1 \quad \Rightarrow \quad \zeta \omega_n > 3.2$$

$$-\zeta \omega_n < -3.2$$



Effect of roots loci on step response

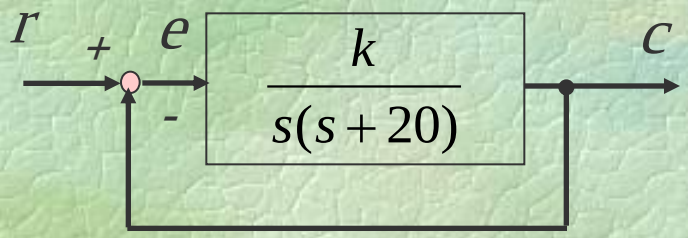
تاثیر مکان ریشه ها بر پاسخ پله



Conjugate root is not shown

ریشه مزدوج نشان داده نشده است

Example 3: Find the poles of the following systems and its corresponding step response for k=75, 100, 200 and 1000



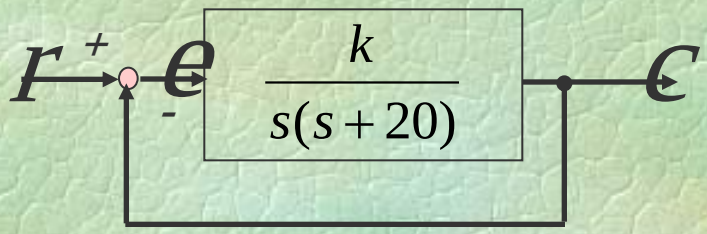
$$M(s)=\frac{c(s)}{r(s)}=\frac{k}{s^2+20s+k}$$

$$k=75 \quad \Rightarrow \quad p_1=-5,\ p_2=-15$$

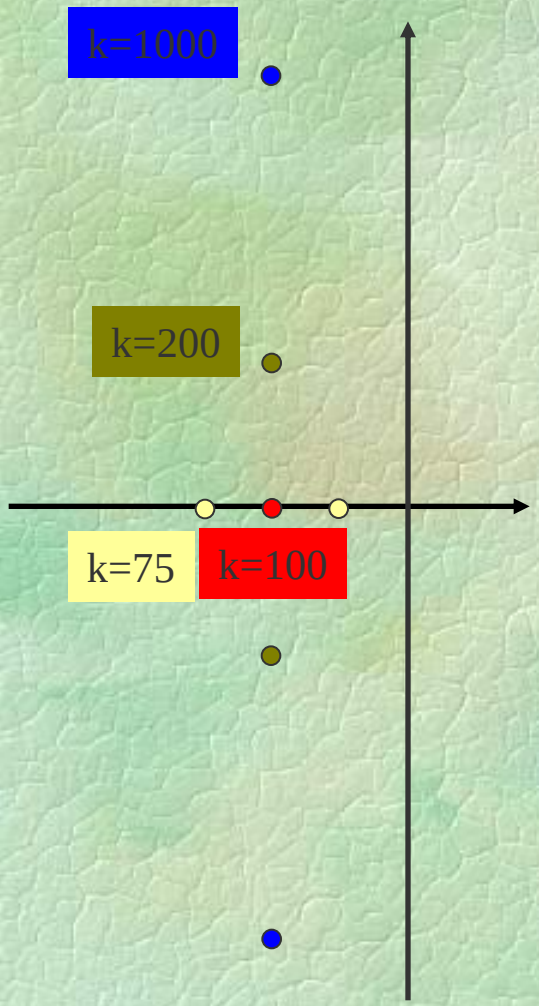
$$k=100 \quad \Rightarrow \quad p_1=-10,\ p_2=-10$$

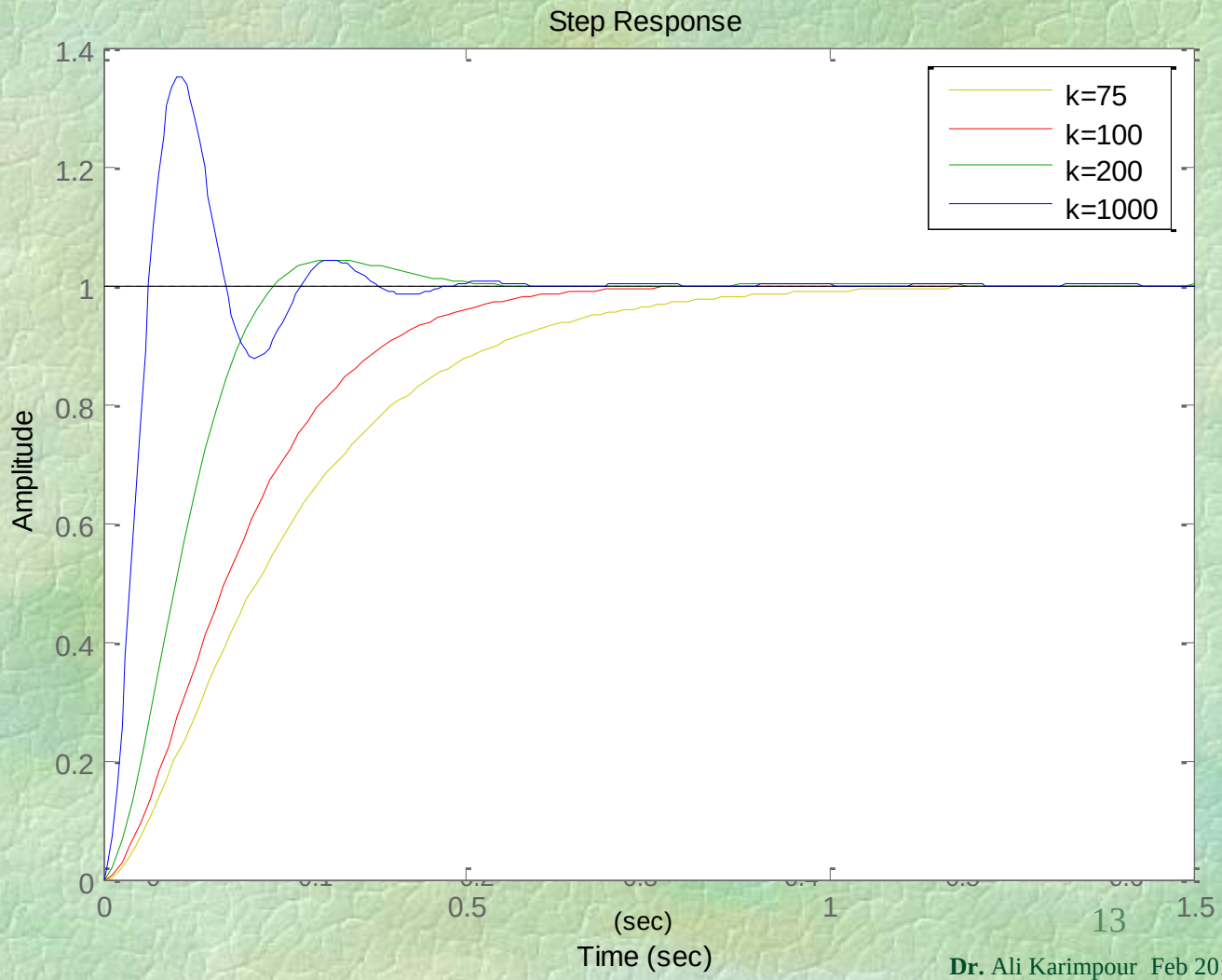
$$k=200 \quad \Rightarrow \quad p_1=-10+10j,\ p_2=-10-10j$$

$$k=1000 \quad \Rightarrow \quad p_1=-10+30j,\ p_2=-10-30j$$



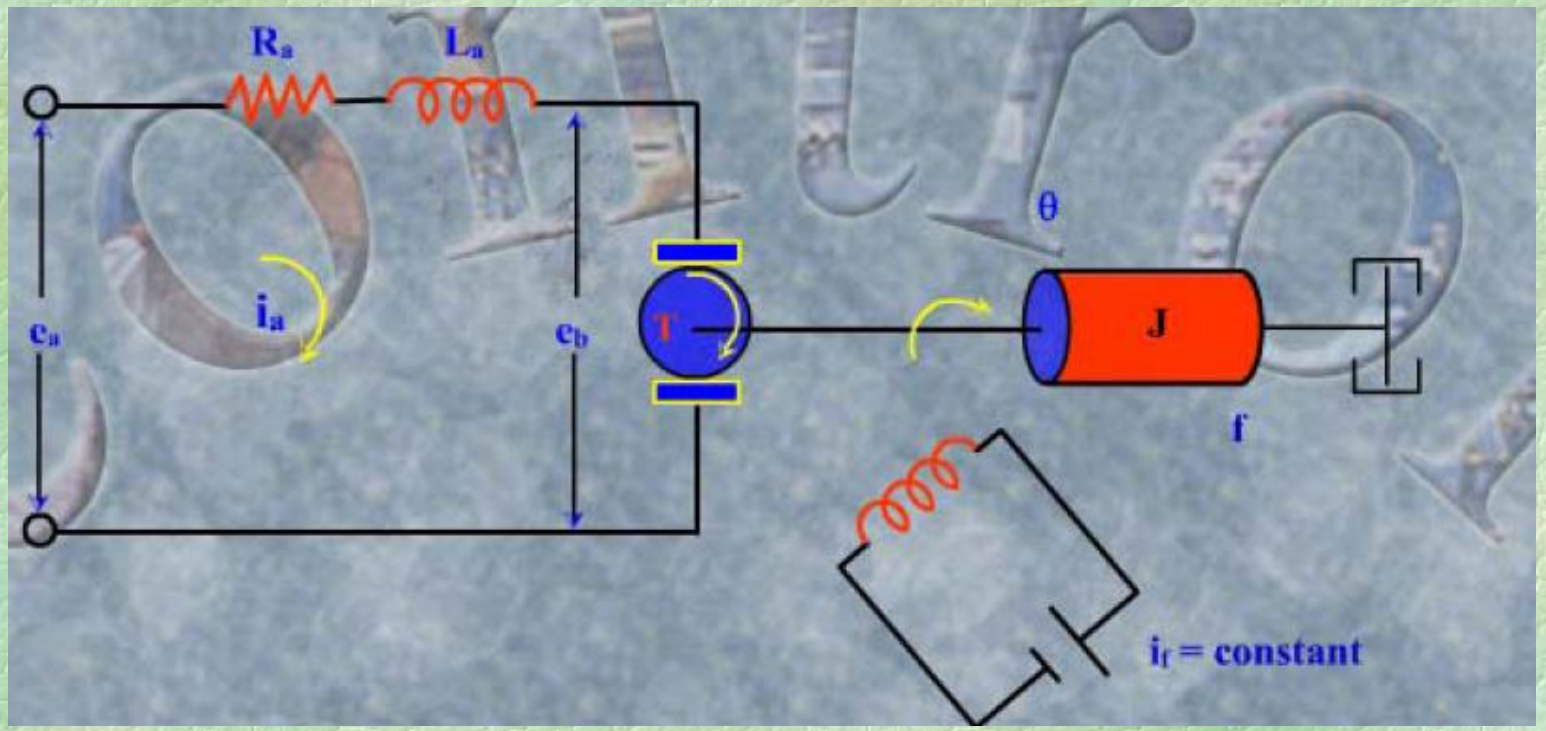
$$M(s) = \frac{c(s)}{r(s)} = \frac{k}{s^2 + 20s + k}$$





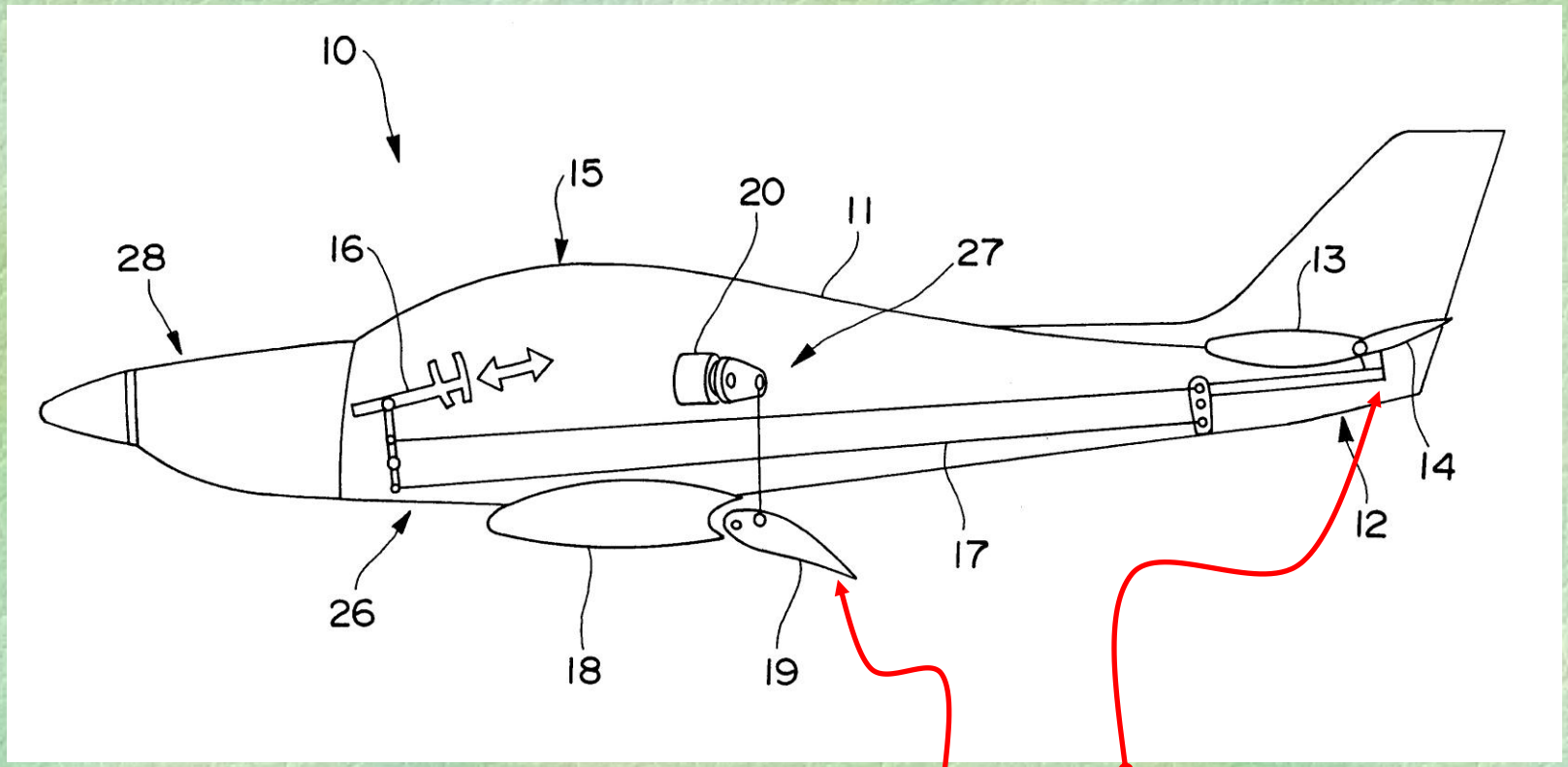
Dynamics of electromechanical system

دینامیک سیستمهای الکترومکانیکی



A simplified aeroplane (position control system)

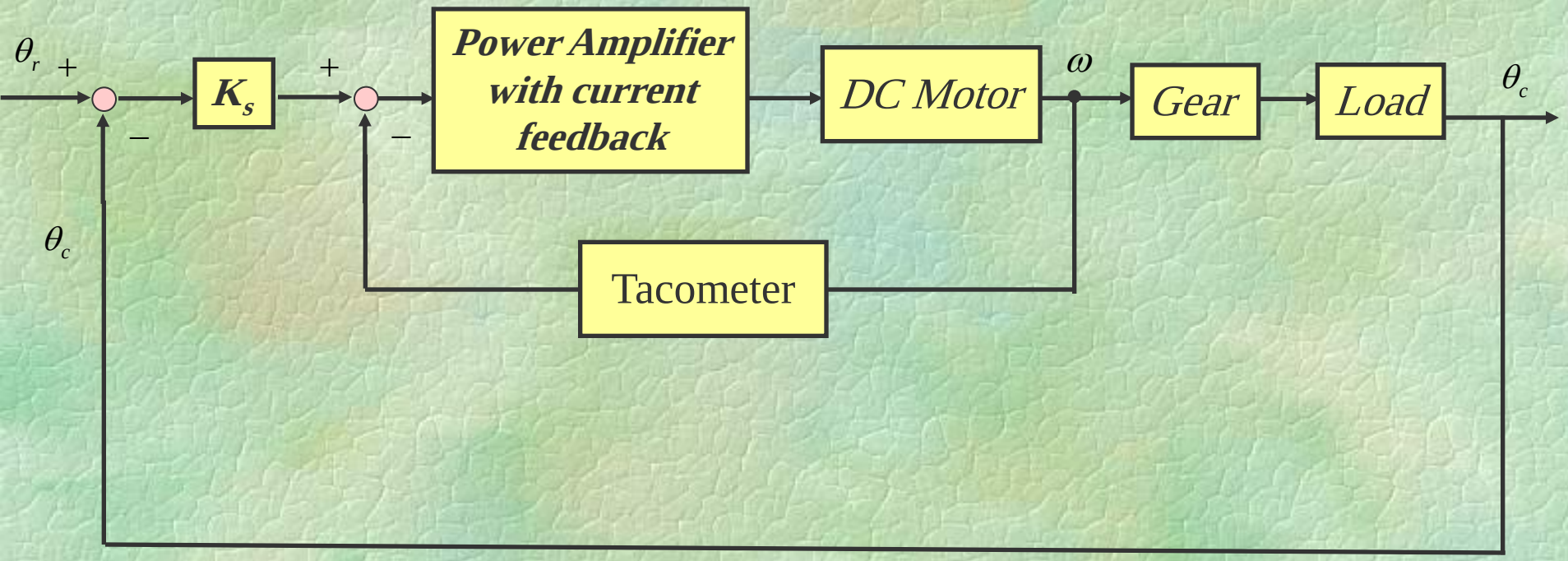
یک هواپیمای ساده شده (سیستم کنترل موقعیت)



Position control

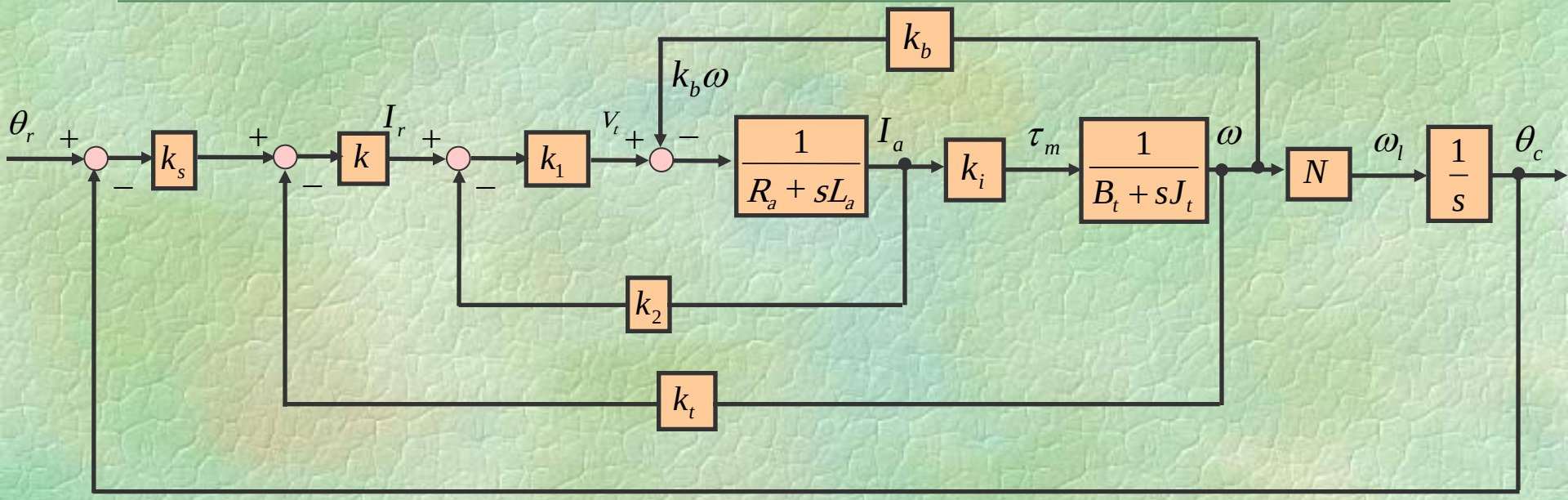
Block diagram of a position control system

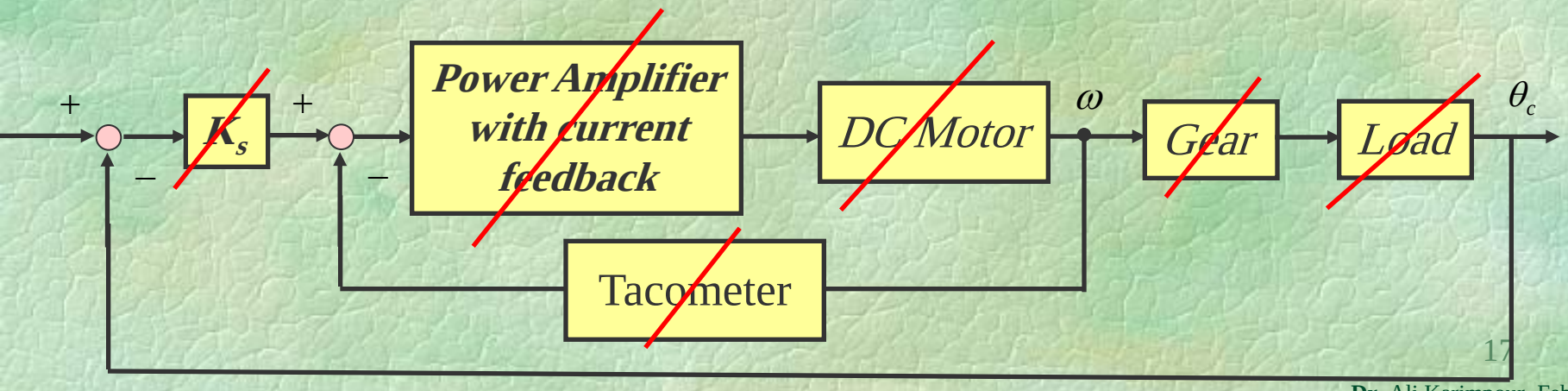
بلوک دیاگرام یک سیستم کنترل موقعیت



Block diagram of a position control system

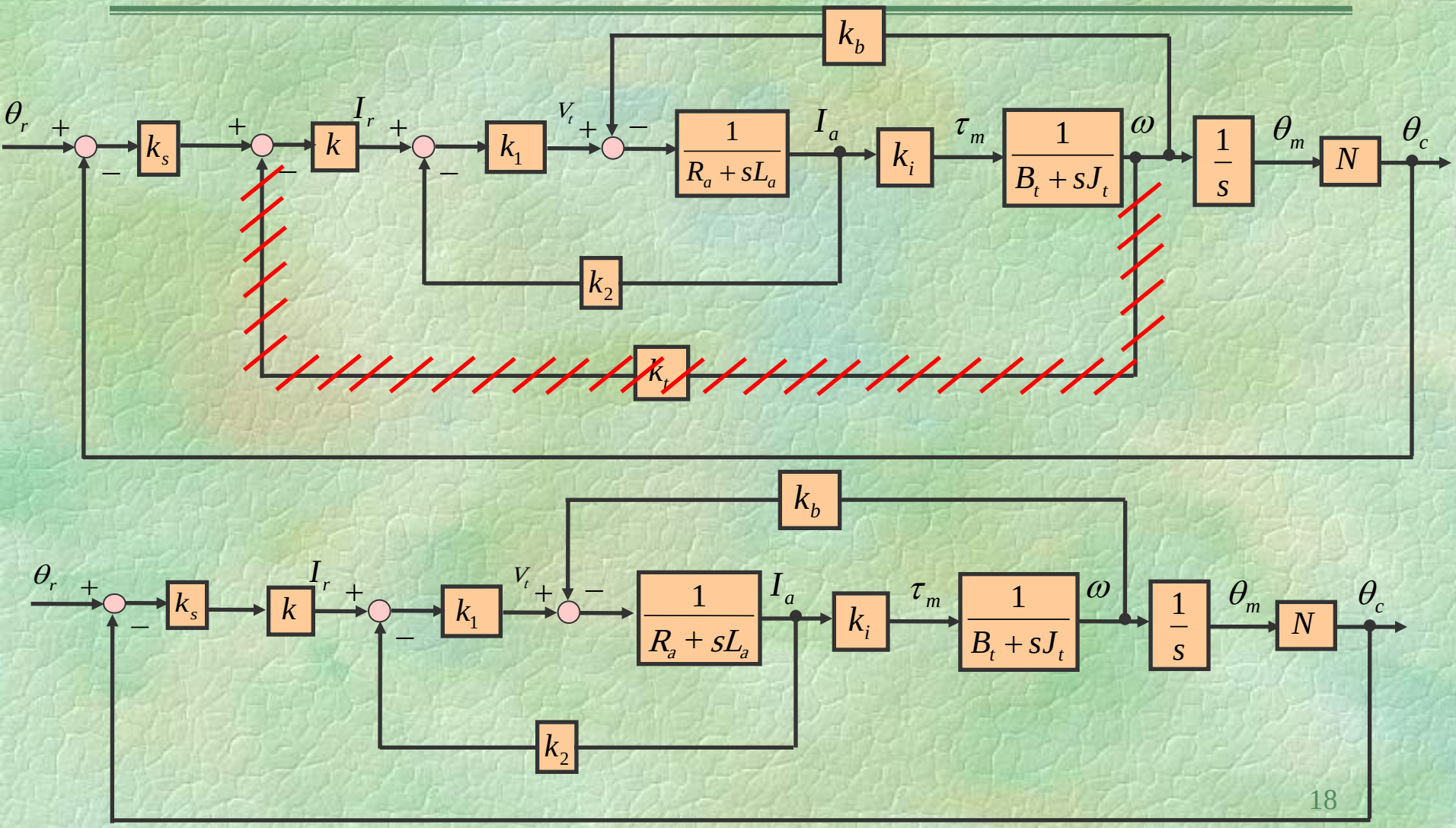
بلوک دیاگرام یک سیستم کنترل موقعیت





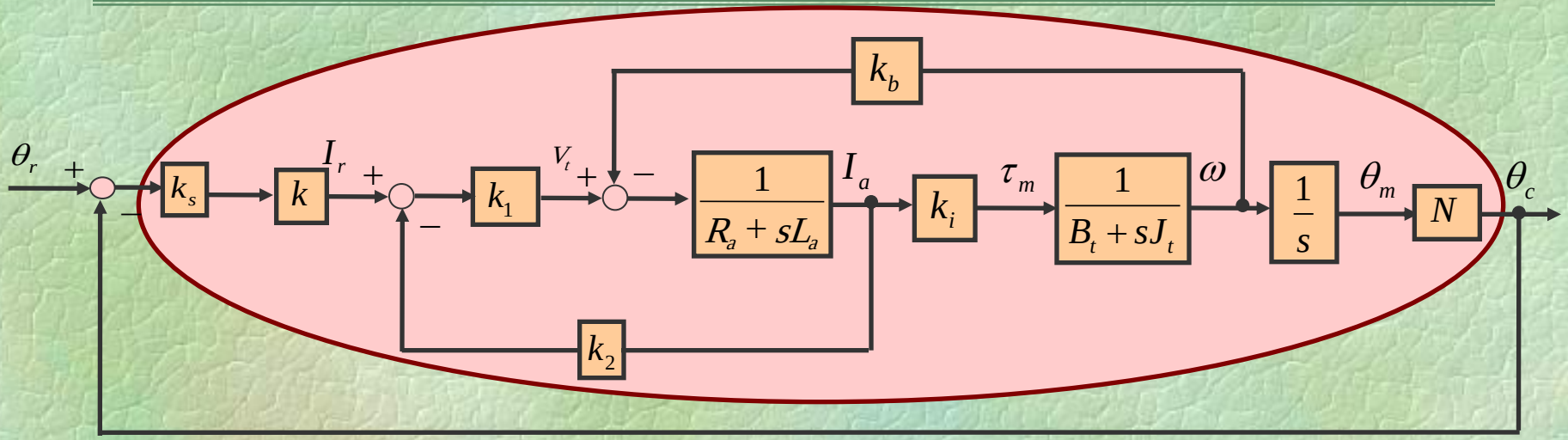
Let ignore the velocity feedback.

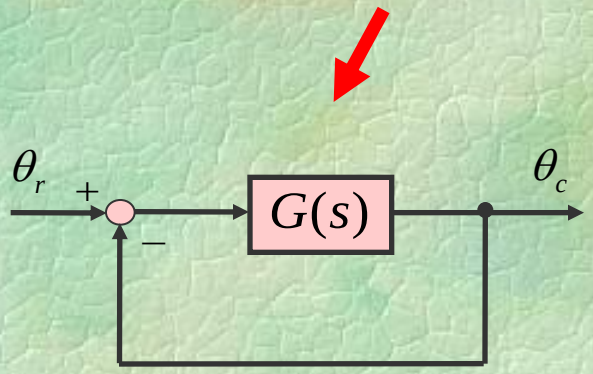
از فیدبک سرعت اغماض کنید.



Simplification

ساده سازی



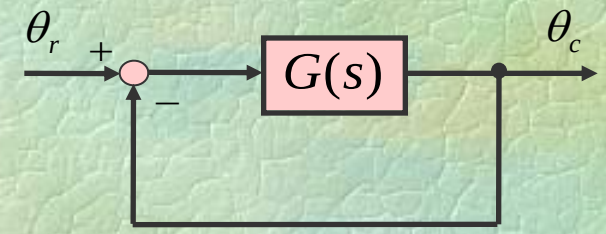


$$G(s) = \frac{\dfrac{k_s k k_1 k_i N}{s(R_a + sL_a)(B_t + sJ_t)}}{1 + \dfrac{k_1 k_2}{R_a + sL_a} + \dfrac{k_i k_b}{(R_a + sL_a)(B_t + sJ_t)}}$$

$$= \frac{k_s k k_1 k_i N}{s\left((R_a + sL_a)(B_t + sJ_t) + k_1 k_2 (B_t + sJ_t) + k_i k_b\right)}$$

Simplification

ساده سازی و عدد گذاری



$$G(s)=\frac{k_s k k_1 k_i N}{s\left((R_a+sL_a)(B_t+sJ_t)+k_1k_2(B_t+sJ_t)+k_ik_b\right)}$$

Let we ignore L_a

با اغماض از L_a داریم

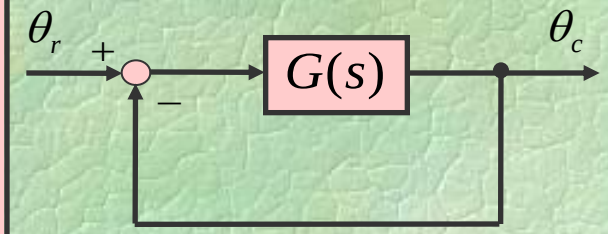
$$\tilde{G}(s)=\frac{\dfrac{k_s k k_1 k_i N}{(R_a+k_1k_2)J_t}}{s\left(s+\dfrac{R_aB_t+k_1k_2B_t+k_ik_b}{(R_a+k_1k_2)J_t}\right)}$$

Simplification

ساده سازی و عدد گذاری

$$G(s)=\frac{k_s k k_1 k_i N}{s\left((R_a+sL_a)(B_t+sJ_t)+k_1k_2(B_t+sJ_t)+k_i k_b\right)}$$

$$\tilde{G}(s)=\frac{\dfrac{k_s k k_1 k_i N}{(R_a+k_1k_2)J_t}}{s\left(s+\dfrac{R_aB_t+k_1k_2B_t+k_ik_b}{(R_a+k_1k_2)J_t}\right)}$$



$$k_1=10 \quad k_2=0.5 \quad k_i=9 \quad k_s=1 \quad k_b=0.0636$$
$$R_a=5 \quad L_a=0.03 \quad N=0.1 \quad B_l=1 \quad B_m=0.005$$
$$J_l=0.01 \quad J_m=0.0001$$

$$B_t=B_m+N^2B_l=0.005+\frac{1}{100}=0.015$$

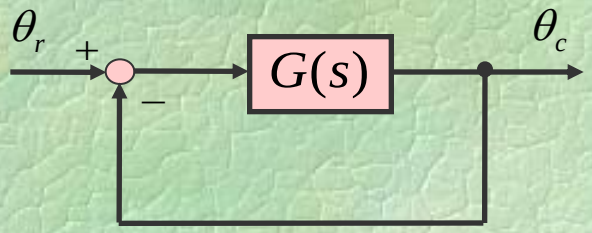
$$J_t=J_m+N^2J_l=0.0001+\frac{0.01}{100}=0.0002$$

Simplification

ساده سازی و عدد گذاری

$$G(s)=\frac{k_s k k_1 k_i N}{s\left((R_a+sL_a)(B_t+sJ_t)+k_1k_2(B_t+sJ_t)+k_i k_b\right)}$$

$$\tilde{G}(s)=\frac{\dfrac{k_s k k_1 k_i N}{(R_a+k_1k_2)J_t}}{s\left(s+\dfrac{R_aB_t+k_1k_2B_t+k_ik_b}{(R_a+k_1k_2)J_t}\right)}$$



$k_1=10$ $k_2=0.5$ $k_i=9$ $k_s=1$

$k_b=0.0636$ $R_a=5$ $L_a=0.003$ $N=0.1$

$B_m=0.0005$ $B_l=1$ $J_l=0.01$ $J_m=0.0001$

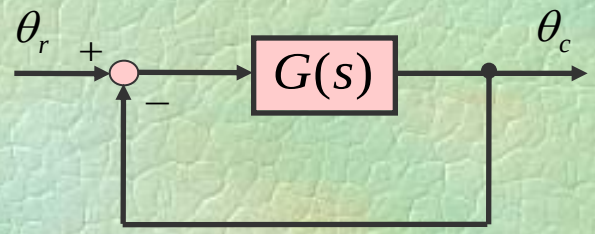
$$B_t=B_m+N^2B_l=0.005+\frac{1}{100}=0.015$$

$$J_t=J_m+N^2J_l=0.0001+\frac{0.01}{100}=0.0002$$

$$G(s)=\frac{1.5\times10^7 k}{s\left(s^2+3408.3s+1204000\right)}=\frac{1.5\times10^7 k}{s(s+400.3)(s+3008)}$$

$$\tilde{G}(s)=\frac{4500k}{s(s+361.2)}$$

Stability



بررسی پایداری

$$G(s)=\frac{1.5\times10^7 k}{s(s+400.3)(s+3008)}$$

$$M(s)=\frac{G(s)}{1+G(s)}=\frac{1.5\times10^7 k}{s(s+400.3)(s+3008)+1.5\times10^7 k}$$

$$s^3+3408.3s^2+1204000s+1.5\times10^7 k=0$$

s^3	1	1204000
s^2	3408.3	$1.5\times10^7 k$
s	$\frac{4100593200-1.5\times10^7 k}{3408.3}$	0
s^0	$1.5\times10^7 k$	

Stable for $0<k<273.57$

$$\tilde{G}(s)=\frac{4500k}{s(s+361.2)}$$

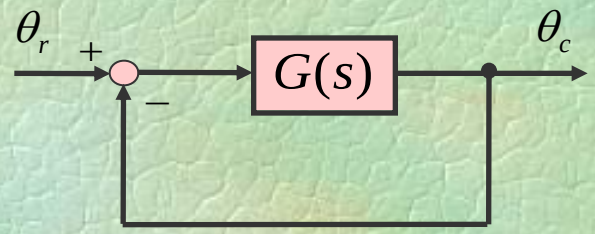
$$M(s)=\frac{4500k}{s^2+361.2s+4500k}$$

$$s^2+361.2s+4500k=0$$

Stable for all $k>0$

Analysis

تجزیه تحلیل



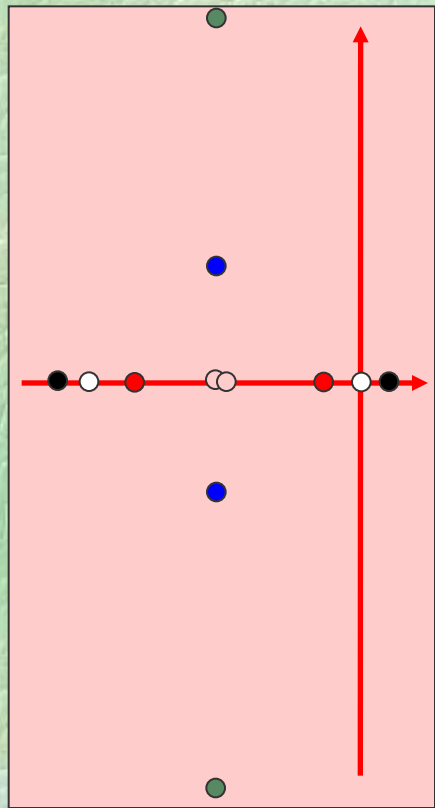
$$\tilde{G}(s) = \frac{4500k}{s(s+361.2)}$$

$$s^2 + 361.2s + 4500k = 0$$

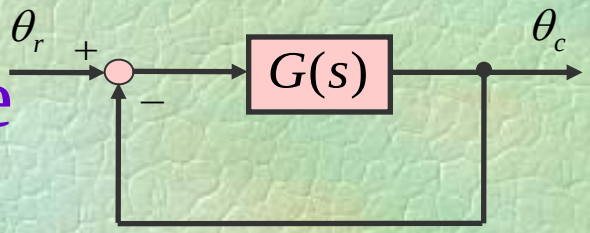
$$\omega_n^{\,2} = 4500k \quad \Rightarrow \quad \omega_n = 30\sqrt{5k}$$

$$2\zeta\omega_n = 361.2 \quad \Rightarrow \quad \zeta = \frac{361.2}{60\sqrt{5k}} = \frac{2.692}{\sqrt{k}}$$

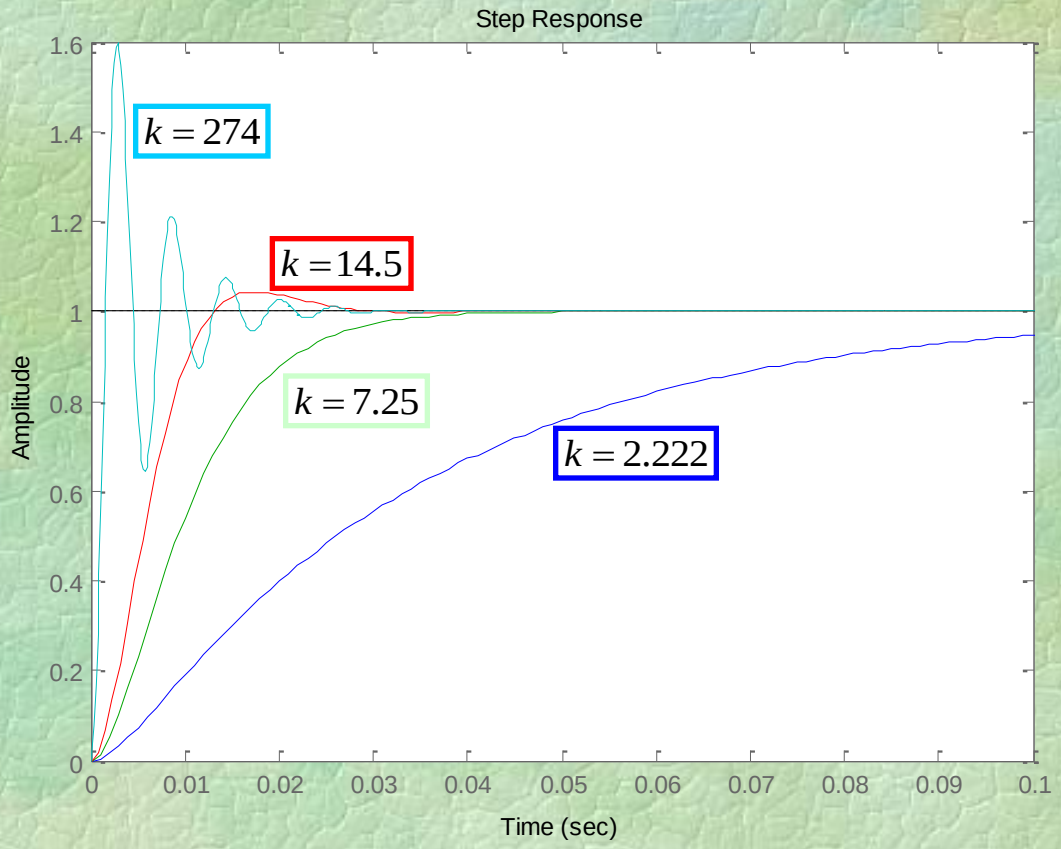
k	p_1	p_2	ζ	ω_n	$P.O.$	t_s
-2	-384	23		Systemis unstable		
0	-361	0		Systemis unstable		
2.222	-331	-30.2	1.81	Systemis over damped		
7.25	-180.6	-180.6	1	Systemis critically damping		
14.5	$-180.6 \pm j180.6$		0.707	255	4.3%	0.022
274	$-180.6 \pm j1096$		0.163	1110	60%	0.022

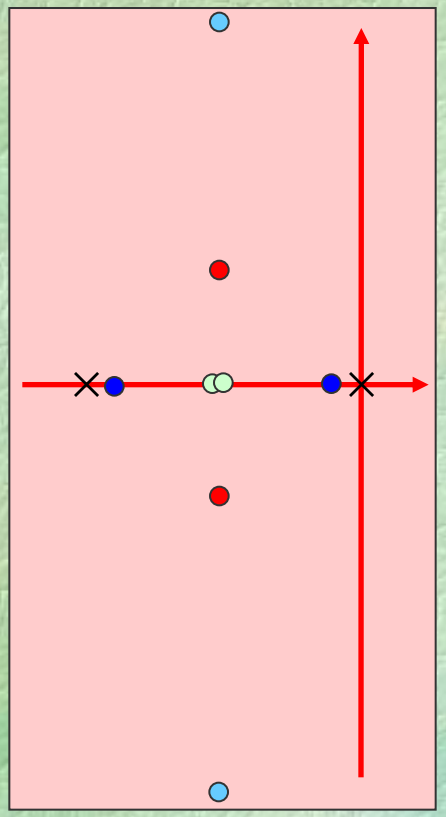


Step response



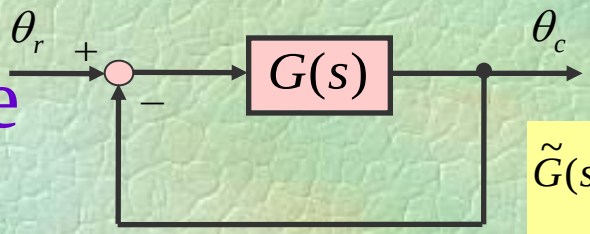
پاسخ پله



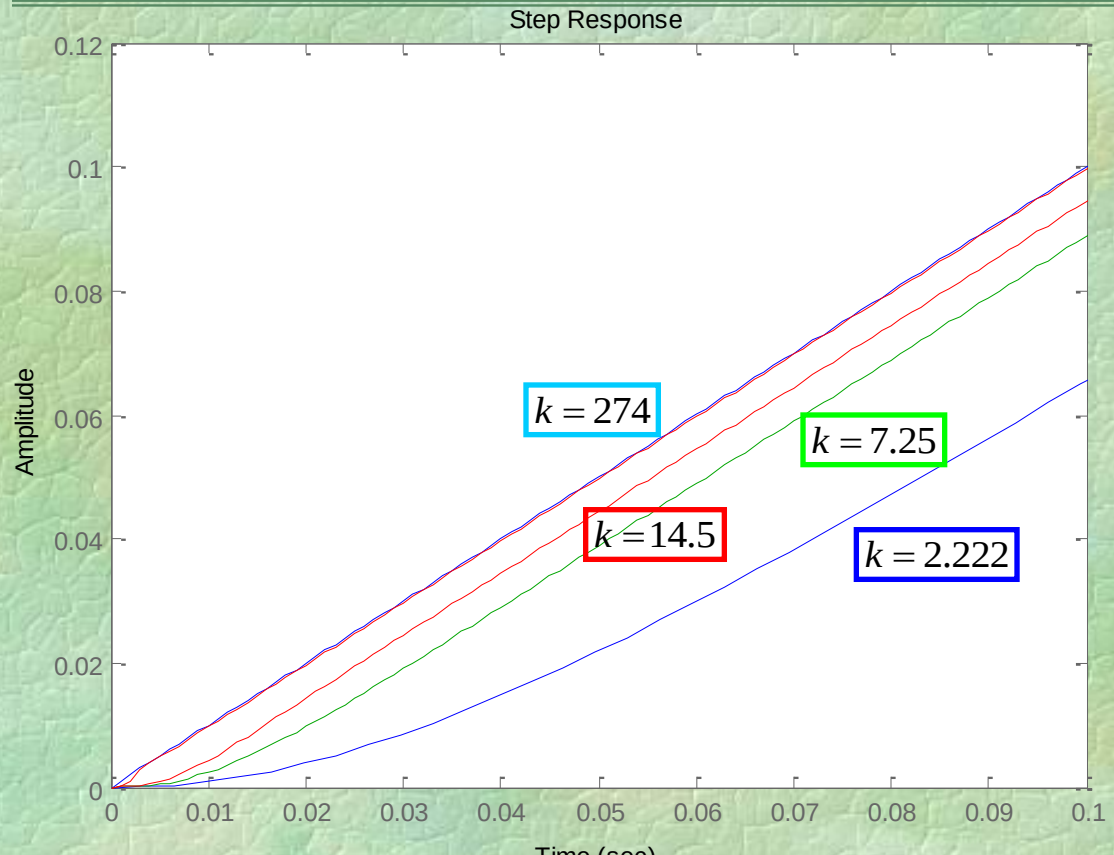


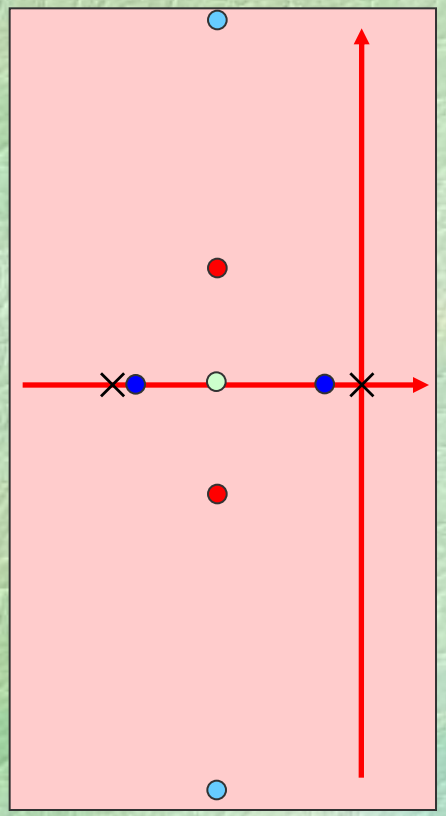
Velocity response

پاسخ شیب



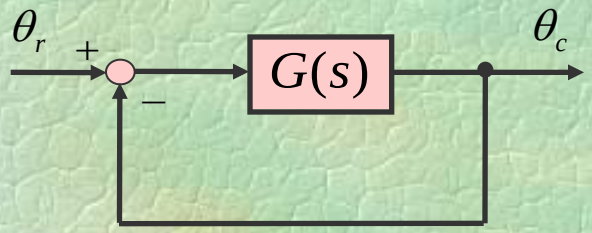
$$\tilde{G}(s) = \frac{4500k}{s(s+361.2)}$$





k	2.222	7.25	14.5	274
k_v	27.68	90.3	180.6	3414
e_{ss}	0.036	0.011	0.005	0.00029

Analysis

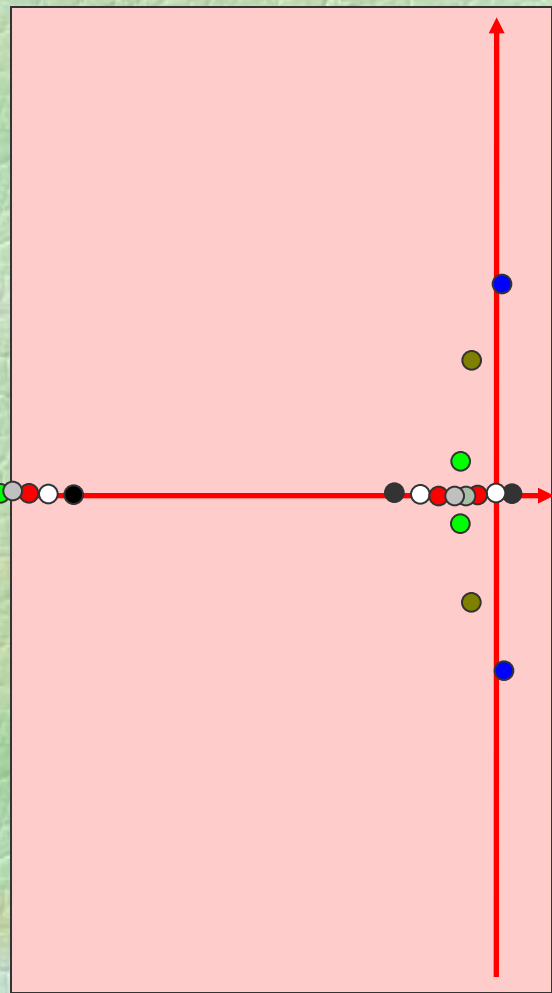


تجزیه تحلیل

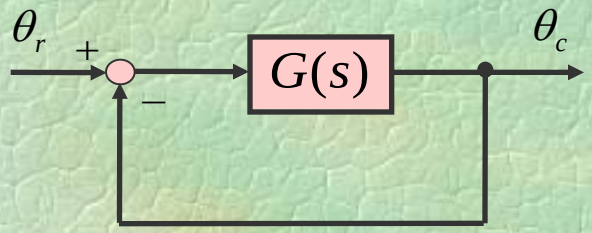
$$G(s)=\frac{1.5\times10^{7}k}{s(s+400.3)(s+3008)}$$

$$s^3+3408.3s^2+1204000s+1.5\times10^{7}k=0$$

k	p_1	p_2	p_3	ζ	ω_n	$P.O.$
-2	-3004	-428	23	Systemis unstable		
0	-3008	-400	0	Systemis unstable		
2.222	-3012	-366	-30	Systemis over damped		
7.250	-3022	-230	-156	Systemis over damped		
14.5	-3035	$-186\pm j192$		0.7	267	4.6%
100	-3178	$-115\pm j677$		0.17	687	59%
274	-3408	$0.2\pm j1098$		Systemis unstable		

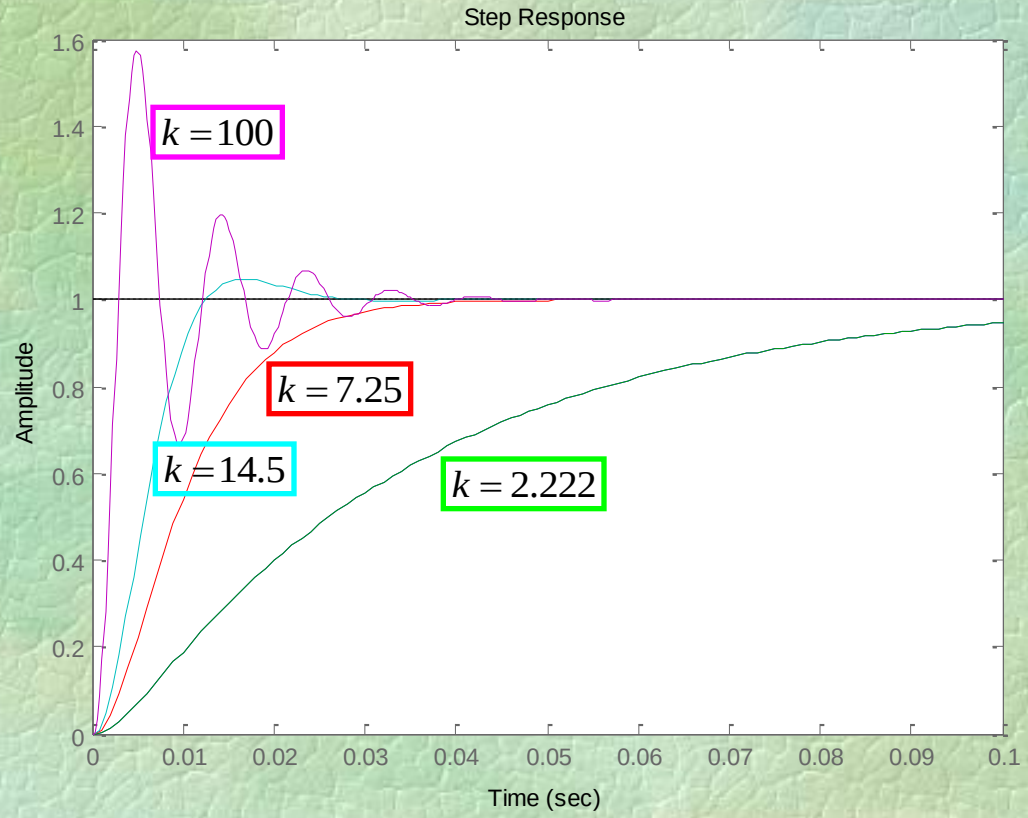


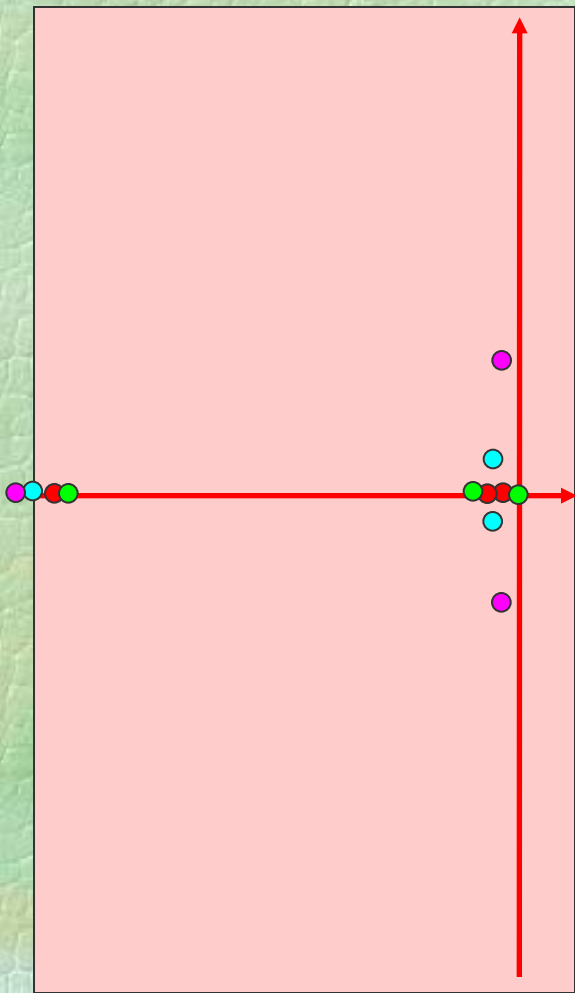
Step response



پاسخ پله

$$G(s) = \frac{1.5\times10^{7}\,k}{s(s+400.3)(s+3008)}$$



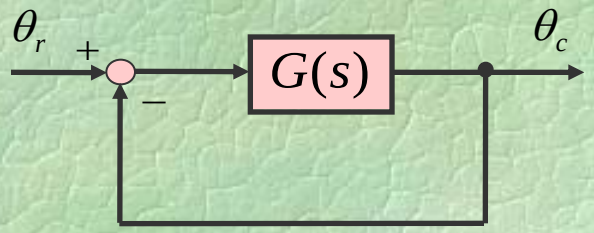


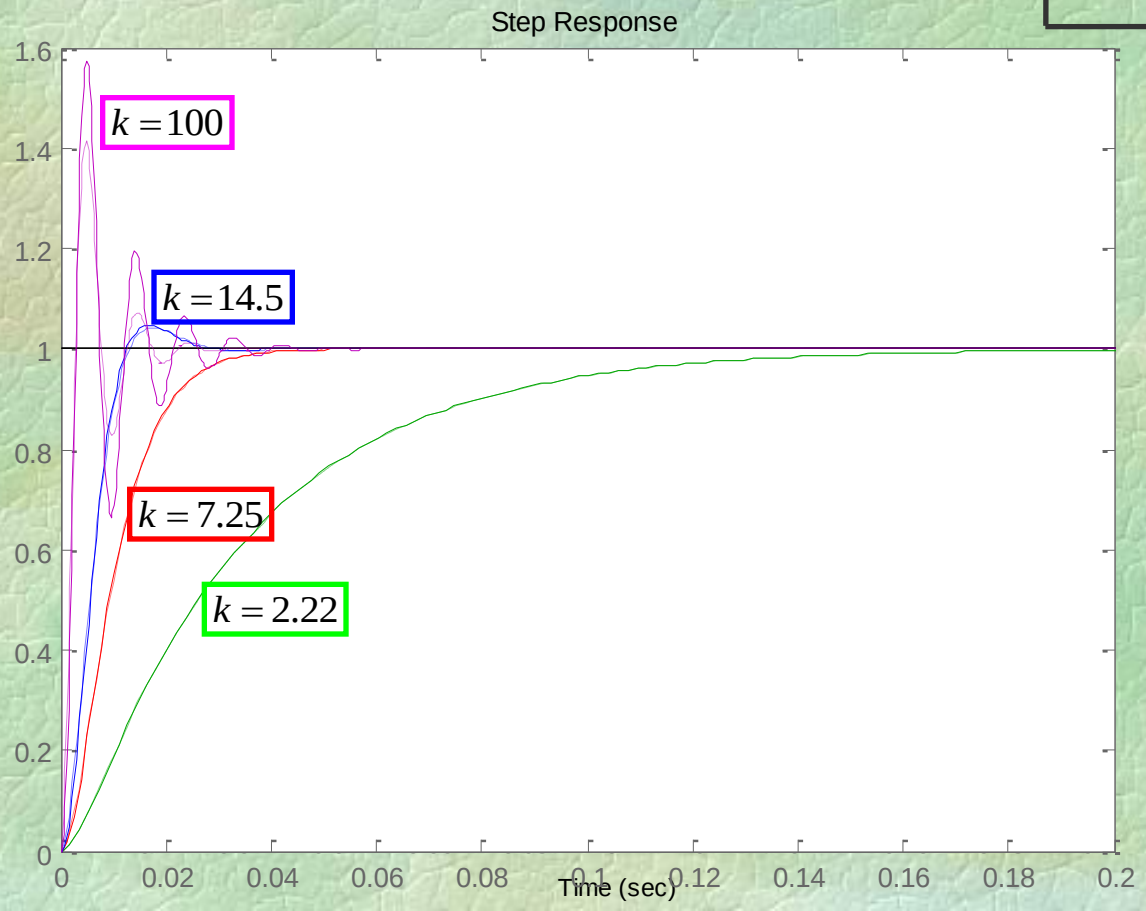
Step response comparison

مقایسه پاسخ پله

$$G(s)=\frac{1.5\times10^{7}\,k}{s(s+400.3)(s+3008)}$$

$$\tilde{G}(s)=\frac{4500k}{s(s+361.2)}$$





Exercises

تمرینها

1- Find the roots of following system for -301<k<301 and show them on the s plane.

$$G(s)=\frac{4500k}{s(s+361.2)}$$

Let k=-300,-280,-260,…….260,280,300

2- Find the roots of following system for -301<k<301 and show them on the s plane.

$$G(s)=\frac{1.5\times10^{7}k}{s(s+400.3)(s+3008)}$$

Let k=-300,-280,-260,…….260,280,300