LINEAR CONTROL SYSTEMS

Ali Karimpour
Associate Professor
Ferdowsi University of Mashhad

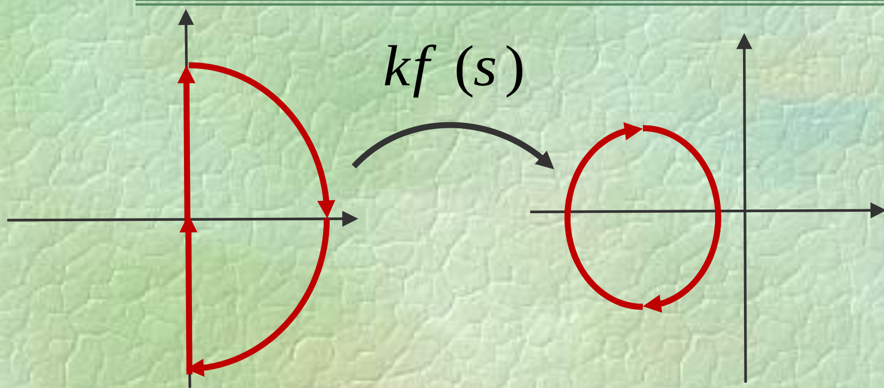
Lecture 19

Nyquist stability criteria (Continue).

Topics to be covered include:

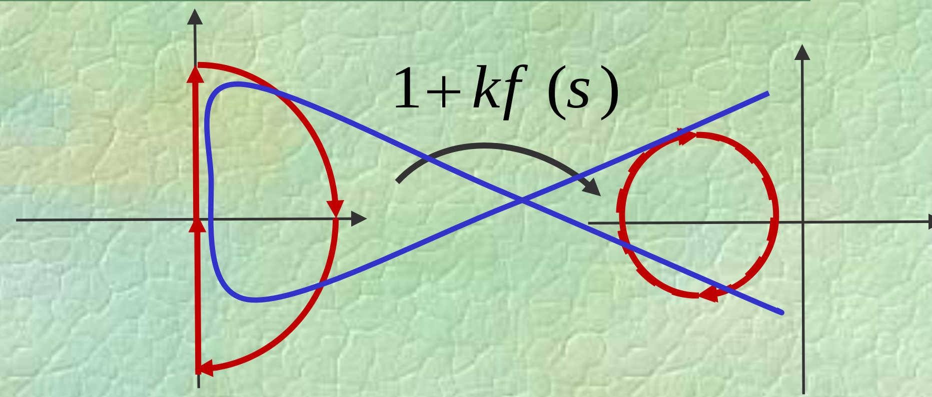
- Nyquist stability criteria (continue).
- Minimum phase systems.
- Simplified Nyquist stability criterion.

Nyquist fundamental



Nyquist path

Nyquist plot



Nyquist path

Nyquist plot

$$Z_0 - P_0 = N_0$$

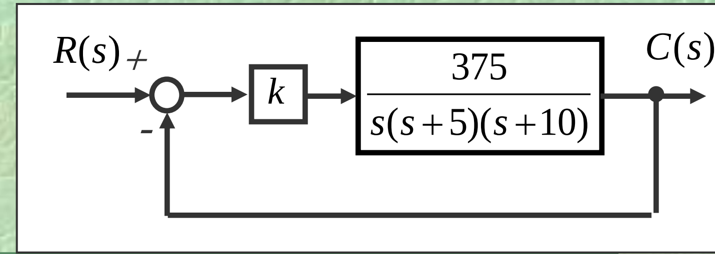
$$Z_{-1} - P_{-1} = N_0 \qquad k>0$$

$$Z_{-1} - P_{-1} = N_{-1} \qquad k>0$$

$$Z_{-1} - P_{-1} = N_1 \qquad k<0$$

Example 1: Check the stability of following system by Nyquist method.

مثال ۱: پایداری سیستم را توسط روش نایکوئیست بررسی کنید.



$$1+\frac{375k}{s(s+5)(s+10)}=0$$

$$f(s)=\frac{375}{s(s+5)(s+10)}$$

$$f(s)=\frac{375}{50s}=\frac{375}{50\varepsilon\angle 0}=\infty\angle 0$$

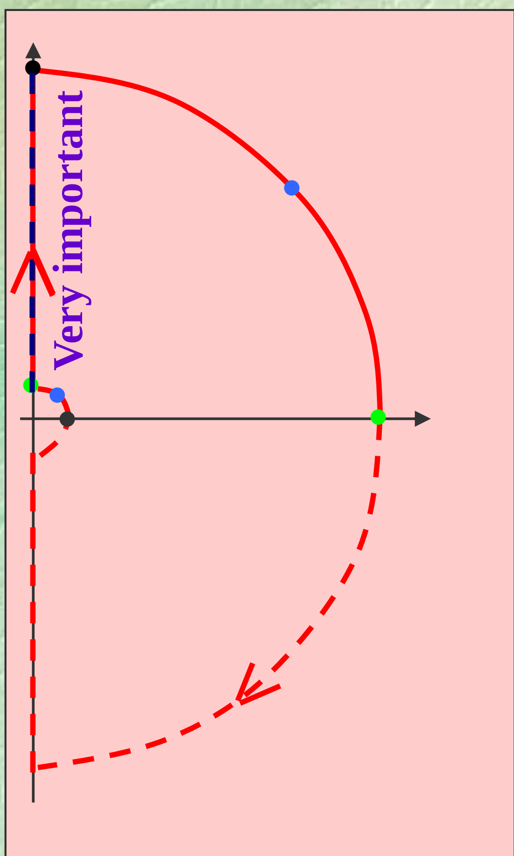
$$f(s)=\frac{375}{50s}=\frac{375}{50\varepsilon\angle 45}=\infty\angle -45$$

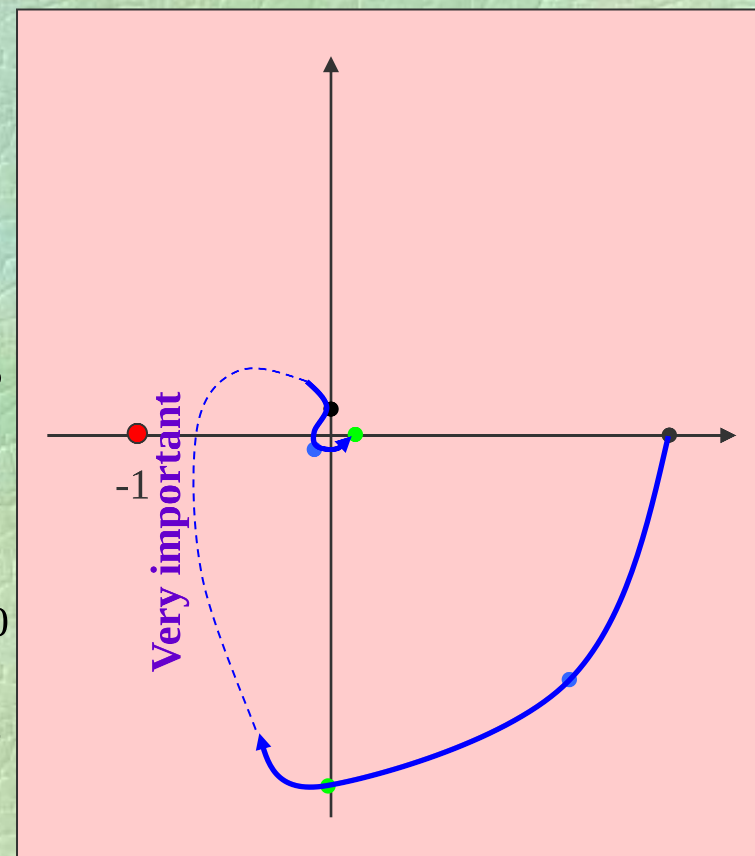
$$f(s)=\frac{375}{50s}=\frac{375}{50\varepsilon\angle 90}=\infty\angle -90$$

$$f(s)=\frac{375}{s^3}=\frac{375}{(\infty\angle 90)^3}=\varepsilon\angle -270$$

$$f(s)=\frac{375}{s^3}=\frac{375}{(\infty\angle 45)^3}=\varepsilon\angle -135$$

$$f(s)=\frac{375}{s^3}=\frac{375}{(\infty\angle 0)^3}=\varepsilon\angle 0$$





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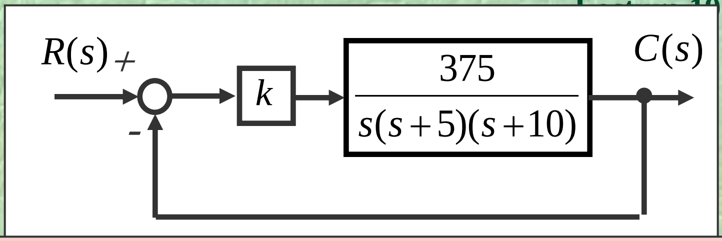
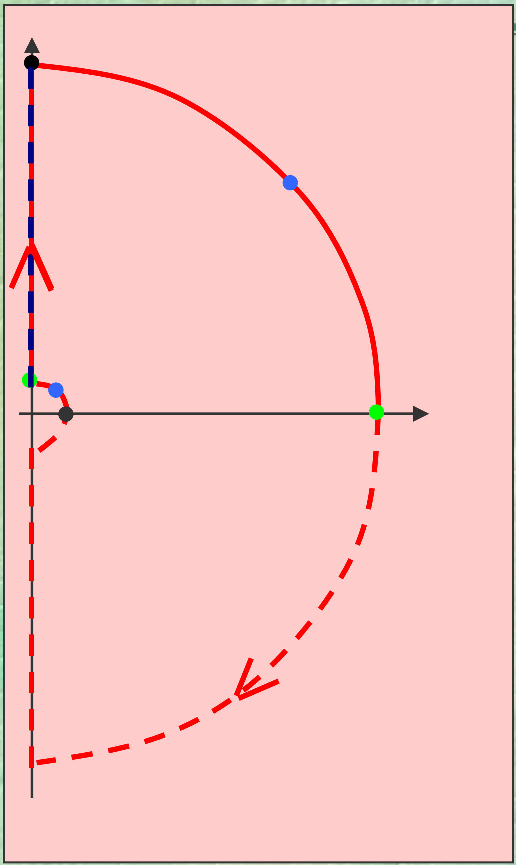


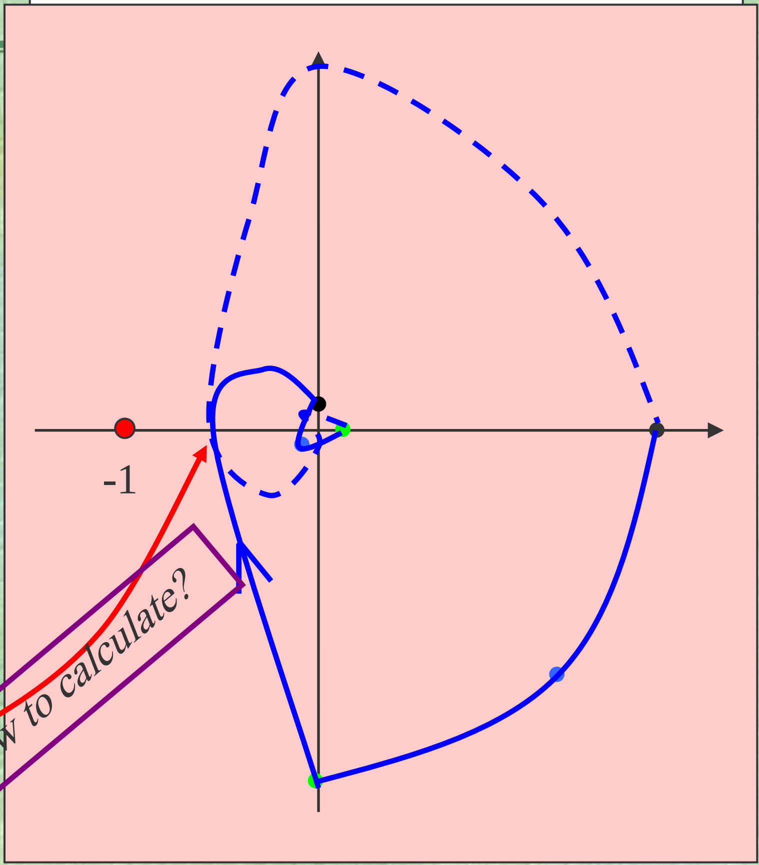
پایداری سیستم را توسط روش نایکوئیست بررسی کنید.



$$1+\frac{375k}{s(s+5)(s+10)}=0$$

$$f(s)=\frac{375}{s(s+5)(s+10)}$$

$$f(j\omega)=\frac{375}{j\omega(j\omega+5)(j\omega+10)}$$

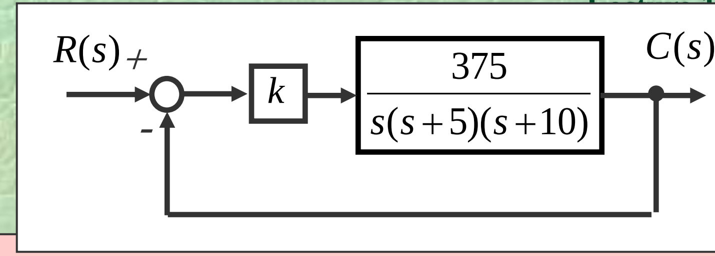


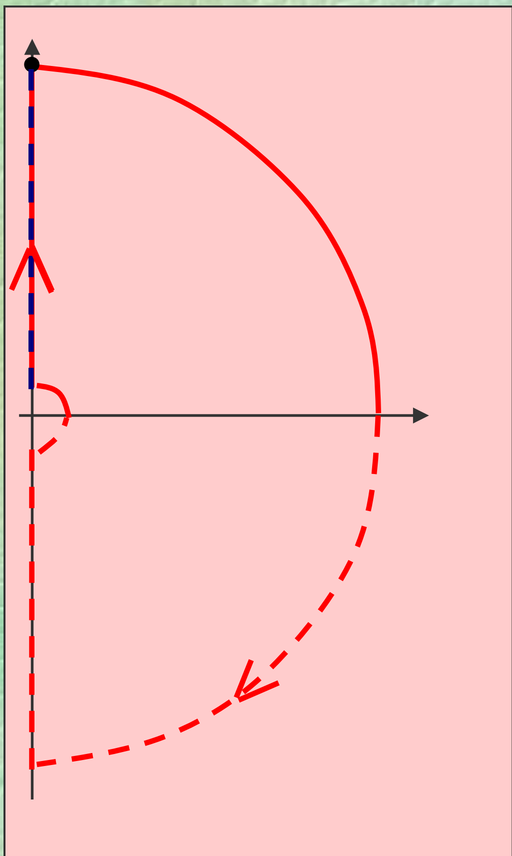
Very important

ω	0	5	7.07	10	20
$f(j\omega)$	$\infty\angle-90$	$0.95\angle-162$	$0.5\angle-180$	$0.24\angle-198$	$0.04\angle-230$

Example 1: Check the stability of following system by Nyquist method.

پایداری سیستم را توسط روش نایکوئیست بررسی کنید.



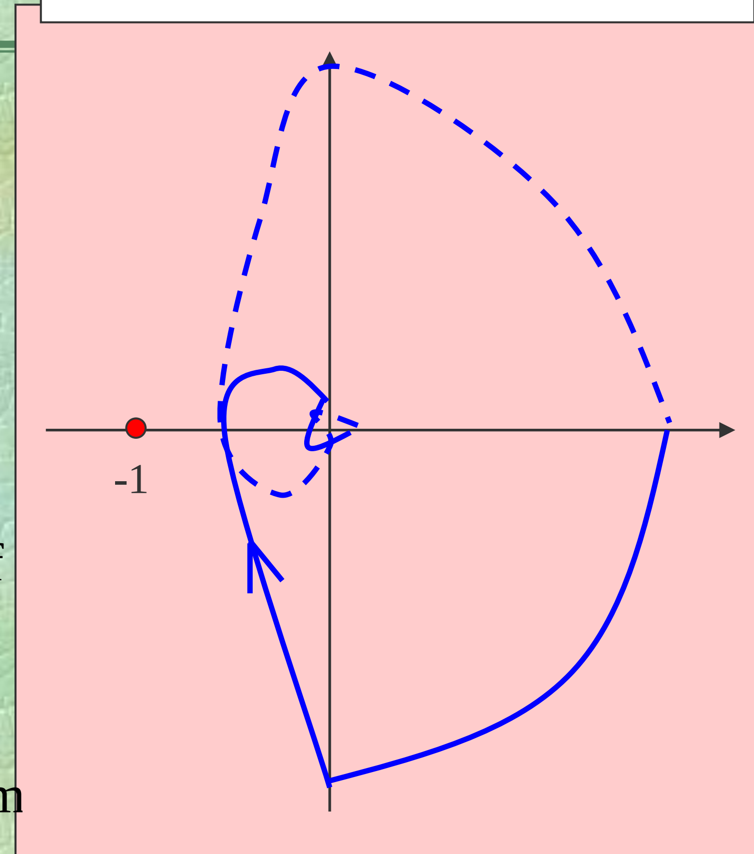


$$f(s) = \frac{375}{s(s+5)(s+10)}$$

Checking the RHP roots of

$$1 + \frac{375k}{s(s+5)(s+10)} = 0$$

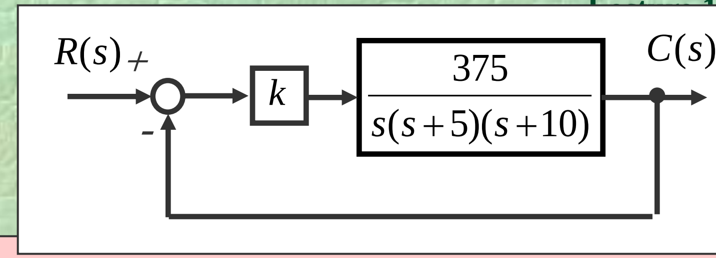
or stability of above system



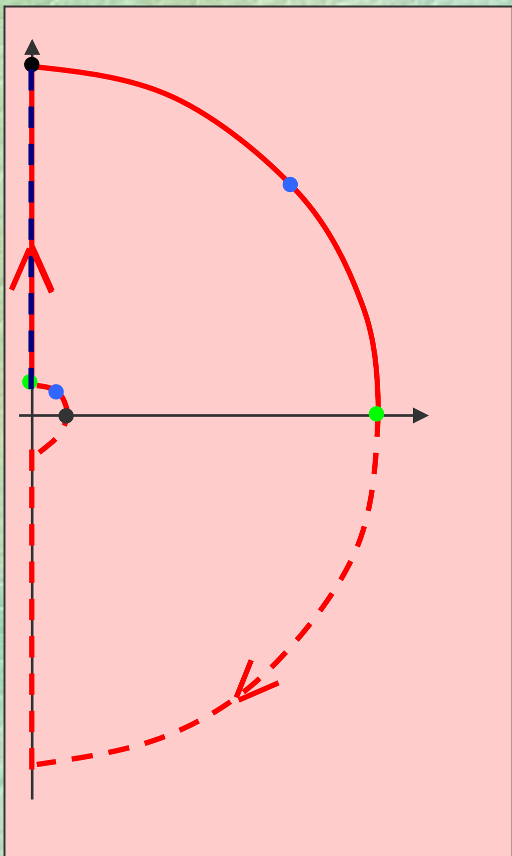
$$k > 0 \quad Z_{-1} - P_{-1} = N_{-1} \qquad Z_{-1} - 0 = N_{-1} \qquad Z_{-1} = N_{-1} = \begin{cases} 0 & \text{Stable for } 0 < k < 2 \\ 2 & \text{Unstable for } k \geq 2 \end{cases}$$

Two RHP roots

Example 1: Check the stability of following system by Nyquist method.



پایداری سیستم را توسط روش نایکوئیست بررسی کنید.



$$f(s)=\frac{375}{s(s+5)(s+10)}$$

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$$1+\frac{375k}{s(s+5)(s+10)}=0$$

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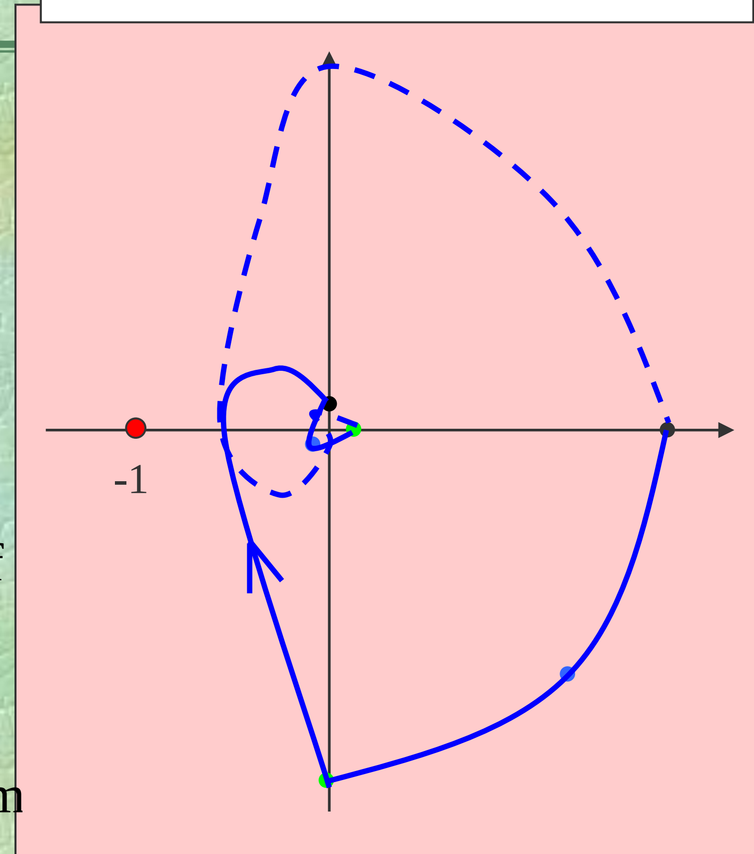


$k\le 0 \qquad Z_{-1}-P_{-1}=N_1 \qquad Z_{-1}-0=N_1 \qquad Z_{-1}=N_1=1+0=1$

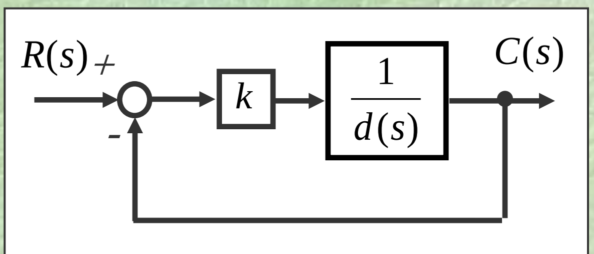
Unstable for $k\le 0$

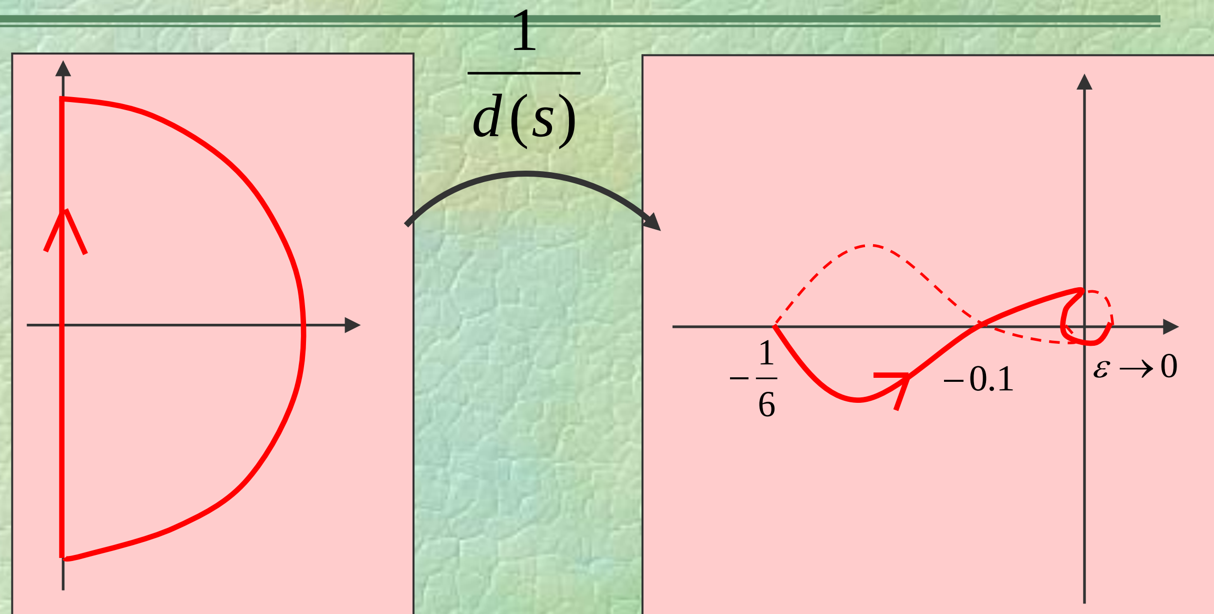
1 *RHP root*

Example 2: Check the stability of following system from the given Nyquist plot.

مثال ۲: پایداری سیستم را با توجه به منحنی نایکوئیست داده شده بررسی کنید.

d(s) is a polynomial.





$Z_0 - P_0 = N_0 \qquad 0 - P_0 = -1 \qquad P_0 = 1 = P_{-1}$

$0 < k < 6 \quad Z_{-1} - P_{-1} = N_{-1} \quad Z_{-1} - 1 = 0 \quad Z_{-1} = 1$ System is unstable(1 RHP zero)

$6 < k < 10 \quad Z_{-1} - P_{-1} = N_{-1} \quad Z_{-1} - 1 = -1 \quad z_{-1} = 0$ System is stable

$k > 10 \quad Z_{-1} - P_{-1} = N_{-1} \quad Z_{-1} - 1 = 1 \quad z_{-1} = 2$ System is unstable(2 RHP zero)

$k < 0 \quad Z_{-1} - P_{-1} = N_1 \quad Z_{-1} - 1 = 0 \quad z_{-1} = 1$ System is unstable(1 RHP zero)

Example 2: Check the stability of following system from the given Nyquist plot.

مثال ۲: پایداری سیستم را با توجه به منحنی نایکوئیست داده شده بررسی کنید.

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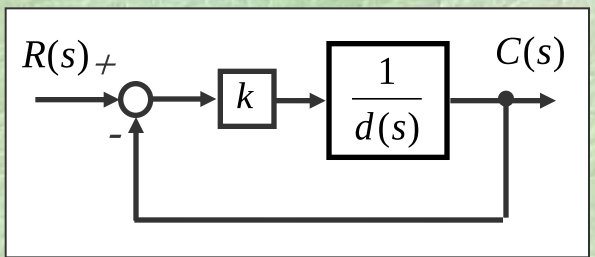
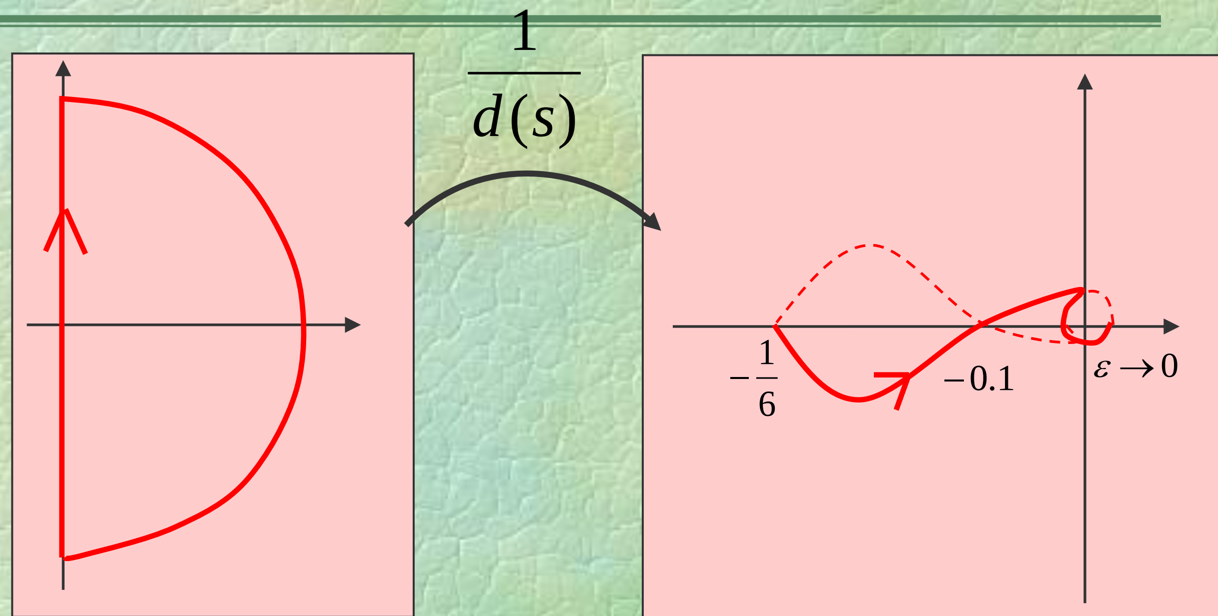




$$Z_0 - P_0 = N_0 \qquad 0 - P_0 = -1 \qquad P_0 = 1 = P_{-1}$$

$$Z_{-1} - P_{-1} = N_{-1} \quad = N_{-1} + P_{-1} = \begin{cases} 0+1 & 0 < k < 6 \\ -1+1 & 6 < k < 10 \\ 1+1 & k > 10 \end{cases}$$

System is unstable(1 RHP zero)

System is stable

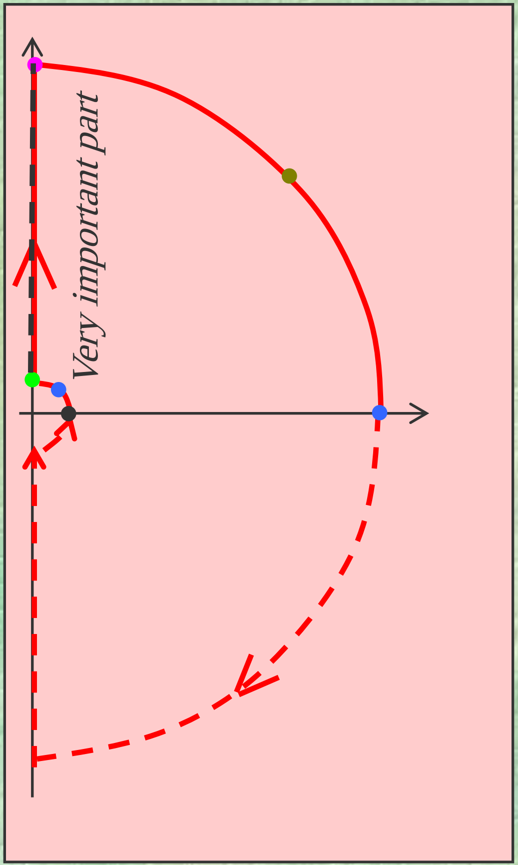
System is unstable(2 RHP zero)

$$k < 0 \qquad Z_{-1} - P_{-1} = N_1 \qquad Z_{-1} - 1 = 0 \qquad z_{-1} = 1$$

System is unstable(1 RHP zero)

Example 3: Discuss about the RHP roots of following system.

$$1+k\frac{2(s-1)}{s(s+1)}=0$$ مثال ۳: در مورد ریشه های سمت راست معادله روبرو بر حسب مقادیر مختلف k بحث کنید.



$$f(s)=\frac{2(s-1)}{s(s+1)}$$

$$f(s)=\frac{-2}{s}=\frac{-2}{\varepsilon\angle 0}=\infty\angle -180$$

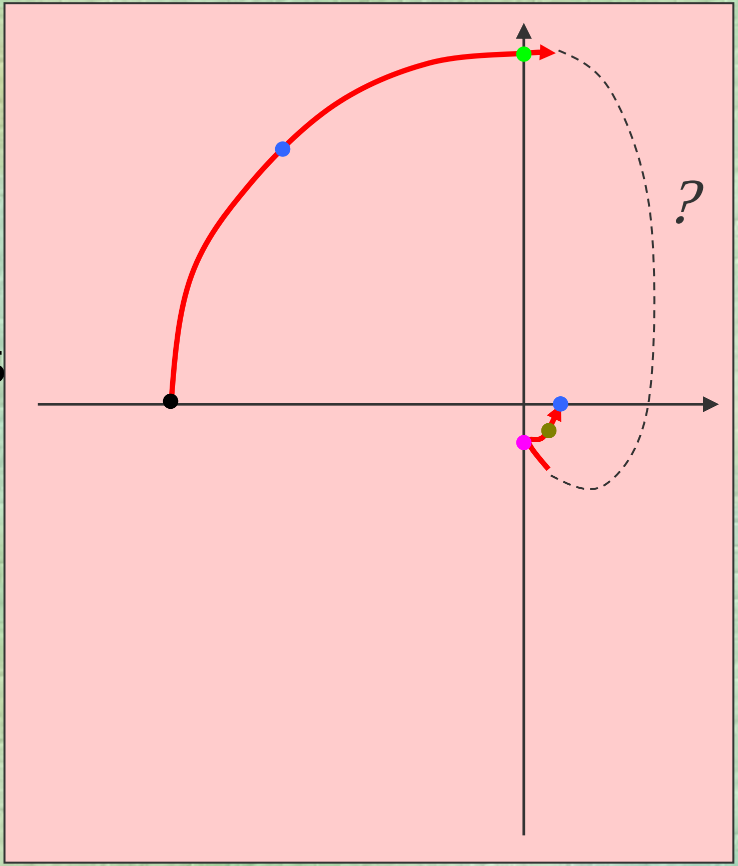
$$f(s)=\frac{-2}{s}=\frac{-2}{\varepsilon\angle 45}=\infty\angle -225$$

$$f(s)=\frac{-2}{s}=\frac{-2}{\varepsilon\angle 90}=\infty\angle -270$$

$$f(s)=\frac{2}{s}=\frac{2}{\infty\angle 90}=\varepsilon\angle -90$$

$$f(s)=\frac{2}{s}=\frac{2}{\infty\angle 45}=\varepsilon\angle -45$$

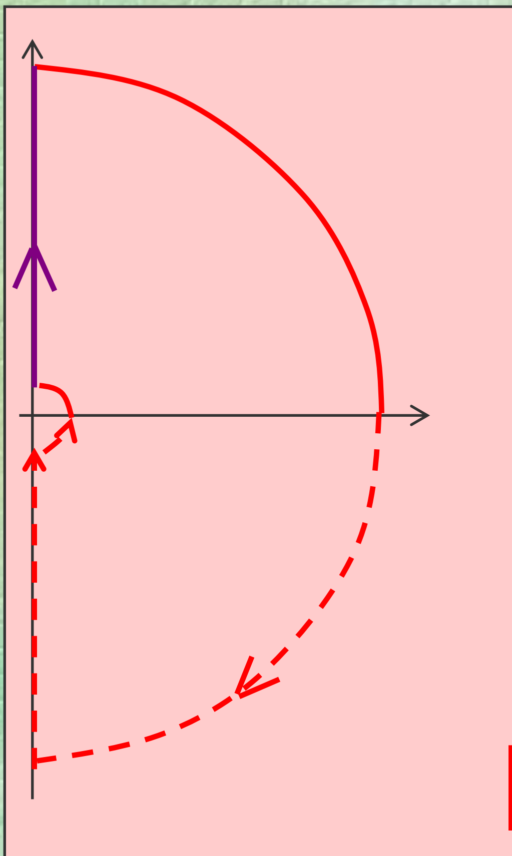
$$f(s)=\frac{2}{s}=\frac{2}{\infty\angle 0}=\varepsilon\angle 0$$



Example 3: Discuss about the RHP roots of the following system.

$$1+k\frac{2(s-1)}{s(s+1)}=0$$

مثال ۳: در مورد ریشه های سمت راست معادله روبرو بر حسب مقادیر مختلف k بحث کنید.

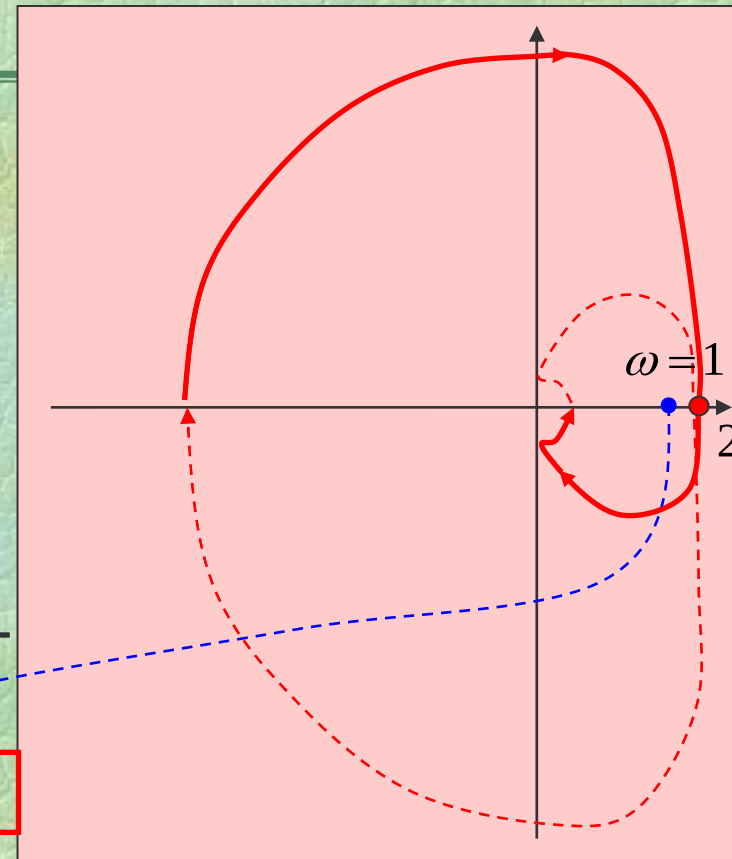


$$f(s)=\frac{2(s-1)}{s(s+1)}$$

$$f(j\omega)=\frac{2(j\omega-1)}{j\omega(j\omega+1)}$$

ω	? 1
$f(j\omega)$	? 2

Which point is very important?



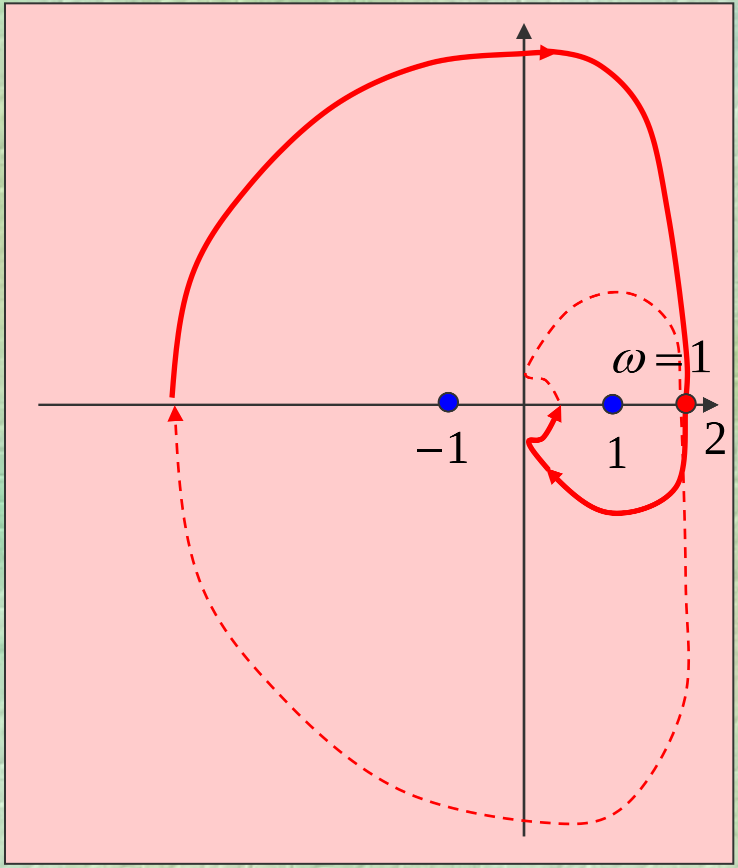
$$\angle f(j\omega)=\angle num-\angle den=\quad 180-\tan^{-1}\omega-(90+\tan^{-1}\omega)\;=90-2\cdot\tan^{-1}\omega$$

$\angle f(j\omega)=0 \quad\rightarrow\quad \omega=1$ Or equivalently let Im(f) = 0 $\quad f(j1)=\dfrac{2(1j-1)}{j(j+1)}=2\angle 0^\circ$

Example 3: Discuss about the RHP roots of the following system for different values of k.

$$1+k\frac{2(s-1)}{s(s+1)}=0$$

مثال ۳: در مورد ریشه های سمت راست معادله روبرو بر حسب مقادیر مختلف k بحث کنید.



$k>0 \quad Z_{-1}-P_{-1}=N_{-1} \qquad Z_{-1}-0=1$

$\Rightarrow \quad Z_{-1}=1$ Unstable(one RHP root)

$k<0 \quad Z_{-1}-P_{-1}=N_{1} \qquad Z_{-1}=N_{1}$

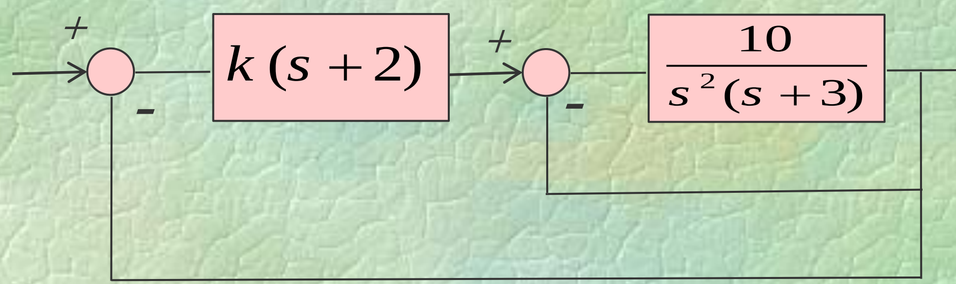
$$\Rightarrow \quad Z_{-1}=\begin{cases}0 & -0.5<k<0 \quad \text{Stable} \\ 2 & k<-0.5 \quad \text{Unstable (two RHP root)}\end{cases}$$

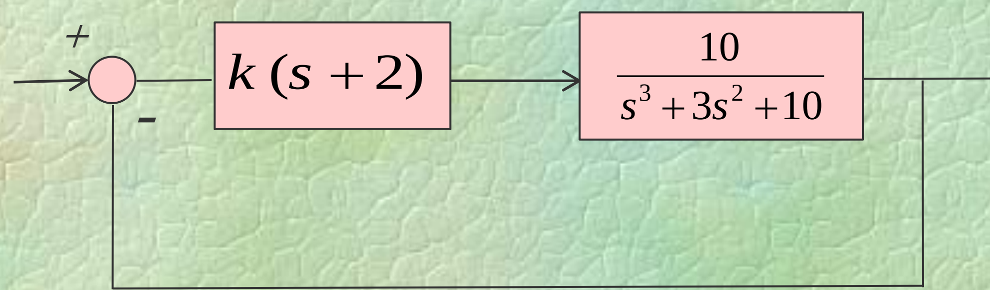
More Study:

$Z_0-P_0=N_0 \qquad 1-0=1$

Example 4: Discuss about the stability of the following system for different values of k.

مثال ۴: پایداری سیستم را بر حسب مقادیر مختلف k بحث کنید.

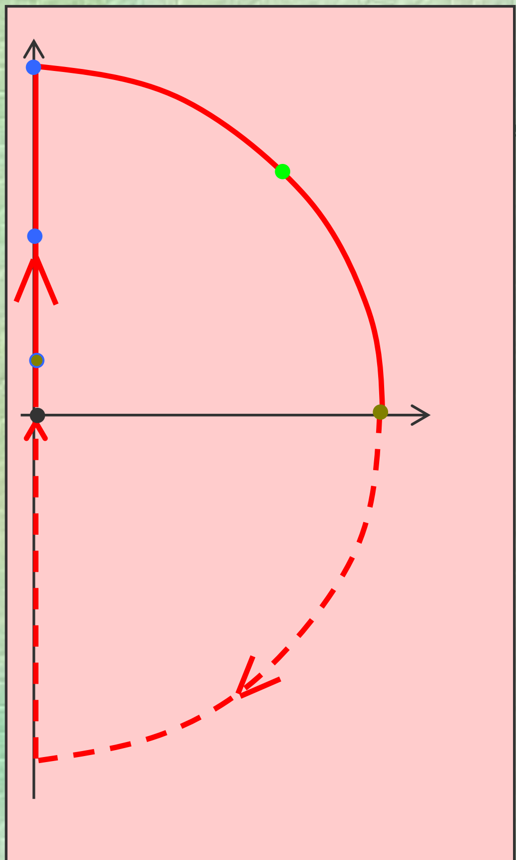




فرم استانداردمعادله به صورت زیر است:

$$1 + k\,\frac{10(s+2)}{s^3 + 3s^2 + 10} = 0$$

Discuss about the RHP roots of: $$1+k\frac{10(s+2)}{s^3+3s^2+10}=0$$



$$f(s)=\frac{10(s+2)}{s^3+3s^2+10}$$

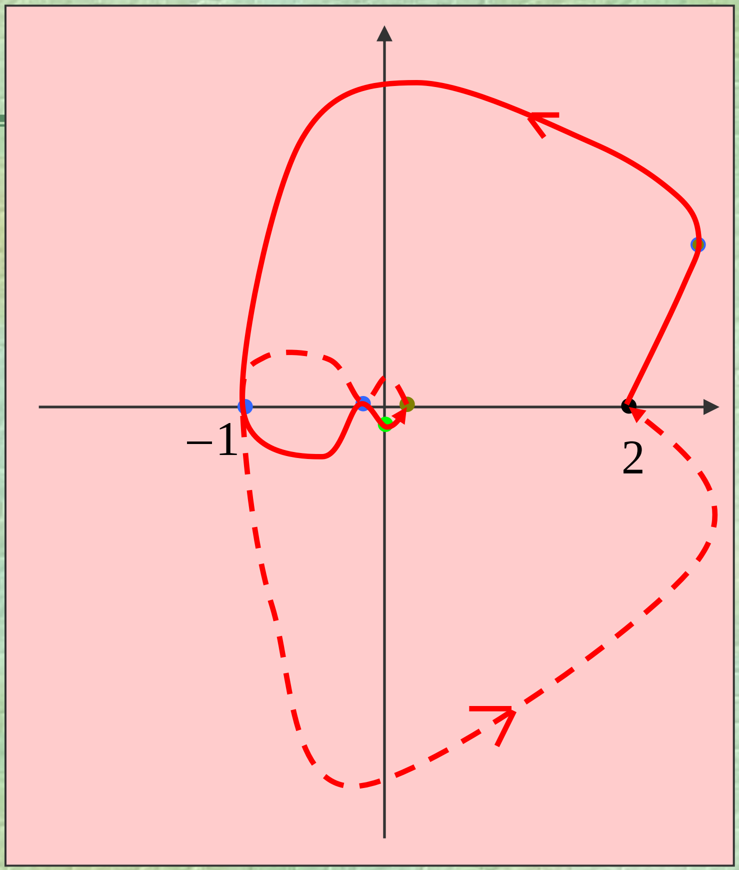
$$f(s)=\frac{10(0+2)}{0+0+10}=2$$

$$f(s)=\frac{10}{s^2}=\frac{10}{(\infty\angle 90)^2}=\varepsilon\angle -180$$

$$f(s)=\frac{10}{s^2}=\frac{10}{(\infty\angle 45)^2}=\varepsilon\angle -90$$

$$f(s)=\frac{10}{s^2}=\frac{10}{(\infty\angle 0)^2}=\varepsilon\angle 0$$

$$f(j\omega)=\frac{10(j\omega+2)}{(10-3\omega^2)-j\omega^3}$$



$$f(j\omega)=\frac{10(j\omega+2)}{(10-3\omega^2)-j\omega^3}\,\frac{(10-3\omega^2)+j\omega^3}{(10-3\omega^2)+j\omega^3}$$

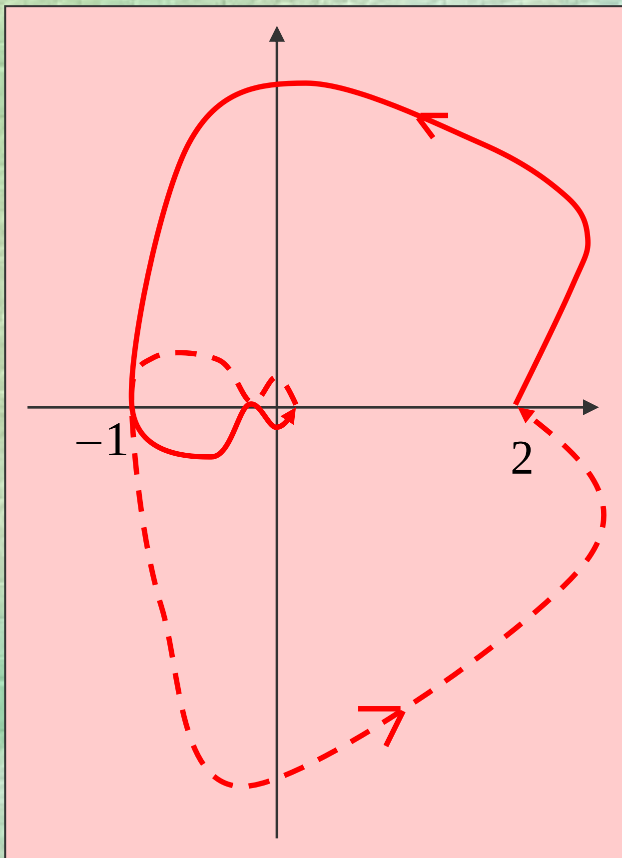
$$\mathrm{Im}(f)=\frac{10\omega(10-3\omega^2)+20\omega^3}{(10-3\omega^2)^2+\omega^6}=0 \qquad \omega=0,\pm\sqrt{10}$$

$$f(j\sqrt{10})=\frac{10(j\sqrt{10}+2)}{(10-30)-j10\sqrt{10}}=1\angle 180^\circ$$

How?

$$f(j1)=\frac{10(j1+2)}{(10-3)-j1}=\frac{20+10j}{7-j1}=2.6+1.8j$$

Discuss about the RHP roots of: $$1+k\frac{10(s+2)}{s^3+3s^2+10}=0$$



$$P_{-1}=P_0=?$$

$$Z_0-P_0=N_0 \qquad 0-P_0=-2 \qquad P_{-1}=P_0=2$$

$k>0$ $\qquad Z_{-1}-P_{-1}=N_{-1} \qquad Z_{-1}-2=N_{-1}$

$$Z_{-1}=N_{-1}+2 = \begin{cases} -2+2=0 & k>1 \\ 0+2=2 & 0<k<1 \end{cases}$$

$k>1$: Stable

$0<k<1$: Unstable, 2 RHP roots

$k<0$ $\qquad Z_{-1}-P_{-1}=N_1 \qquad Z_{-1}-2=N_1 \qquad Z_{-1}=N_1+2$

$$Z_{-1}=N_1+2 = \begin{cases} 0+2=2 & -0.5<k<0 \\ -1+2=1 & k<-0.5 \end{cases}$$

$-0.5<k<0$: Unstable (2 RHP roots)

$k<-0.5$: Unstable (1 RHP roots)

Minimum Phase systems

توابع حداقل فاز

f(s) is said to be minimum phase if it has no poles and zeros on the RHP and on the jω axis (origin is an exception) and there is no delay.

تابع f(s) را حداقل فاز گویند اگر این تابع هیچ قطب و صفری در RHP و روی محور jω نداشته (مبدا استثنا است) و دارای تاخیر نباشد.

$$f(s)=\frac{\prod_{i=1}^{n_z}(s+z_i)}{s^{T_y}\prod_{j=1}^{n_p}(s+p_j)}$$

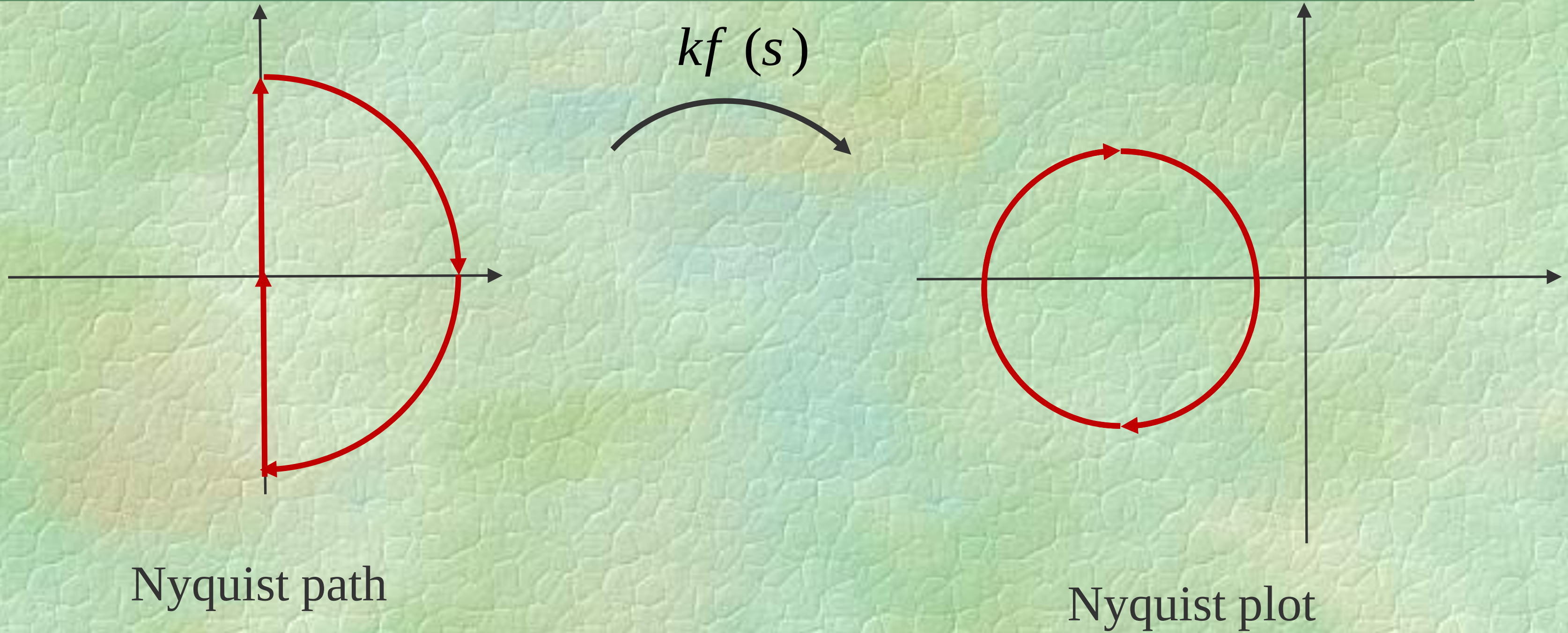
If it was minimum phase

$$z_i\ ,\ p_j>0$$

T_y is type of system

Important note: If f(s) is minimum phase then $Z_0=P_{-1}=P_0=0$

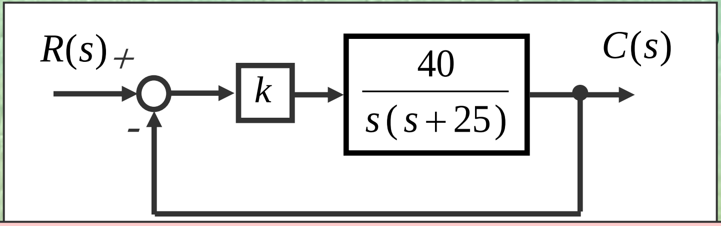
Nyquist fundamental for minimum phase systems

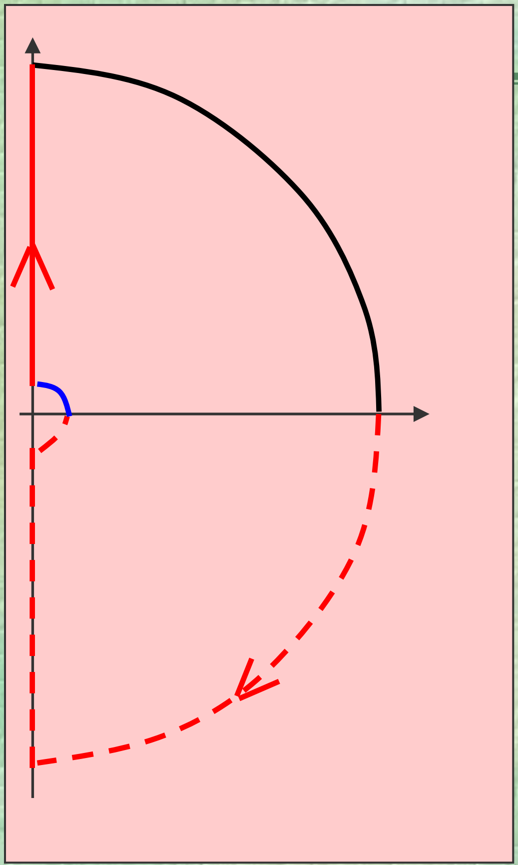


$k > 0 \qquad Z_{-1} - P_{-1} = N_{-1} \qquad Z_{-1} = N_{-1}$

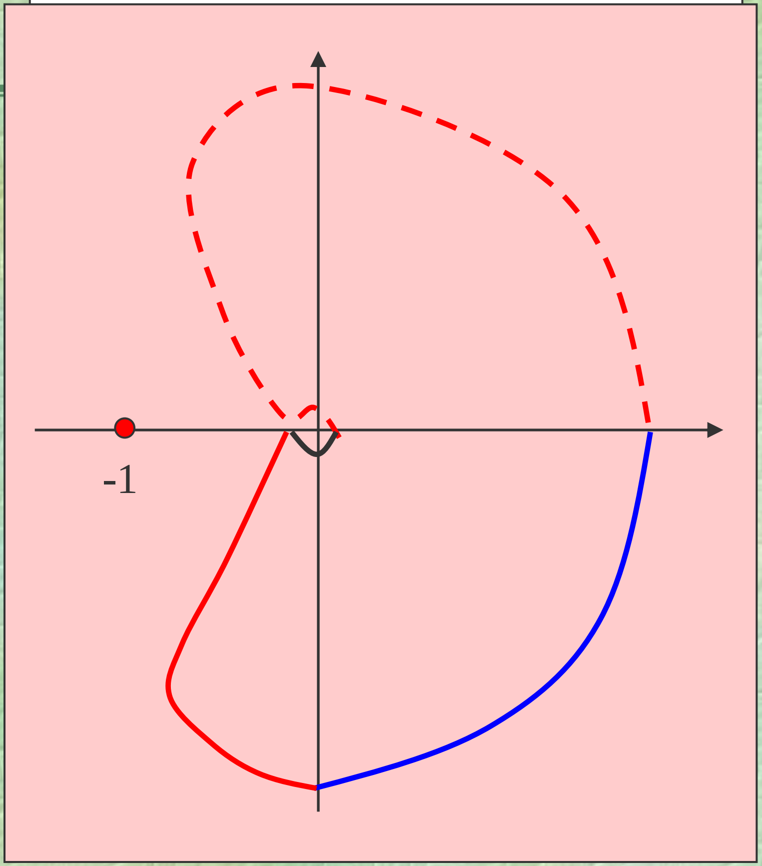
$k < 0 \qquad Z_{-1} - P_{-1} = N_{1} \qquad Z_{-1} = N_{1}$

Check the stability of following system by Nyquist method.





$$f(s)=\frac{40}{s(s+25)}$$



System is minimum phase

$k>0$ $$Z_{-1}=N_{-1}=\frac{2(90T_y+\varphi_{-1})}{360^\circ}=\frac{2(90^\circ-90^\circ)}{360^\circ}=0$$

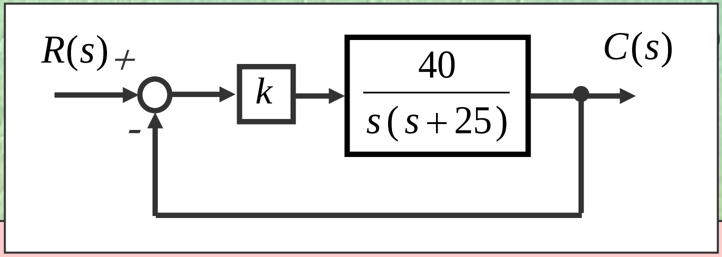
Stable for $k>0$

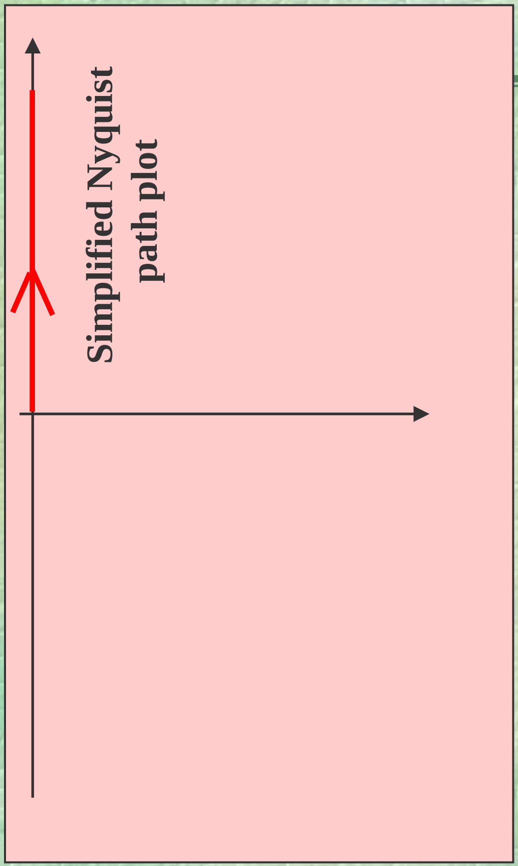
$k<0$ $$Z_{-1}=N_{1}=\frac{2(90T_y+\varphi_{1})}{360^\circ}=\frac{2(90^\circ+90^\circ)}{360^\circ}=1$$

Unstable for $k\le 0$

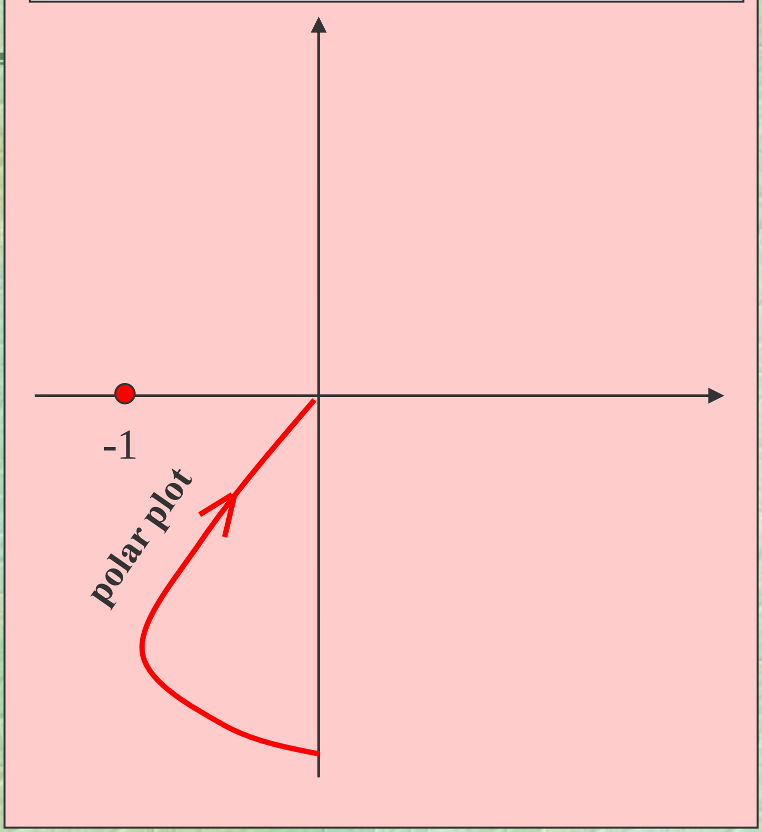
One RHP zero

Check the stability of following system by Nyquist method.





$$f(s)=\frac{40}{s(s+25)}$$



System is minimum phase

$k>0$ $\quad Z_{-1}=N_{-1}=\dfrac{2(90T_y+\varphi_{-1})}{360^\circ}$

φ_{-1} is the angle of polar plot around -1

$k<0$ $\quad Z_{-1}=N_{1}=\dfrac{2(90T_y+\varphi_{1})}{360^\circ}$

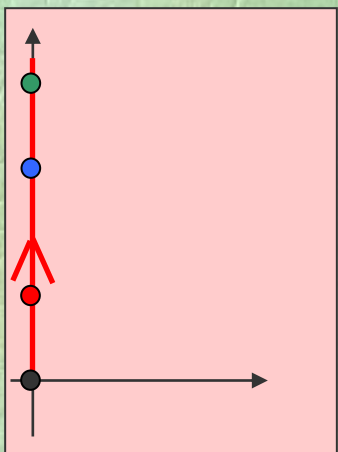
φ_{1} is the angle of polar plot around 1

Example 5: Discuss about the RHP roots of the following system for different values of k.

مثال ۵: در مورد ریشه های RHP سیستم زیر بر حسب مقادیر مختلف k بحث کنید.

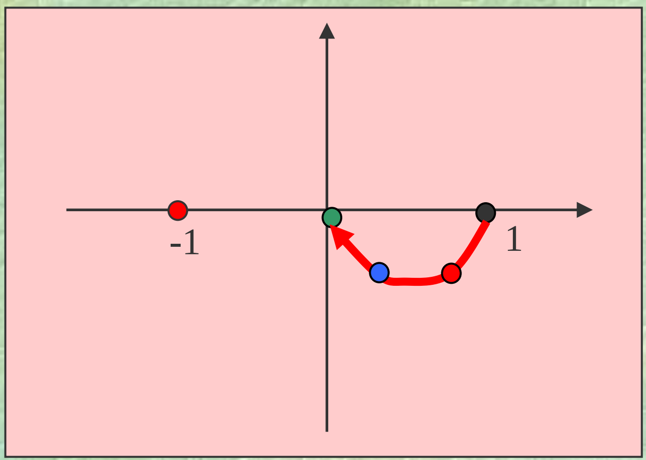
$$1 + k\frac{1}{1+\tau s} = 0 \qquad \tau > 0$$

Clearly System is minimum phase so we use simplified Nyquist method



Simplified Nyquist path plot

$$f(s) = \frac{1}{1+\tau s}$$



Polar plot

$$k > 0 \qquad Z_{-1} = N_{-1} = \frac{2(90T_y + \varphi_{-1})}{360^\circ} = \frac{2(0+0)}{360^\circ} = 0$$

No RHP root.

$$k < 0 \qquad Z_{-1} = N_1 = \frac{2(90T_y + \varphi_1)}{360^\circ} = \frac{\varphi_1}{180^\circ} = \begin{cases} 180/180 = 1 & k < -1 \\ 0/180 = 0 & -1 < k < 0 \end{cases}$$

$k < -1$: **One RHP root.**

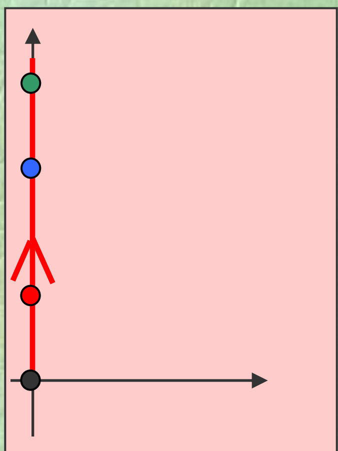
$-1 < k < 0$: **No RHP root.**

Example 7: Discuss about the RHP roots of the following system for different values of k.

مثال ۶: در مورد ریشه های RHP سیستم زیر بر حسب مقادیر مختلف k بحث کنید.

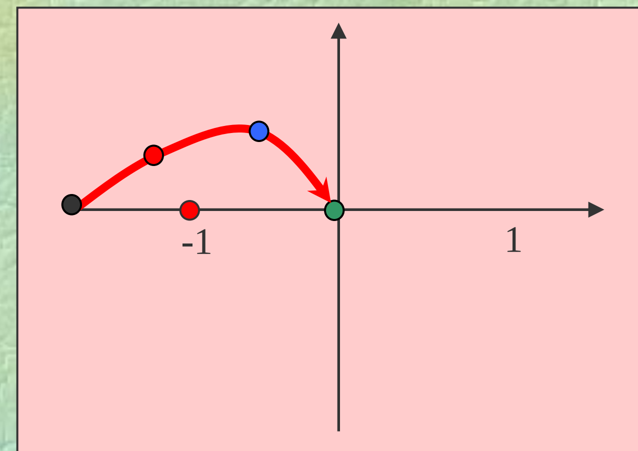
$$1+k\frac{1}{s^2(1+\tau s)}=0 \qquad \tau>0$$

Clearly System is minimum phase so we use simplified Nyquist method



Simplified Nyquist path plot

$$f(s)=\frac{1}{s^2(1+\tau s)}$$

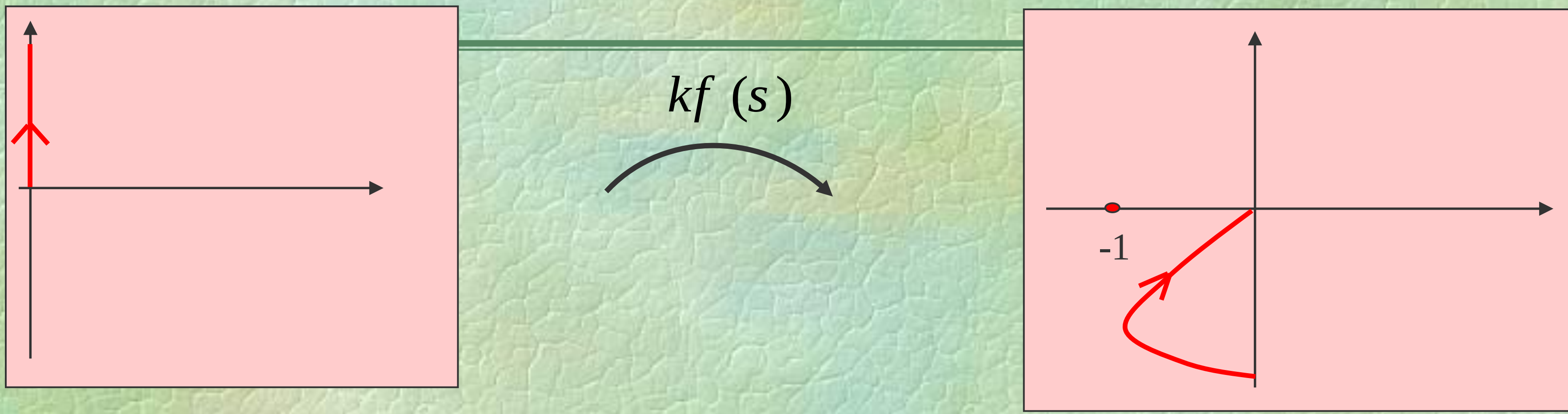


Polar plot

$$k>0 \qquad Z_{-1}=N_{-1}=\frac{2(90T_y+\varphi_{-1})}{360^\circ}=\frac{2(180+180)}{360^\circ}=2$$ Two RHP roots.

$$k<0 \qquad Z_{-1}=N_{1}=\frac{2(90T_y+\varphi_{1})}{360^\circ}=\frac{2(180+0)}{360^\circ}=1$$ One RHP root.

Simplified Nyquist method



Simplified Nyquist path plot

System is minimum phase

Polar plot

$k>0$ $\quad Z_{-1}=N_{-1}=\dfrac{2(90T_y+\varphi_{-1})}{360^\circ}$

φ_{-1} is the angle of polar plot around -1

$k<0$ $\quad Z_{-1}=N_{1}=\dfrac{2(90T_y+\varphi_{1})}{360^\circ}$

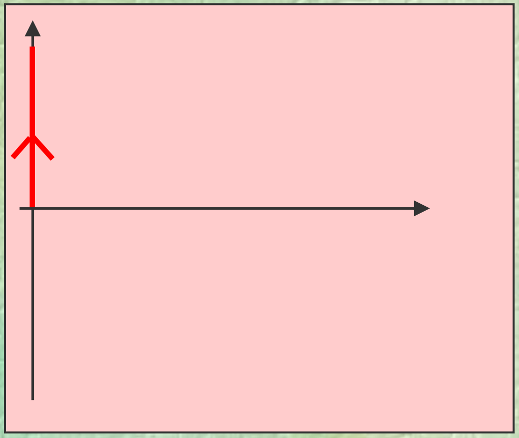
φ_{1} is the angle of polar plot around 1

Important remark: If any of φ_{-1} or φ_1 is greater than zero the system is unstable but if they were less than zero one must check it!

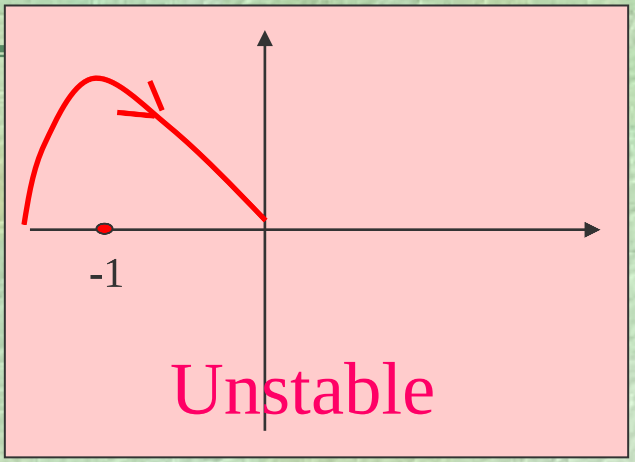
Simplified Nyquist method

$kf(s)$

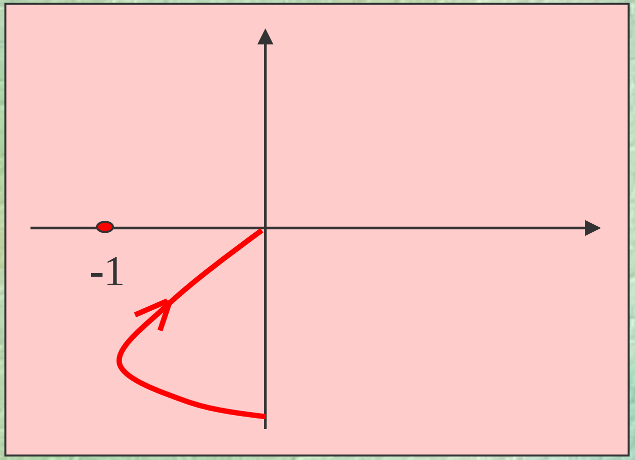
$kf(s)$



Simplified Nyquist path plot



Polar plot



Polar plot

Stability depends on T_y

Exercises

تمرينها

1- The open loop transfer function of a unity-feedback (negative sign) is:

$$G_p(s) = \frac{k}{(s+5)^n}$$

Apply the Nyquist criterion to determine the range of k for stability. Let n=1,2,3 and 4

2- The characteristic equation of a linear control system is:

$$s^3 + 2s^2 + 20s + 10k = 0$$

Apply the Nyquist criterion to determine the range of k for stability.

Exercises

تمرينها

3- The open loop transfer function of a unity-feedback (negative sign) with PD controller is:

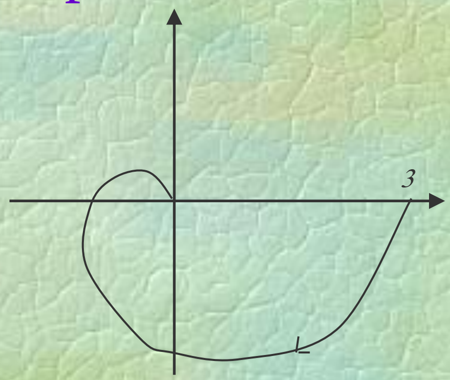
$$G_p(s) = \frac{10(K_p + K_d s)}{s^2}$$

Select the value of K_p so that the parabolic error constant be 100. Find the equivalent open-loop transfer function $G_{eq}(s)$ for stability analysis with K_d as a gain factor. Sketch the Nyquist plot and check the stability for different values of K_d.

Exercises

تمرینها

4- The polar plot of an open loop transfer function of a minimum phase system is:



Determine the steady state error of the system to a unit step. $e_{ss}=\frac{1}{4}$:جواب

5- The open loop transfer function of a unity-feedback (negative sign) is:

$$G(s)=\frac{ke^{-Ts}}{s+1} \qquad (k>1)$$

Derive an expression that make the system stable.

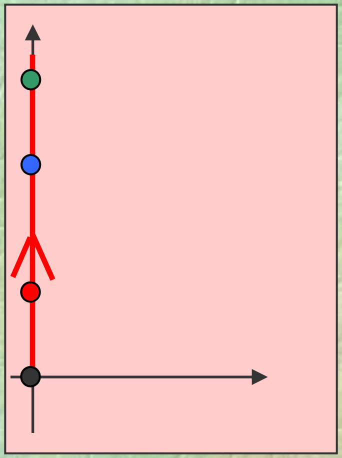
$[T\sqrt{k^2-1}+\tan^{-1}\sqrt{k^2-1}]<\Pi$:جواب

Example 6: Discuss about the RHP roots of the following system for different values of k.

مثال 7: در مورد ریشه های RHP سیستم زیر بر حسب مقادیر مختلف k بحث کنید.

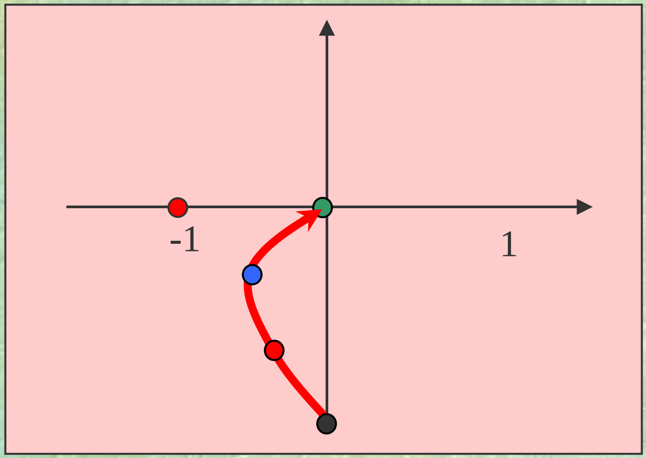
$$1+k\frac{1}{s(1+\tau s)}=0 \qquad \tau>0$$

Clearly System is minimum phase so we use simplified Nyquist method



Simplified Nyquist path plot

$$f(s)=\frac{1}{s(1+\tau s)}$$



Polar plot

$$k>0 \qquad Z_{-1}=N_{-1}=\frac{2(90T_y+\varphi_{-1})}{360^\circ}=\frac{2(90-90)}{360^\circ}=0$$

No RHP root.

$$k<0 \qquad Z_{-1}=N_{1}=\frac{2(90T_y+\varphi_{1})}{360^\circ}=\frac{2(90+90)}{360^\circ}=1$$

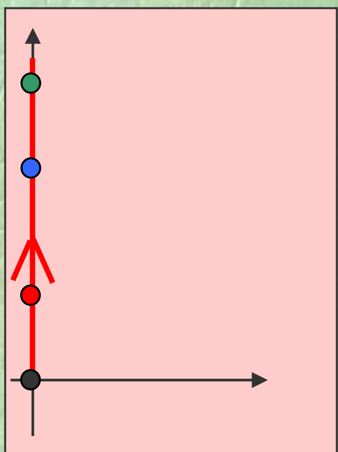
One RHP root.

Example 8: Discuss about the RHP roots of following system for different value of k.

مثال ۸: در مورد ریشه های RHP سیستم زیر بر حسب مقادیر مختلف k بحث کنید.

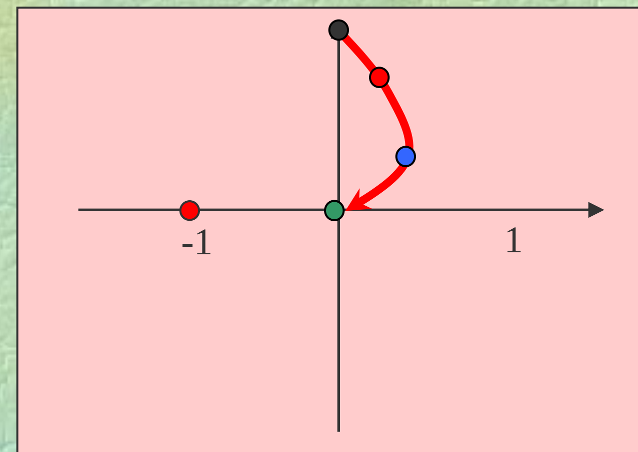
$$1 + k\frac{1}{s^3(1+\tau s)} = 0 \qquad \tau > 0$$

Clearly System is minimum phase so we use simplified Nyquist method



Simplified Nyquist path plot

$$f(s) = \frac{1}{s^3(1+\tau s)}$$



Polar plot

$$k > 0 \qquad Z_{-1} = N_{-1} = \frac{2(90T_y + \varphi_{-1})}{360^\circ} = \frac{2(270+90)}{360^\circ} = 2$$ Two RHP roots.

$$k < 0 \qquad Z_{-1} = N_{1} = \frac{2(90T_y + \varphi_{1})}{360^\circ} = \frac{2(270-90)}{360^\circ} = 1$$ One RHP root.