LINEAR CONTROL SYSTEMS

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Lecture 17

Time domain design of control systems

Topics to be covered include:

Time domain design of the lead and lag controllers.

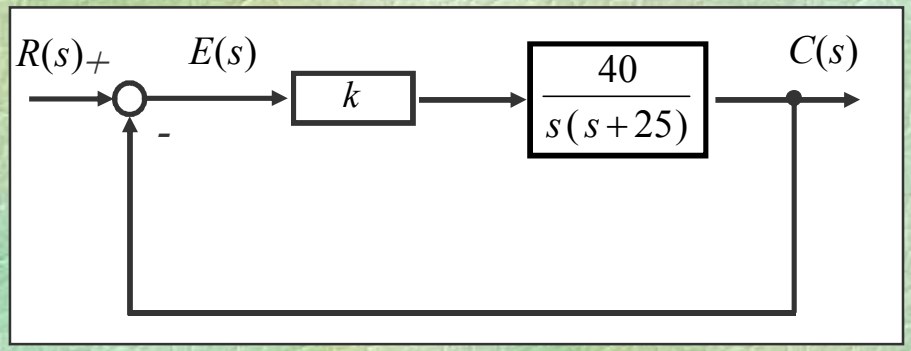
- Design of lag controllers.
- Design of lead controllers.

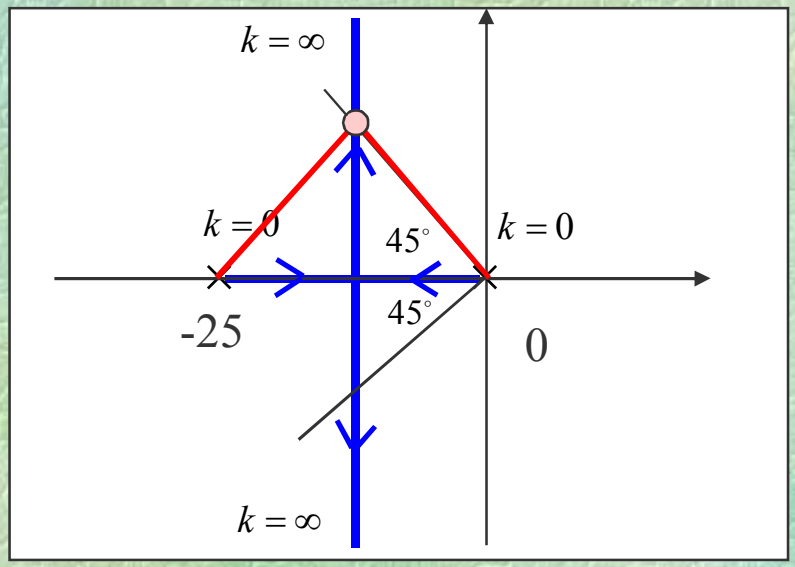
Time domain design

طراحی حوزه زمانی

Is it possible to set the value of *k* such that the damping ratio of complex poles be 0.707? Yes

در سیستم زیر آیا می توان k را بگونه ای تنظیم کرد که نسبت میرائی قطبهای مختلط سیستم 0.707 گردد؟ بله





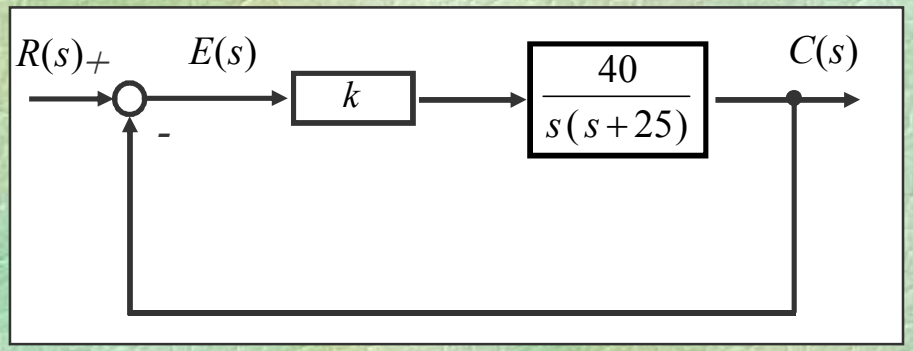
$$40k = \frac{12.5\sqrt{2} \bullet 12.5\sqrt{2}}{1} \rightarrow k = 7.8125$$

Time domain design

طراحی حوزه زمانی

Is it possible to set the value of k such that ramp error constant be 100? Yes

در سیستم زیر آیا می توان k را بگونه ای تنظیم کرد که ثابت خطای شیب معادل ۱۰۰ گردد؟ بله



$$k_v = 100$$

$$\rightarrow \lim_{s\to 0} s\frac{40k}{s(s+25)} = 100$$

$$\rightarrow 40k = 2500$$

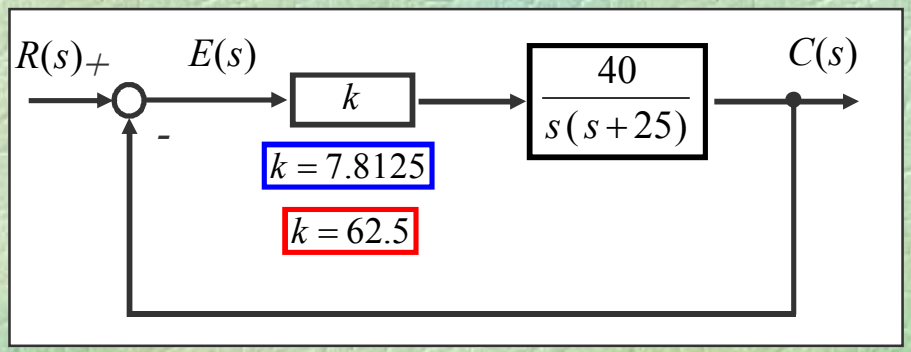
$$\rightarrow k = 62.5$$

Time domain design

طراحی حوزه زمانی

Is it possible to set the value of k such that the damping ratio of complex poles be 0.707 and ramp error constant be 100 ?

در سیستم زیر آیا می توان k را بگونه ای تنظیم کرد که نسبت میرائی قطبهای مختلط سیستم 0.707 و ثابت خطای شیب معادل ۱۰۰ گردد ؟



Clearly the design is not possible

???!!!???

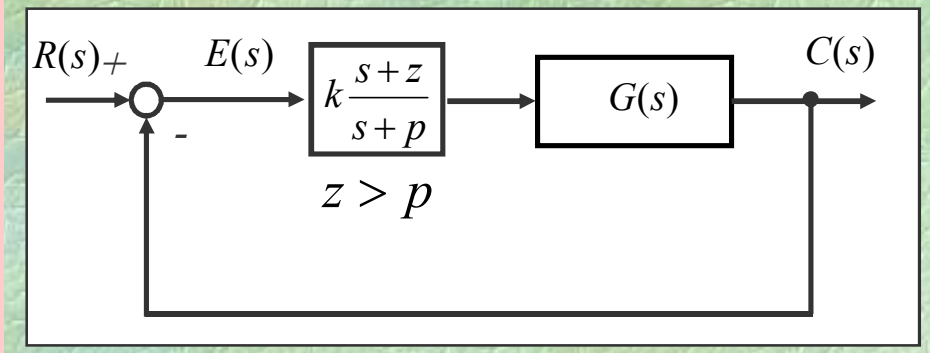
Other controllers

Lag controller design procedure

رویه طراحی کنترلر پس فاز

1- Obtain the root-locus (without controller) and determine the gain k_0 to satisfy the desired damping ratio or ……

2- Find the gain k to satisfy the desired steady state error (without controller). If k is in conflict with k_0 continue.



3- Evaluate the needed controller gain

$$\text{needed gain} = \frac{k}{k_0} = \frac{\text{Gain to satisfy the desired steady-state error}}{\text{Gain to satisfy the desired damping ratio or ...}}$$

Why?

4- Choose pole and zero of controller near origin such that:

$$\frac{z}{p} = \text{needed gain} = \frac{k}{k_0}$$

5 - Choose the controller as:

$$G_c(s) = k_0 \frac{s+z}{s+p}$$

What is near?

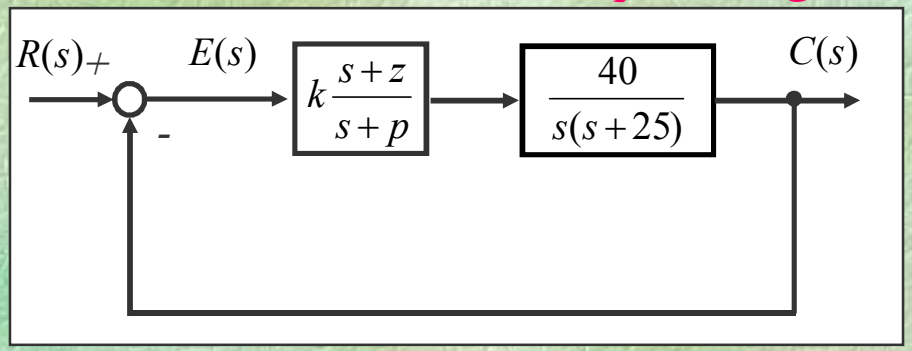
6 - Check the controller.

Example 1: Designing lag controller

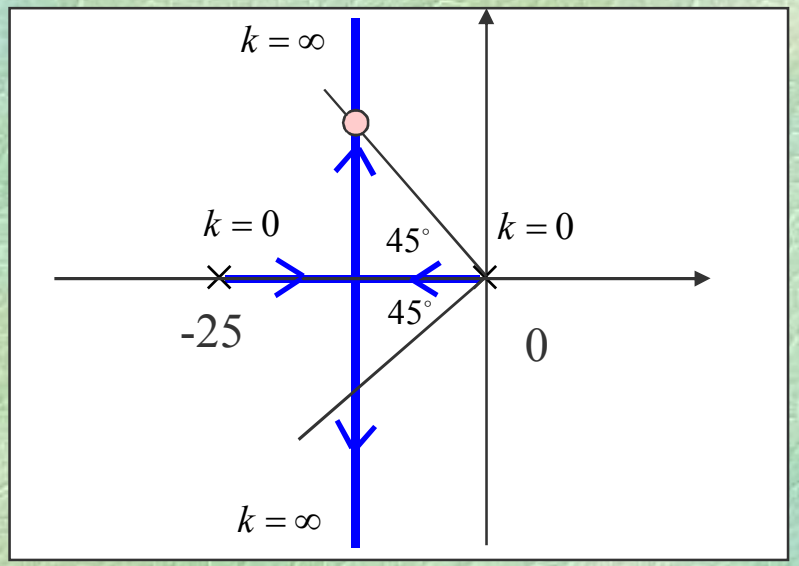
مثال ۱: طراحی کنترلر پس فاز

Is it possible to set the value of k such that the damping ratio of complex poles be 0.707 and ramp error constant be 100 ?

در سیستم زیر آیا می توان k را بگونه ای تنظیم کرد که نسبت میرائی قطبهای مختلط سیستم 0.707 و ثابت خطای شیب معادل ۱۰۰ گردد ؟



1- Obtain the root-locus (without controller) and determine the gain k_0 to satisfy the desired damping ratio or ……



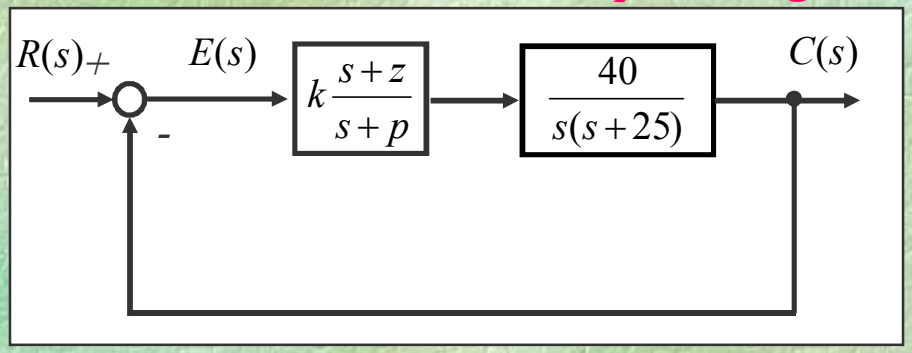
$$40k_0 = \frac{12.5\sqrt{2} \bullet 12.5\sqrt{2}}{1} \rightarrow k_0 = 7.8125$$

Example 1: Designing lag controller

مثال ۱: طراحی کنترلر پس فاز

Determine the controller coefficient such that the damping ratio of complex poles be 0.707 and ramp error constant be 100 ?

در سیستم ضرایب کنترلر را بگونه ای تنظیم کنید که نسبت میرائی قطبهای مختلط سیستم 0.707 و ثابت خطای شیب معادل ۱۰۰ گردد ؟



2- Determine the gain k to satisfy the desired steady-state error (without controller).
If there is a problem then continue.

$$k_v = 100$$

$$\rightarrow \lim_{s\to 0} s\frac{40k}{s(s+25)} = 100$$

$$\rightarrow 40k = 2500$$

$$\rightarrow k = 62.5$$

Example 1: Designing lag controller

مثال ۱: طراحی کنترلر پس فاز

Determine the controller coefficient such that the damping ratio of complex poles be 0.707 and ramp error constant be 100 ?

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3- Evaluate the needed controller gain

$$\text{needed gain} = \frac{k}{k_0} = \frac{\text{Gain to satisfy the desired steady-state error}}{\text{Gain to satisfy the desired damping ratio or ...}} = \frac{62.5}{7.8125} = 8$$

4- Choose pole and zero of controller near origin such that:

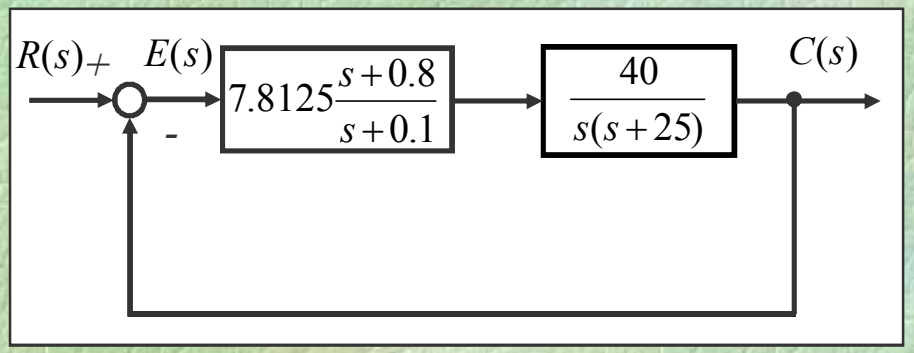
$$\frac{z}{p} = \text{needed gain} = \frac{k}{k_0} = 8 \qquad p = 0.1 \qquad z = 0.8$$

5 - Choose the controller as:

$$G_c(s) = k_0 \frac{s+z}{s+p} = 7.8125 \frac{s+0.8}{s+0.1}$$

Example 1: Designing lag controller

مثال ۱: طراحی کنترلر پس فاز



6 - Check the controller.

$$k_v = \lim_{s\to 0} s\,7.8125\,\frac{s+0.8}{s+0.1}\,\frac{40}{s(s+25)} = 100$$

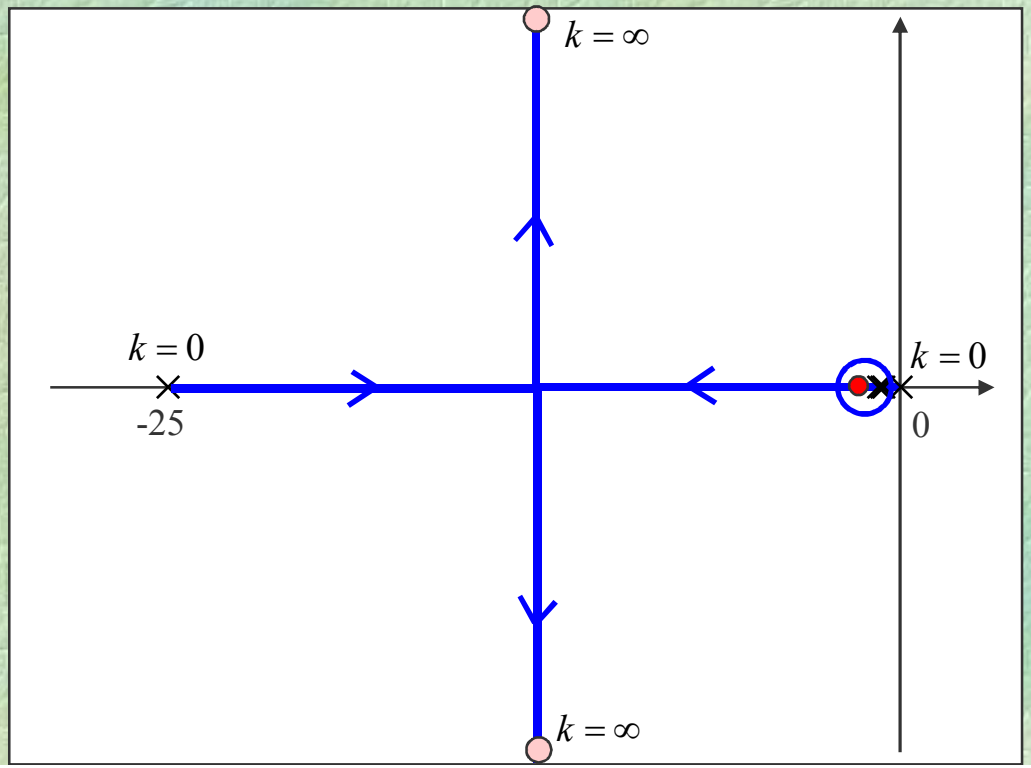
$\zeta = 0.707$

Why?

$$\frac{C(s)}{R(s)} = \frac{312.5(s+0.8)}{s(s+25)(s+0.1)+312.5(s+0.8)}$$

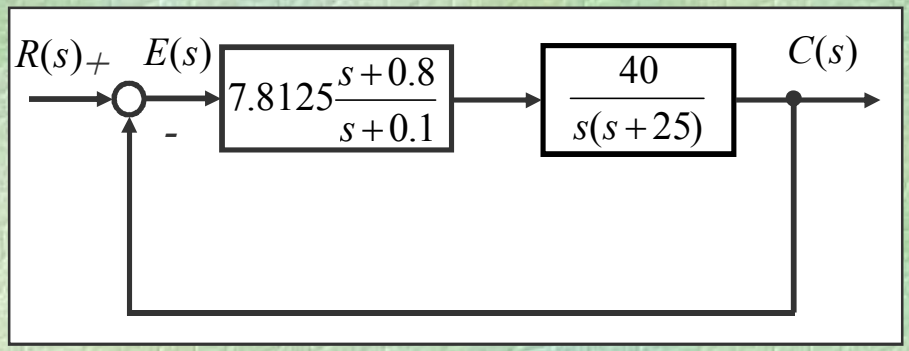
$$\frac{C(s)}{R(s)} = \frac{312.5(s+0.8)}{s^3+25.1s^2+315s+250}$$

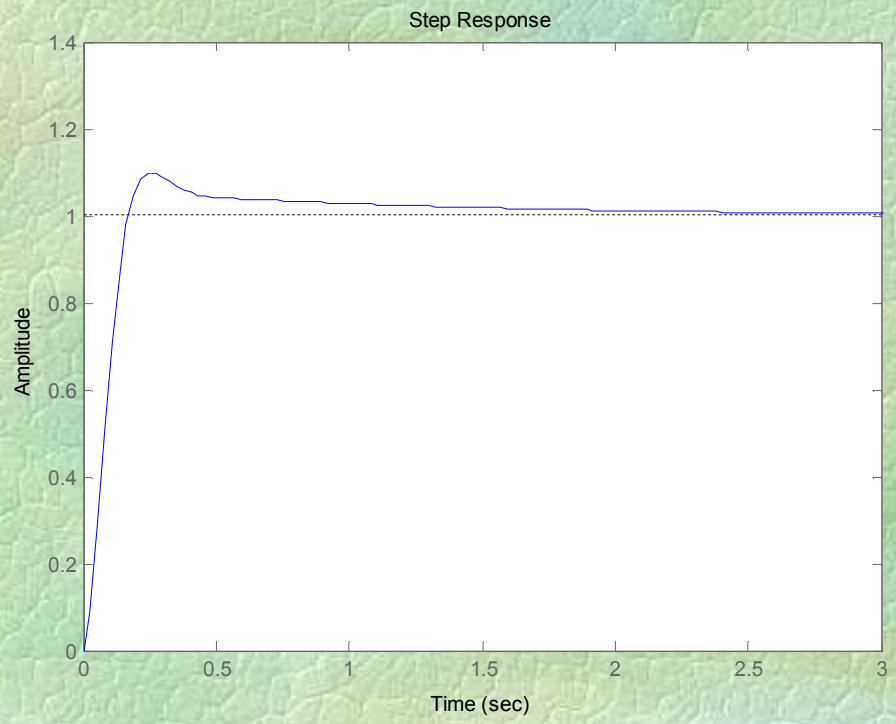
Poles are: $-12.13 \pm 12.14j, -0.85$



Example 1: Designing lag controller and its step response

مثال ۱: طراحی کنترلر پس فاز و پاسخ پله



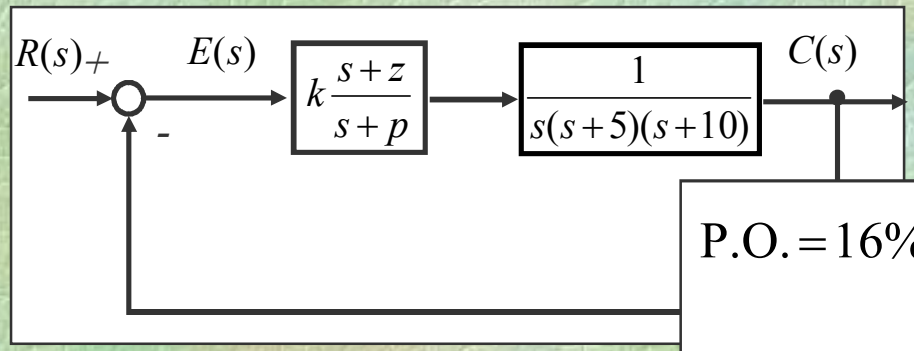


Example 2: Designing lag controller

مثال ۲: طراحی کنترلر پس فاز

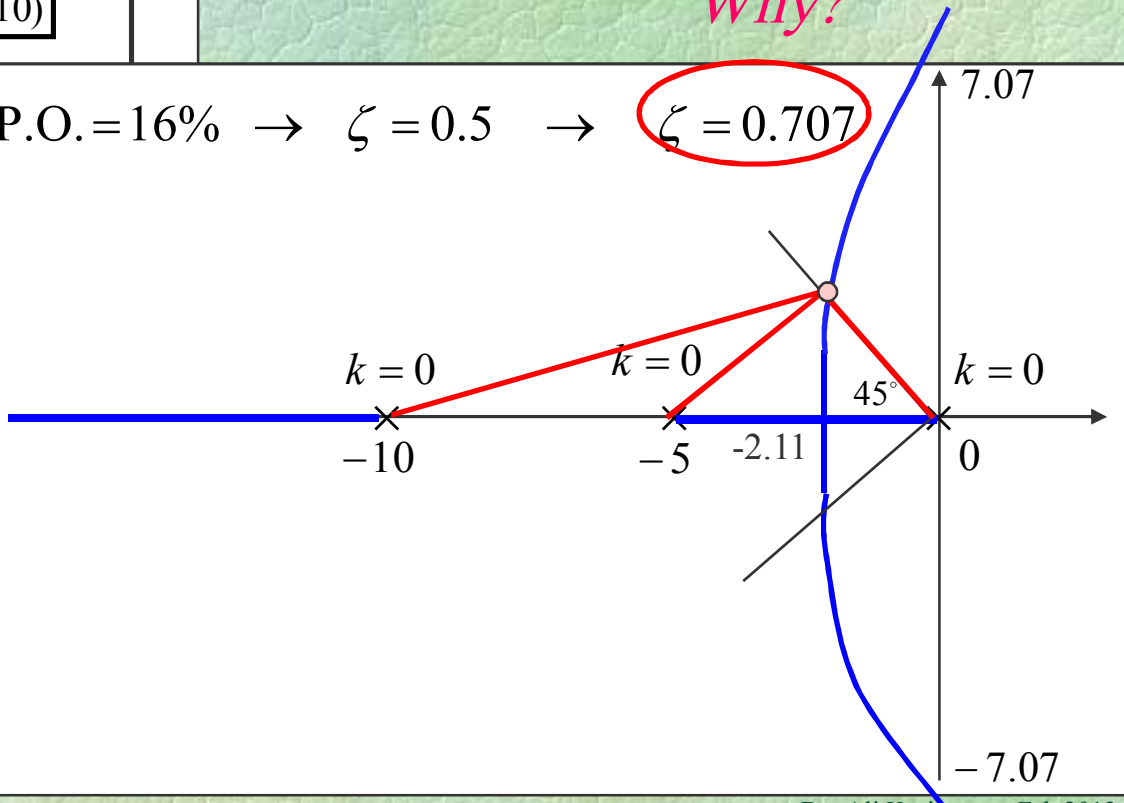
Design a lag controller such that the ramp error constant be 50 and P.O. be 16 % .

کنترلی طراحی کنید که ثابت خطای شیب معادل ۵۰ گردد و درصد فراجهش ۱۶٪ گردد.



Why?

$$\text{P.O.} = 16\% \rightarrow \zeta = 0.5 \rightarrow \zeta = 0.707$$



1- Obtain the root-locus and determine the gain k_0 to satisfy the desired damping ratio or …...

$$k_0 = \frac{2\sqrt{2} \bullet \sqrt{3^2+2^2} \bullet \sqrt{8^2+2^2}}{1}$$

$$k_0 = 84$$

Example 2: Designing lag controller

مثال ۲: طراحی کنترلر پس فاز

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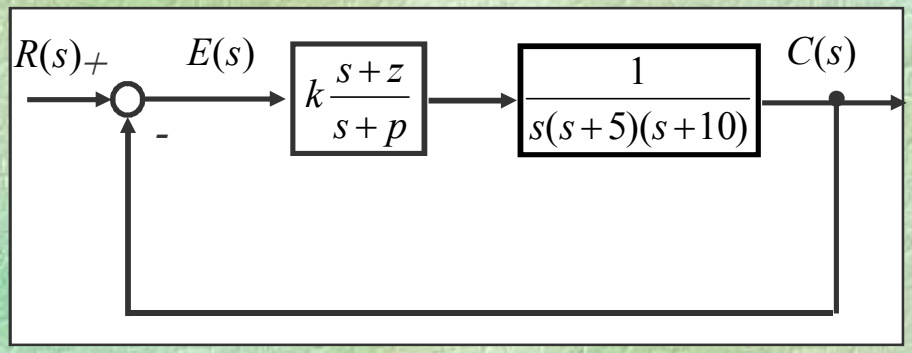

2- Determine the gain k to satisfy the desired steady-state error. If there is a problem then Continue. (without controller)

$$k_v = 50$$

$$\rightarrow \quad \lim_{s\to 0} s\frac{k}{s(s+5)(s+10)} = 50$$

$$\rightarrow \quad k = 2500$$

Example 2: Designing lag controller

مثال ۲: طراحی کنترلر پس فاز

Design a lag controller such that the ramp error constant be 50 and P.O. be 16 % .

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3- Evaluate the needed controller gain

$$\text{needed gain} = \frac{k}{k_0} = \frac{\text{Gain to satisfy the desired steady-state error}}{\text{Gain to satisfy the desired damping ratio or ...}} = \frac{2500}{84} = 29.76$$

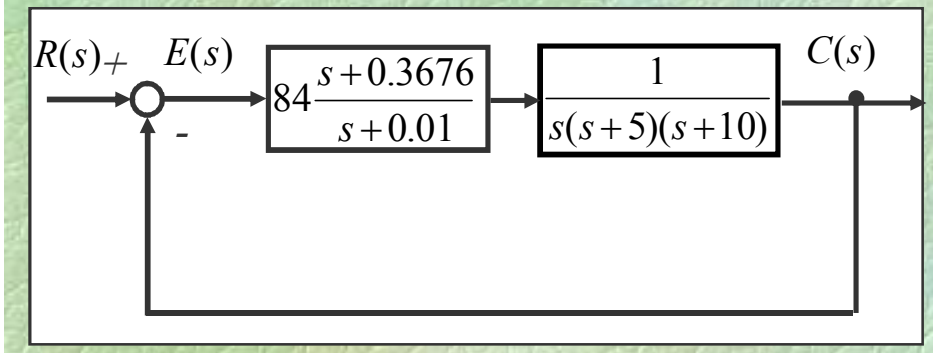
4- Choose pole and zero of controller near origin such that:

$$\frac{z}{p} = \text{needed gain} = \frac{k}{k_0} = 29.76 \qquad p = 0.01 \qquad z = 0.2976$$

5 - Choose the controller as: $G_c(s) = k_0 \frac{s+z}{s+p} = 84\frac{s+0.2976}{s+0.01}$

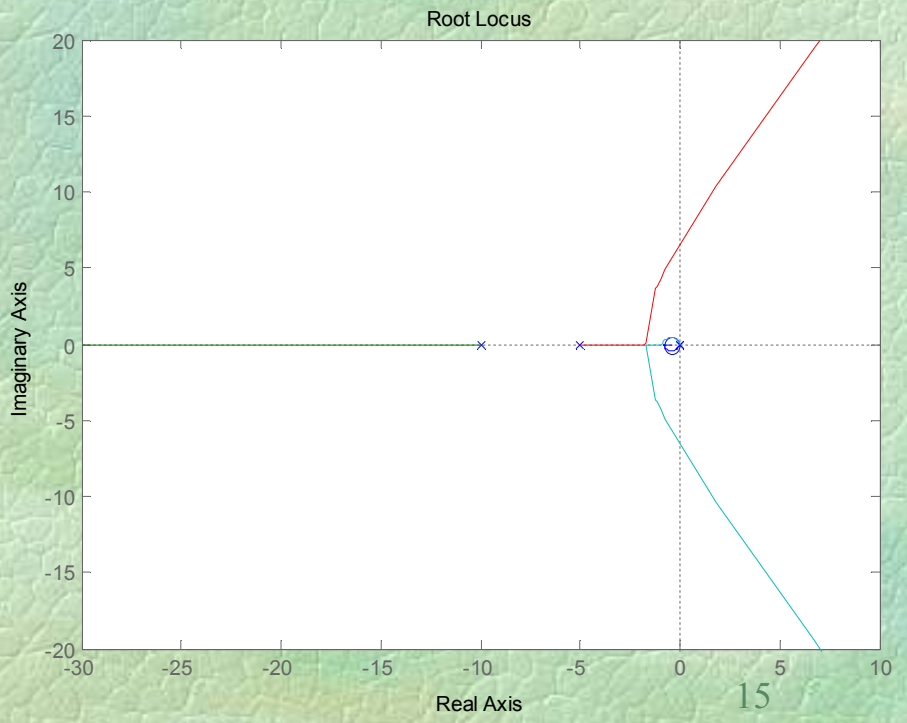
Example 2: Designing lag controller

مثال ۲: طراحی کنترلر پس فاز



6 - Check the controller.

$$k_v = \lim_{s\to 0} s84\frac{s+0.2976}{s+0.01}\frac{1}{s(s+5)(s+10)} = 50$$



$$\zeta = 0.707$$

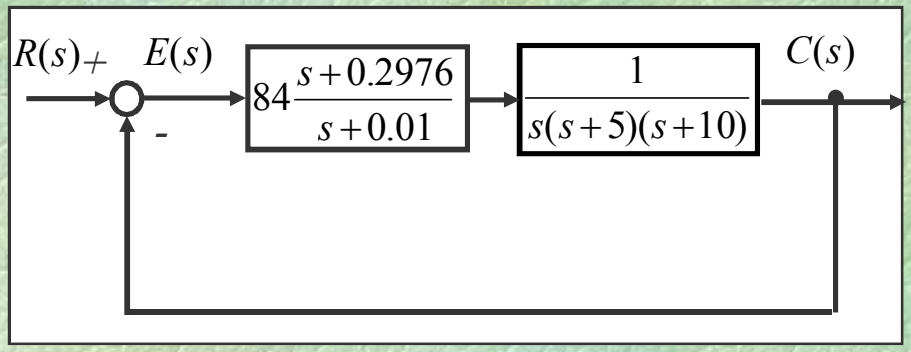
$$\frac{C(s)}{R(s)} = \frac{84(s+0.2976)}{s(s+5)(s+10)(s+0.01)+84(s+0.2976)}$$

$$\frac{C(s)}{R(s)} = \frac{84(s+0.2976)}{s^4+15.01s^3+50.15s^2+84.5s+25}$$

Poles are: $-1.73\pm1.76j, -0.37, -11.18$

Example 2: Designing lag controller

مثال ۲: طراحی کنترلر پس فاز



P.O. is not ok

Exercise 1: Tune the controller to derive the performance

Designing lag controller

طراحی کنترلر پس فاز

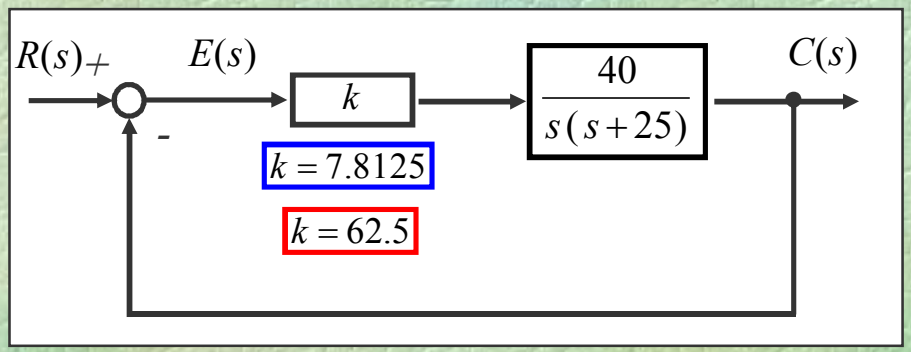
When the design of lag controller is not possible?

Time domain design

طراحی حوزه زمانی

Is it possible to set the value of k such that the damping ratio of complex poles be 0.707 and ramp error constant be 100 ?

در سیستم زیر آیا می توان k را بگونه ای تنظیم کرد که نسبت میرائی قطبهای مختلط سیستم 0.707 و ثابت خطای شیب معادل ۱۰۰ گردد ؟



Clearly the design is not possible

???!!!???

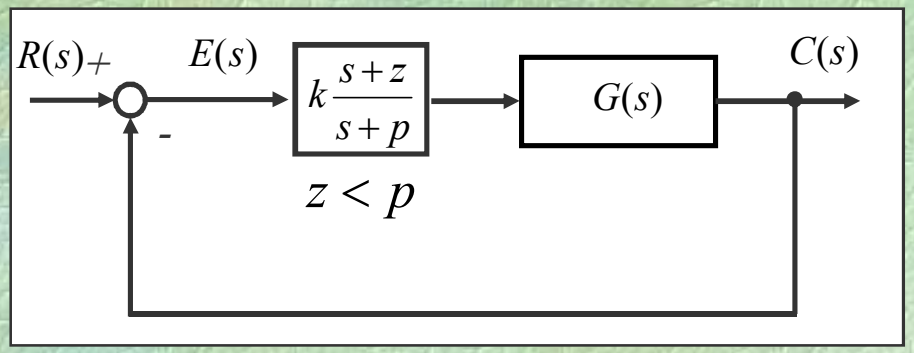
Other controllers

Lead controller design procedure

رویه طراحی کنترلر پیش فاز

1- From the time-domain specifications obtain the desired location of the closed-loop dominant poles.

2- Select the zero of controller. Place the zero on the real value of desired location of the closed-loop dominant poles or on the pole for pole-zero cancellation.



3- Locate the compensator pole so that the angle criterion is satisfied.

4- Determine the compensator gain k, such that the magnitude criterion is satisfied.

5 - Choose the controller as: $$G_c(s) = k\frac{s+z}{s+p}$$

6 - Check the controller.

7 - If the overall response rise time, overshoot and settling time is not satisfactory, choose another location of the closed-loop dominant poles.

Example 3: Designing lead controller

مثال ۳: طراحی کنترلر پیش فاز

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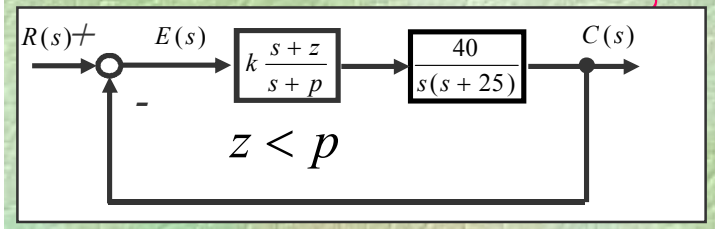
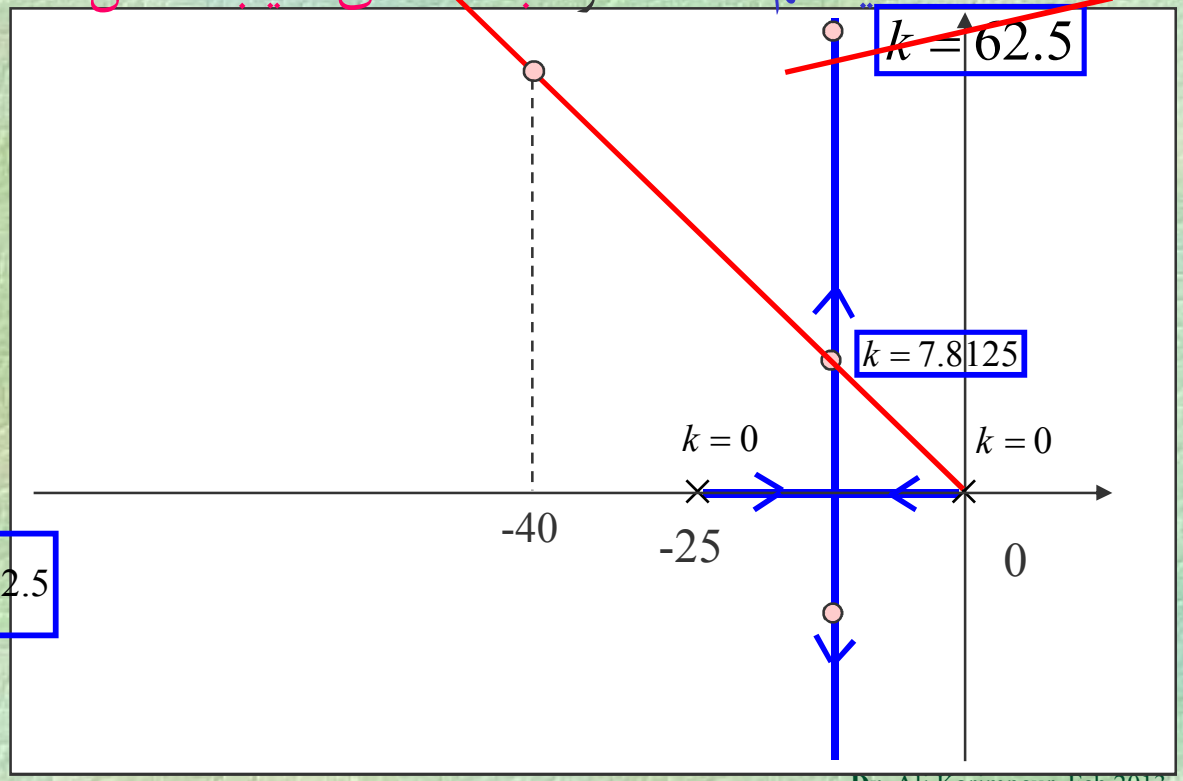


1- From the time-domain specifications obtain the desired location of the closed-loop dominant poles.

$k = 7.8125$ → $k_v = \frac{7.8125 \bullet 40}{25} = 12.5$

$k_v = 100$ → $k = 62.5$



Example 3: Designing lead controller

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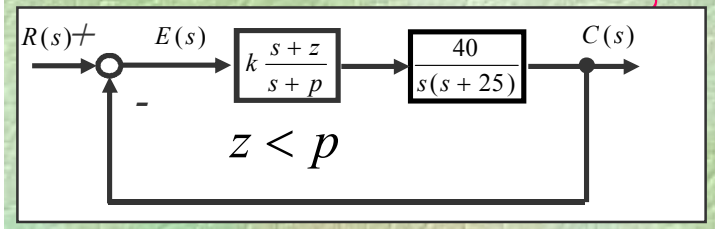
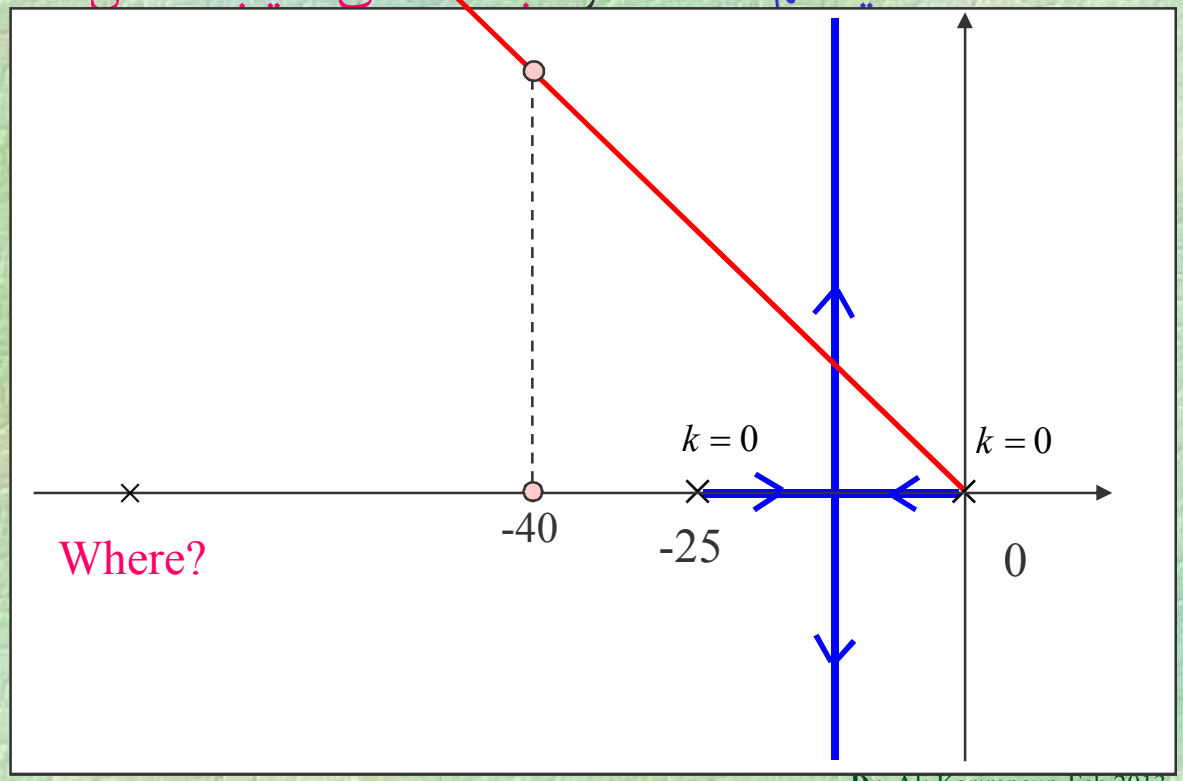


2- Select the zero of controller. Place the zero on the real value of desired location of the closed-loop dominant poles or on the pole for pole-zero cancellation.



Example 3: Designing lead controller

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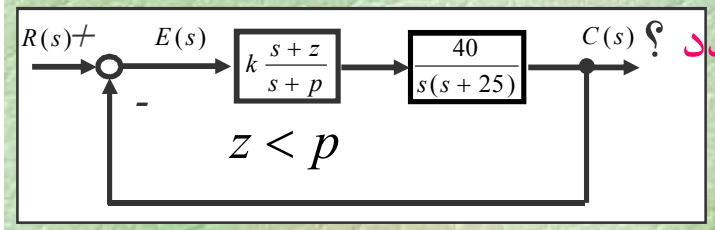


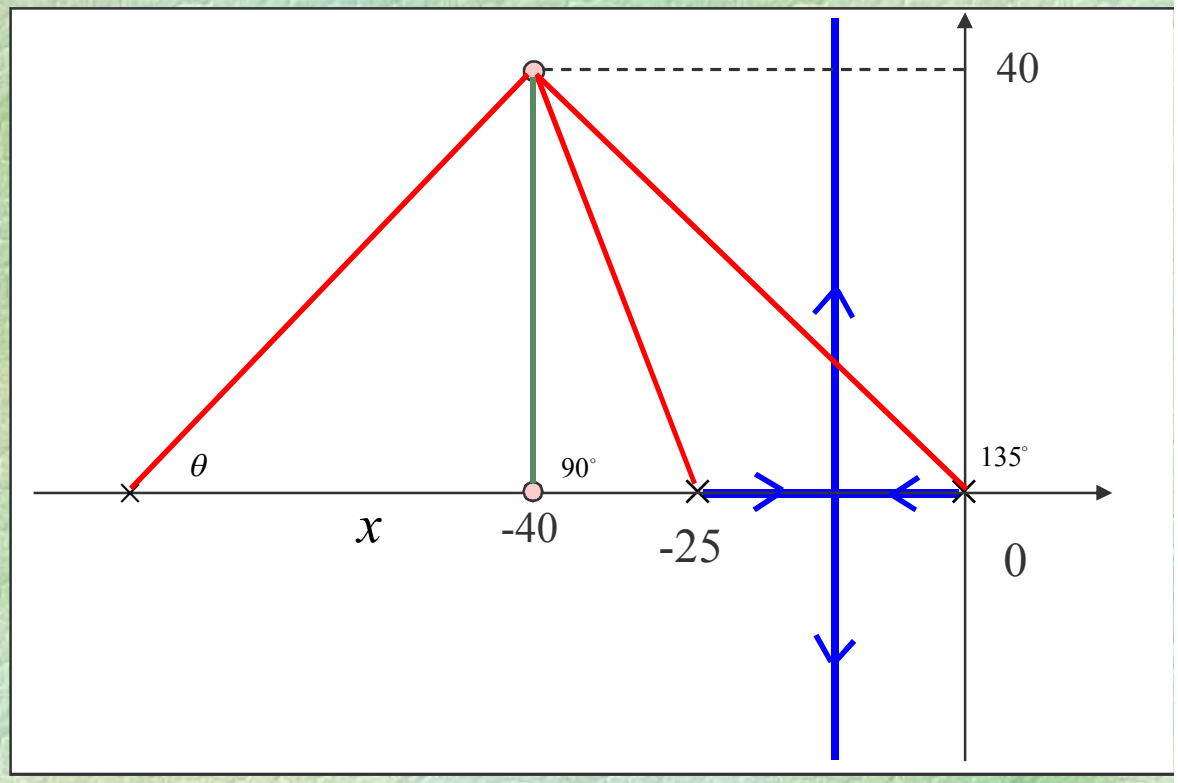
3- Locate the compensator pole so that the angle criterion is satisfied.

$$90^\circ - 135^\circ - (180 - tg^{-1}\frac{40}{15})$$

$$-\theta = \pm 180$$

$$\theta = 24.44^\circ \quad tg\theta = 0.4545 = \frac{40}{x}$$

$$x = 88 \qquad p = 128$$



Example 3: Designing lead controller

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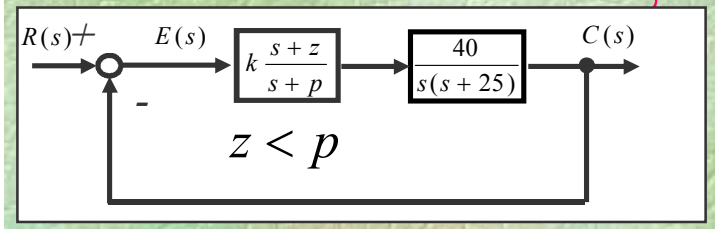
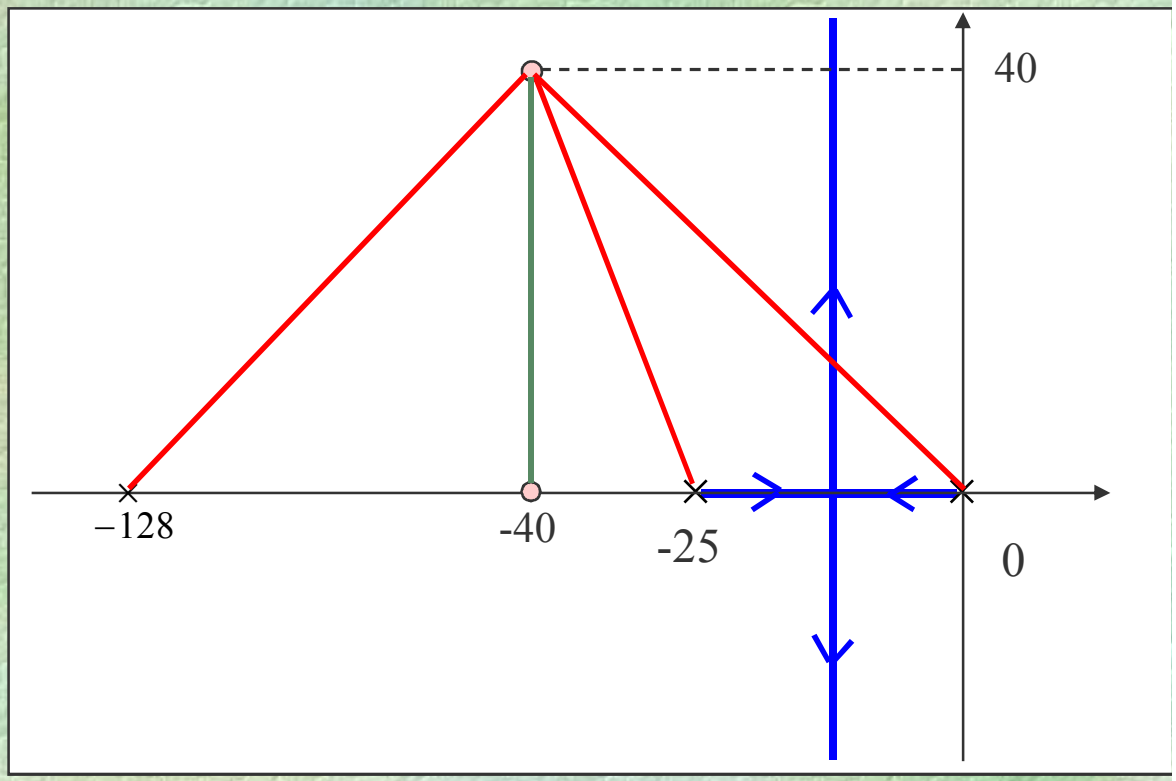


4- Determine the compensator gain k, such that the magnitude criterion is satisfied.

$$40k = \frac{40\sqrt{2} \bullet \sqrt{40^2 + 15^2} \bullet \sqrt{40^2 + 88^2}}{40}$$

$$k = 146$$



Example 3: Designing lead controller

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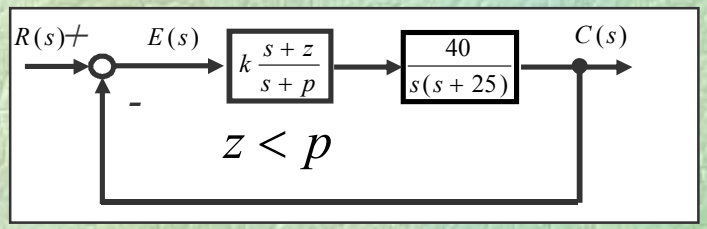
در سیستم زیر آیا می توان k را بگونه ای تنظیم کرد که نسبت میرائی قطبهای مختلط سیستم 0.707 و ثابت خطای شیب معادل ۱۰۰ گردد ؟



5 - Choose the controller as:

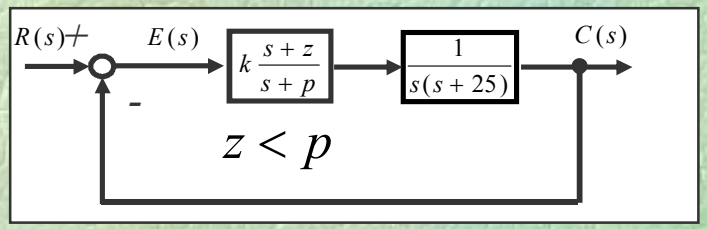
$$G_c(s) = 146\frac{s+40}{s+128}$$

Example 3: Designing lead controller

مثال ۳: طراحی کنترلر پیش فاز

Is it possible to set the value of k such that the damping ratio of complex poles be 0.707 and ramp error constant be 100 ?

در سیستم زیر آیا می توان k را بگونه ای تنظیم کرد که نسبت میرائی قطبهای مختلط سیستم 0.707 و ثابت خطای شیب معادل ۱۰۰ گردد ؟



6 - Check the controller.

Clearly $\zeta = 0.707$

$$k_v = \lim_{s\to 0} s\left(146\frac{s+40}{s+128}\frac{40}{s(s+25)}\right)$$

~~$k_v = 73$~~

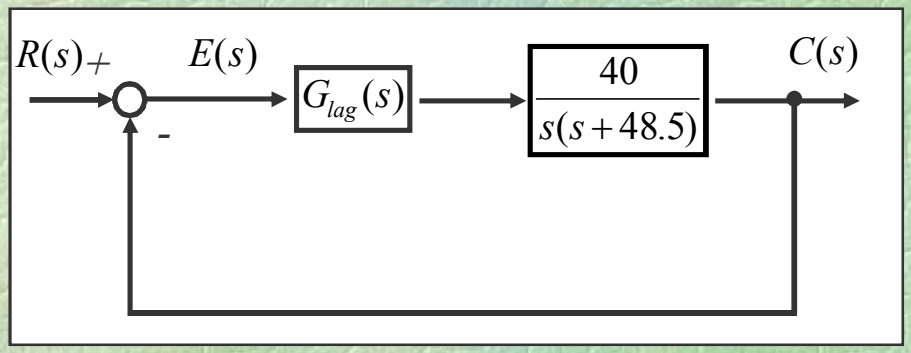
It is not ok we must try other closed-loop dominant poles

Exercise 2: Tune the controller to derive the requested performance

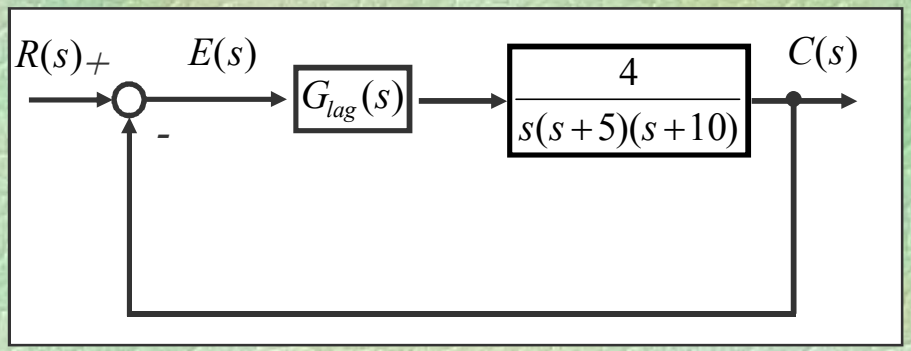
Exercises

تمرینها

1 In the following system design a lag controller such that the damping ratio of complex poles be 0.6 and ramp error constant be 80.



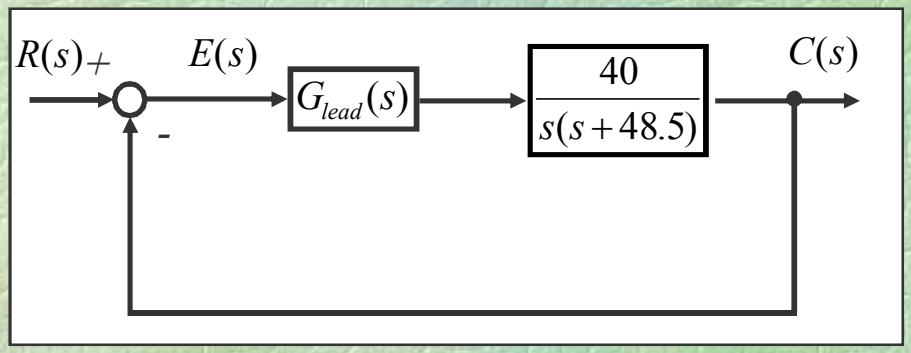
2 In the following system design a lag controller such that the damping ratio of complex poles be 0.6 and ramp error constant be 80.



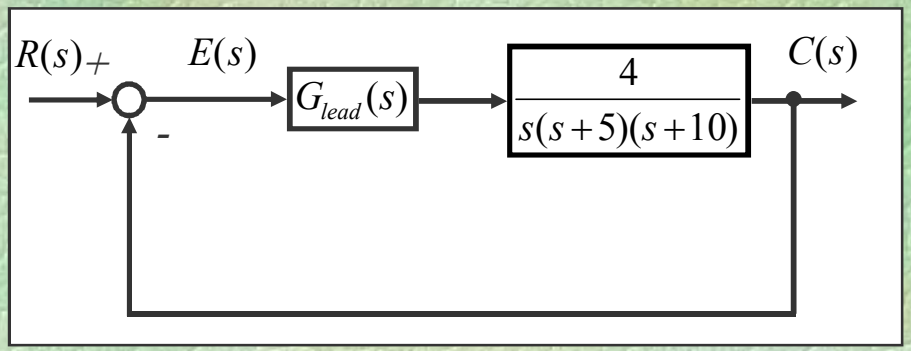
Exercises

تمرینها

3 In the following system design a Lead controller such that the damping ratio of complex poles be 0.6 and ramp error constant be 80.



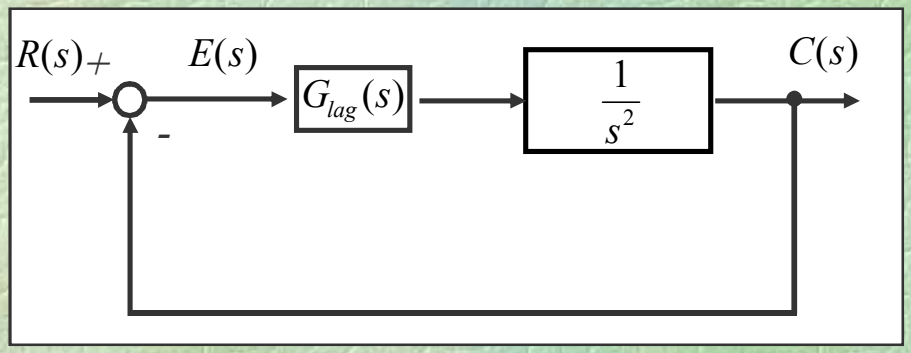
4 In the following system design a Lead controller such that the damping ratio of complex poles be 0.6 and ramp error constant be 80.



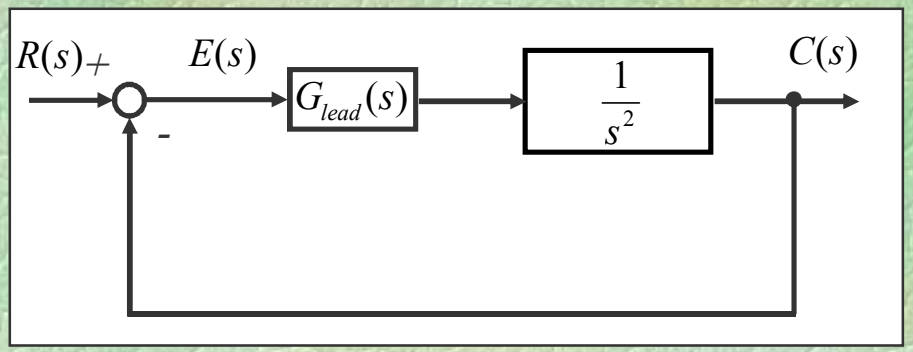
Exercises(Continue)

تمرینها(ادامه)

5 In the following system design a lag controller such that that the settling time be less than 3 sec.



6 In the following system design a lead controller such that that the settling time be less than 3 sec.



Exercises(Continue)

تمرینها(ادامه)

5 In the following system design a Lead controller such that the damping ratio of complex poles be 0.4.(Final 1390)