LINEAR CONTROL SYSTEMS

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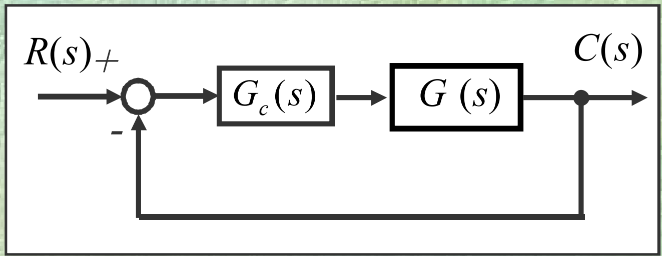
Lecture 21

Frequency domain charts

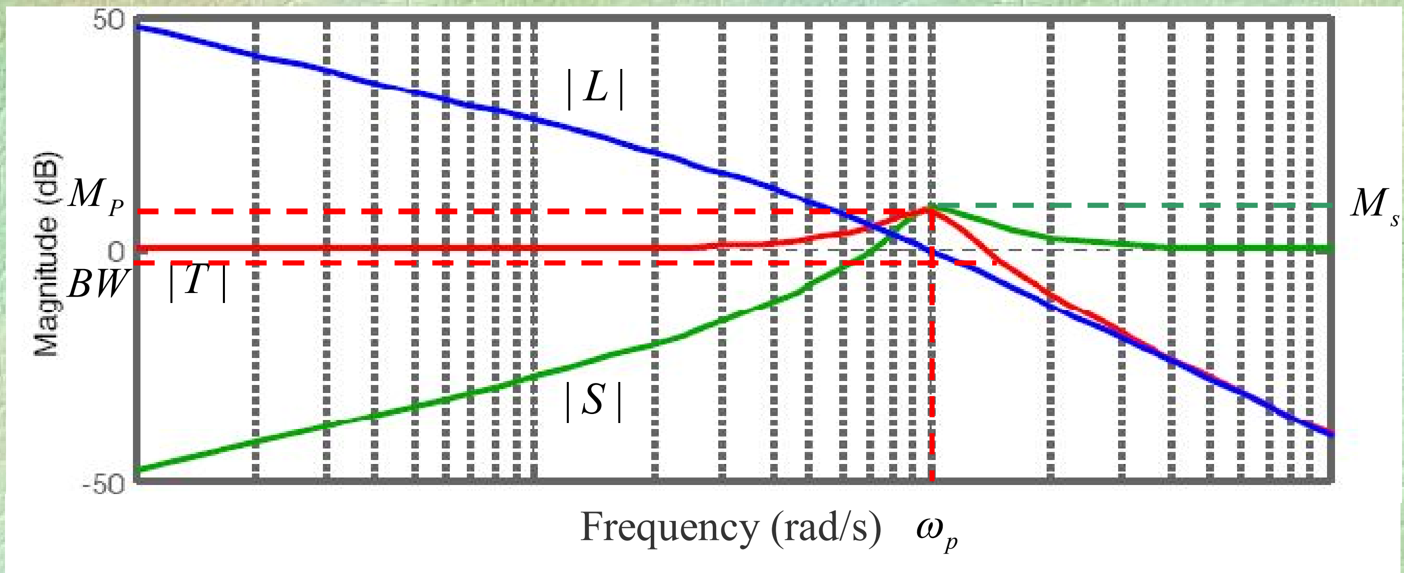
Topics to be covered include:

- Nyquist chart.
  - Constant M loci.
  - Constant N loci.
- Nichols chart.
  - Constant gain loci.
  - Constant phase loci.
  - Nichols chart specification.
- Effect of adding poles and zeros on loop transfer function.

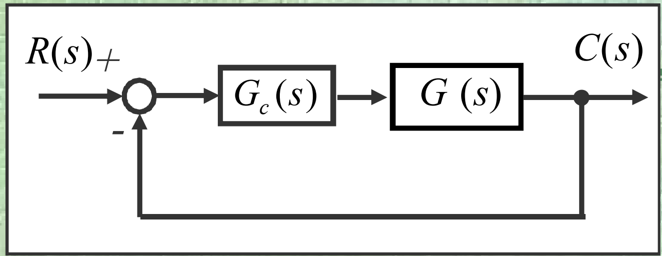
Closed loop transfer functions



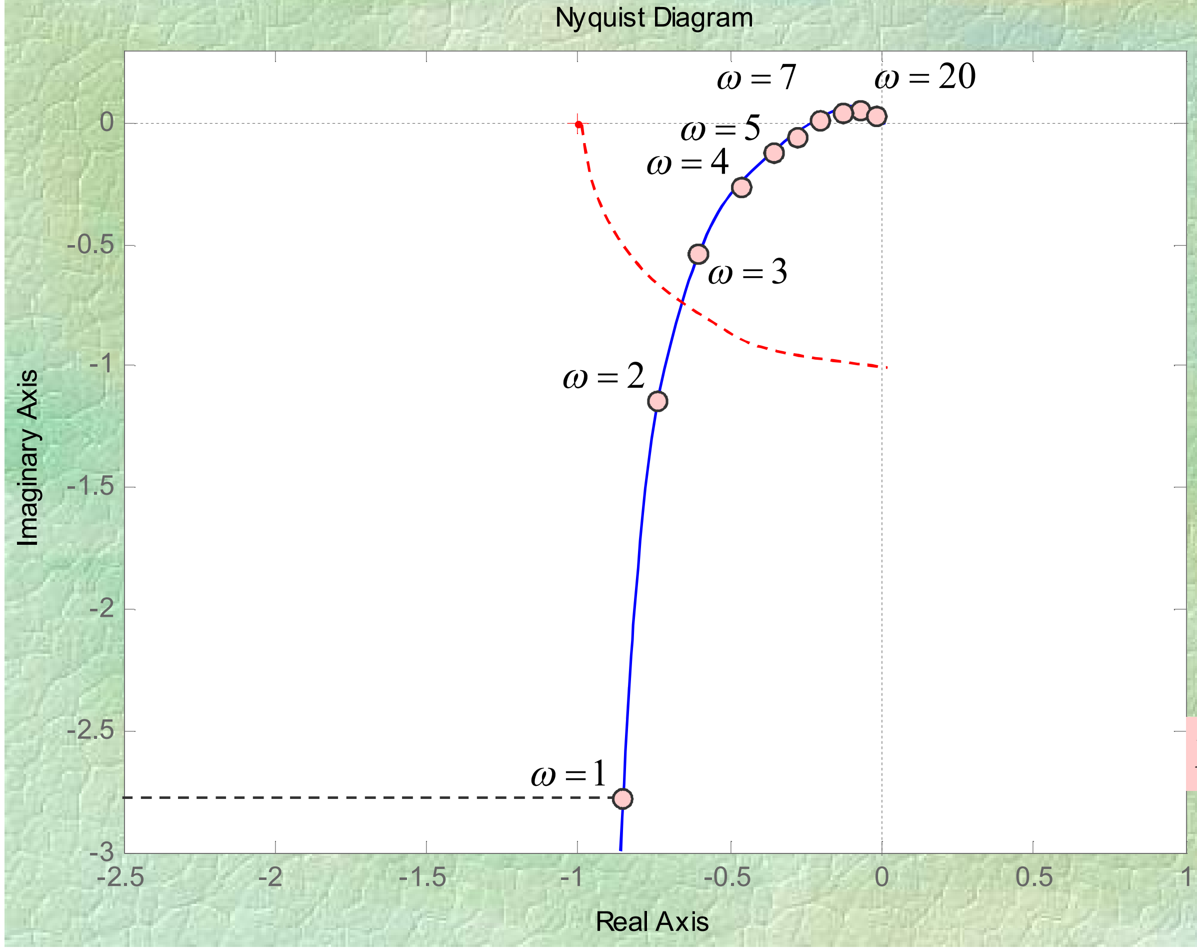
$$L(s) = G_c(s)G(s) \qquad T(s) = \frac{G_c(s)G(s)}{1+G_c(s)G(s)} = \frac{L(s)}{1+L(s)} \qquad S(s) = \frac{1}{1+G_c(s)G(s)} = \frac{1}{1+L(s)}$$



Closed loop values from Nyquist chart



$$\text{Let}\quad G_c(s)G(s)=\frac{150}{s(s+5)(s+10)}$$



$$T(j1)=?\qquad \omega=1$$

$$\Rightarrow\quad G(j1)G_c(j1)=-0.86-2.8j$$

$$T(j1)=?$$

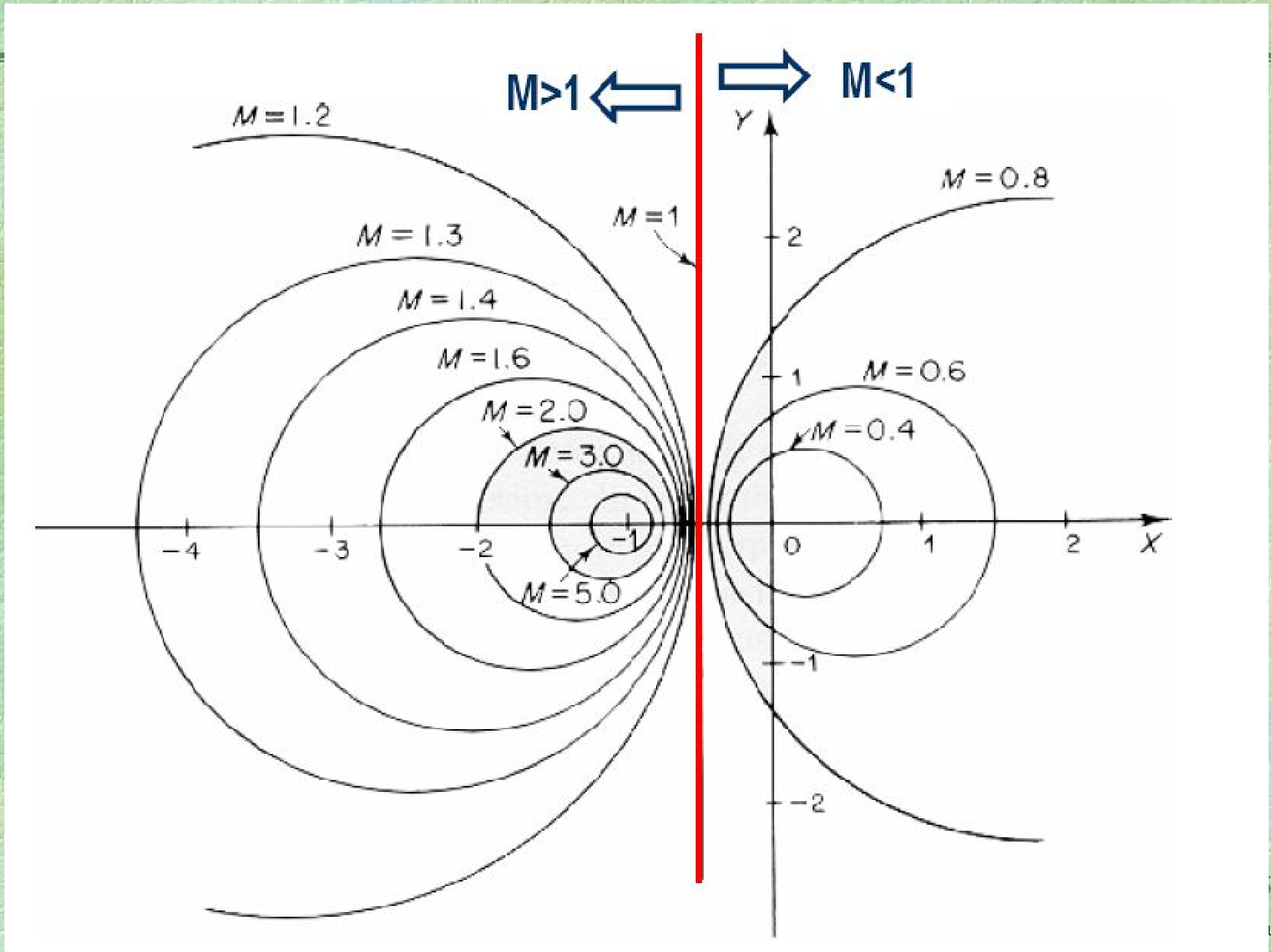
$$T(j1)=\frac{L(j1)}{1+L(j1)}=\frac{-0.86-2.8j}{1-0.86-2.8j}$$

$$T(j1)=1.04\angle-20$$

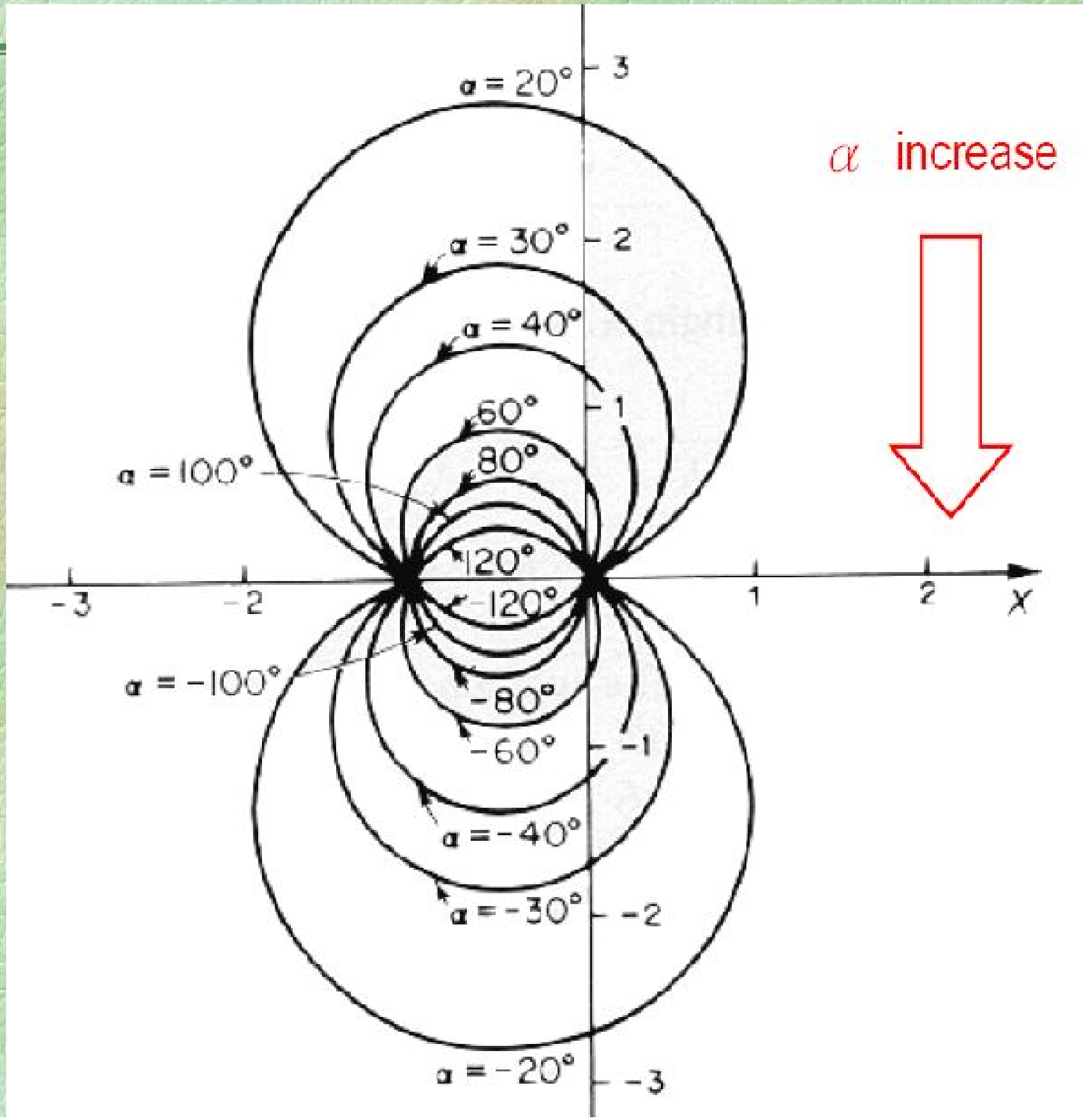
$$T(j2)=?\qquad T(j3)=?\qquad T(j\omega)=?$$

$$M_p=?$$

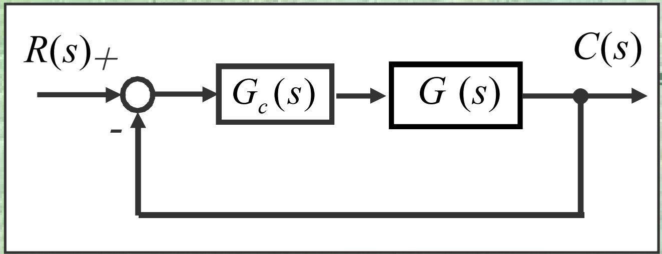
M circles (constant magnitude of T)



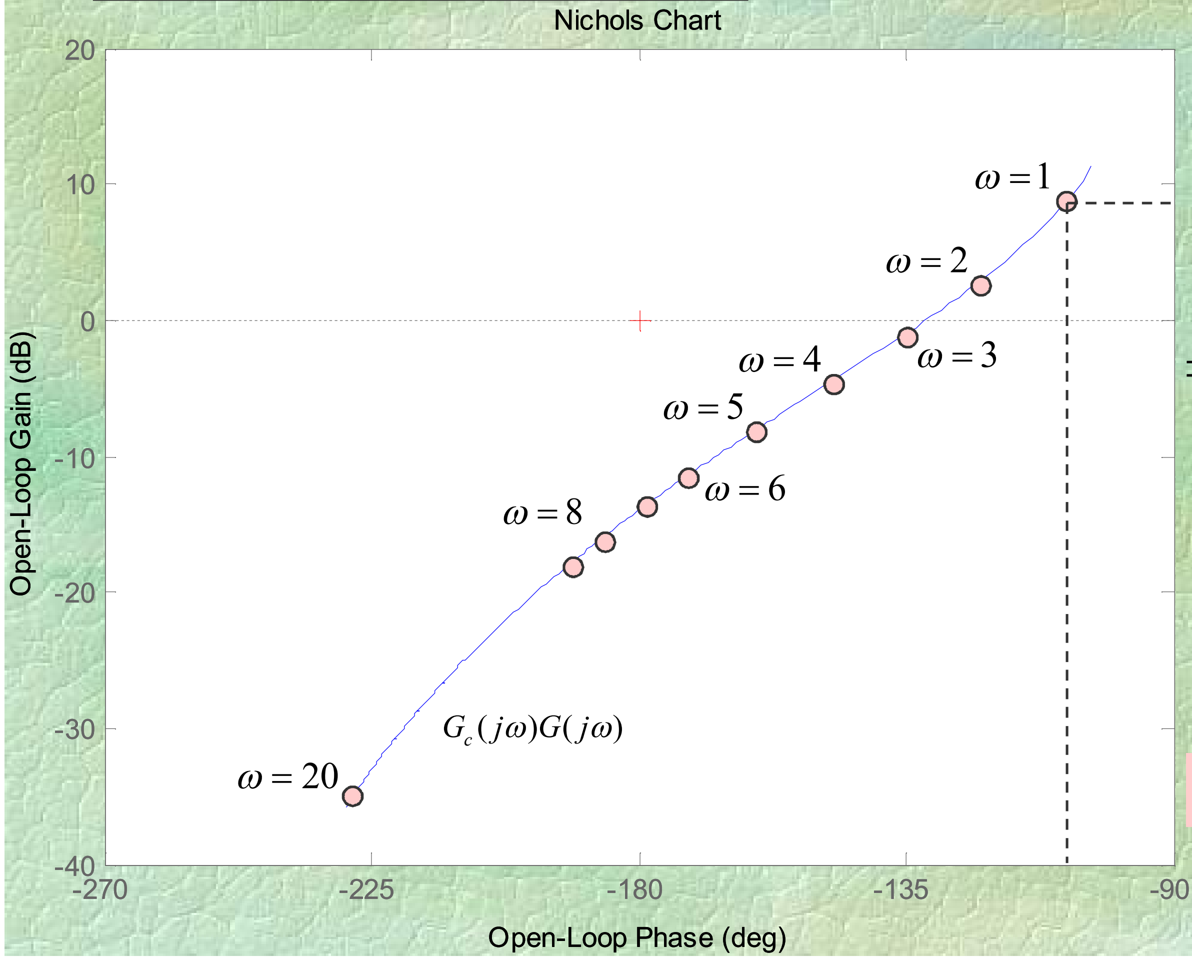
N circles (constant phase of T)



Closed loop values from Nichols chart



Let $G_c(s)G(s)=\dfrac{150}{s(s+5)(s+10)}$



$T(j1)=?$ $\quad \omega=1$

$$20\log\left|G_c(j\omega)G(j\omega)\right|=9.43$$

$$\angle G_c(j\omega)G(j\omega)=-107$$

$$\Rightarrow \quad G(j1)G_c(j1)=2.93\angle-107^\circ$$

$$T(j1)=?$$

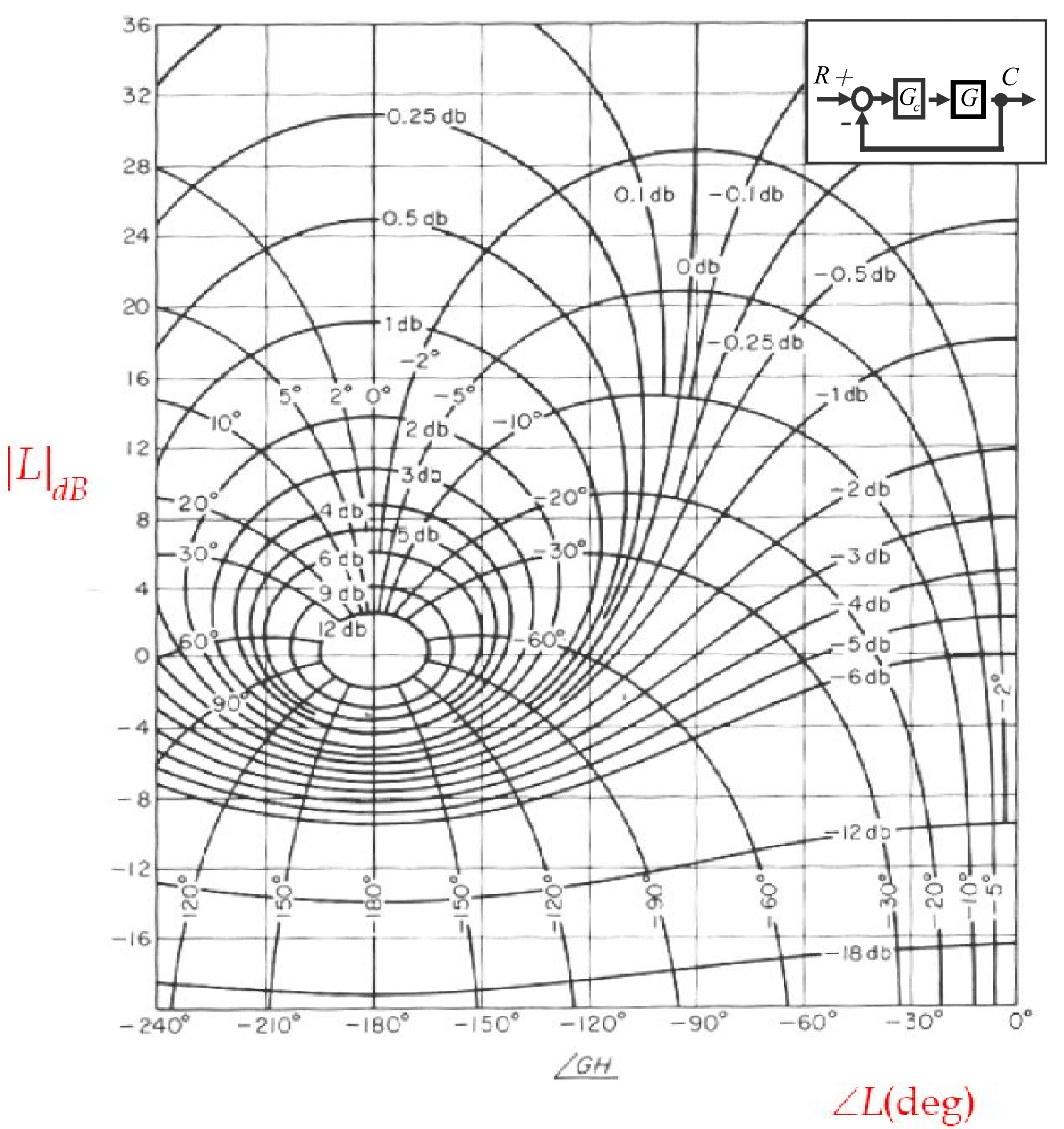
$$T(j1)=\frac{L(j1)}{1+L(j1)}=\frac{2.93\angle-107}{1+2.93\angle-107}$$

$$T(j1)=1.04\angle-20$$

$T(j2)=?$ $\quad T(j3)=?$ $\quad T(j\omega)=?$

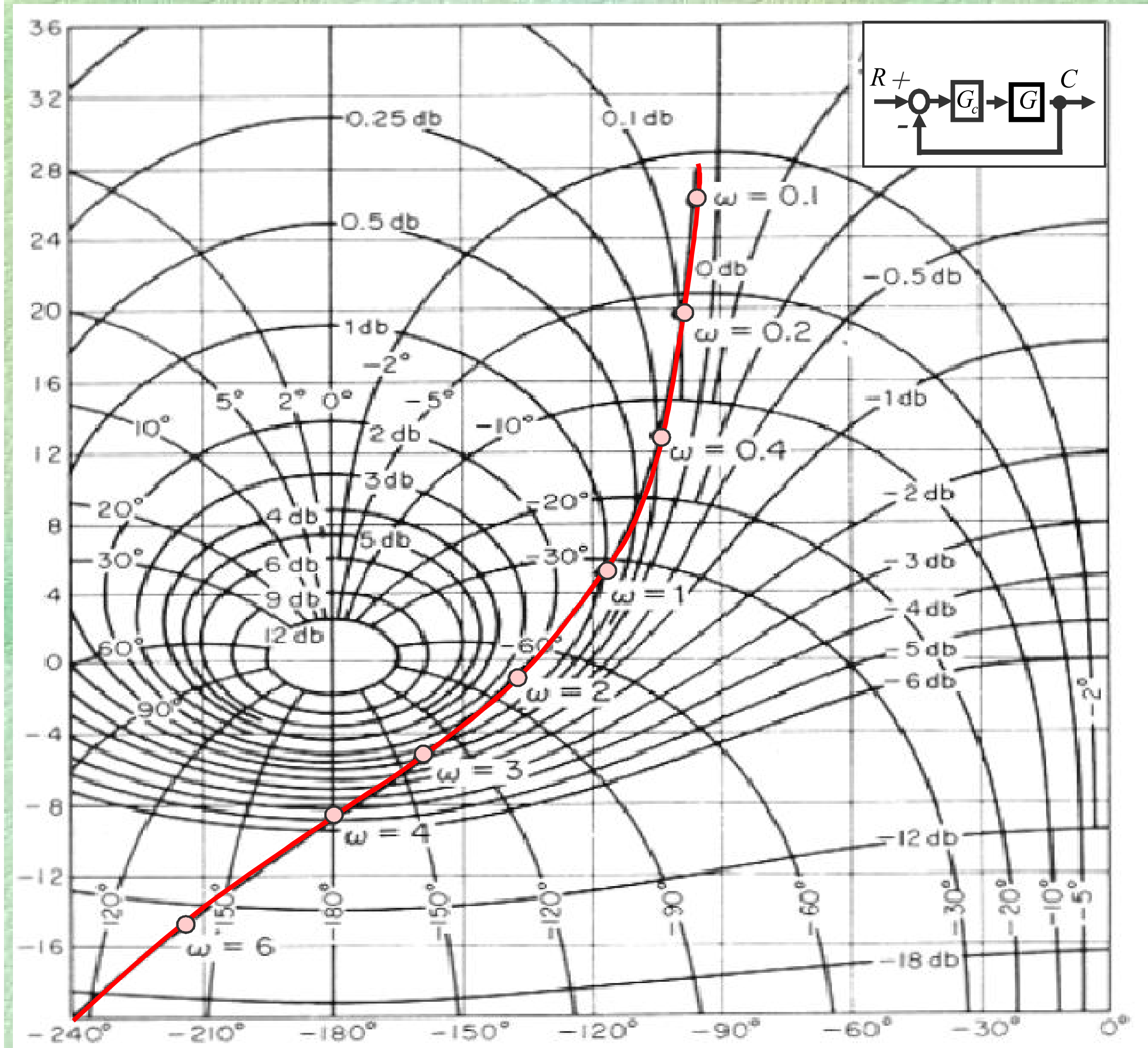
$M_p=?$

Constant gain and phase loci in Nichols chart



M circles and N circles on Nichols chart

Nichols chart specification



How to plot |T| versus frequency?

How to plot <T versus frequency?

How to derive cross over frequencies?

How to derive φ_m and GM?

How to derive open loop bandwidth?

How to derive closed loop bandwidth?

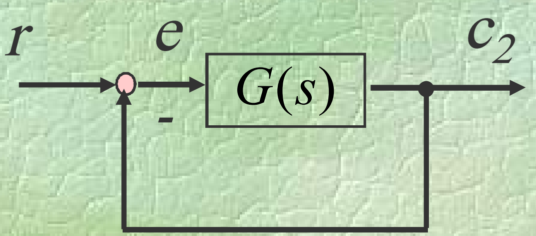
How to derive M_p?

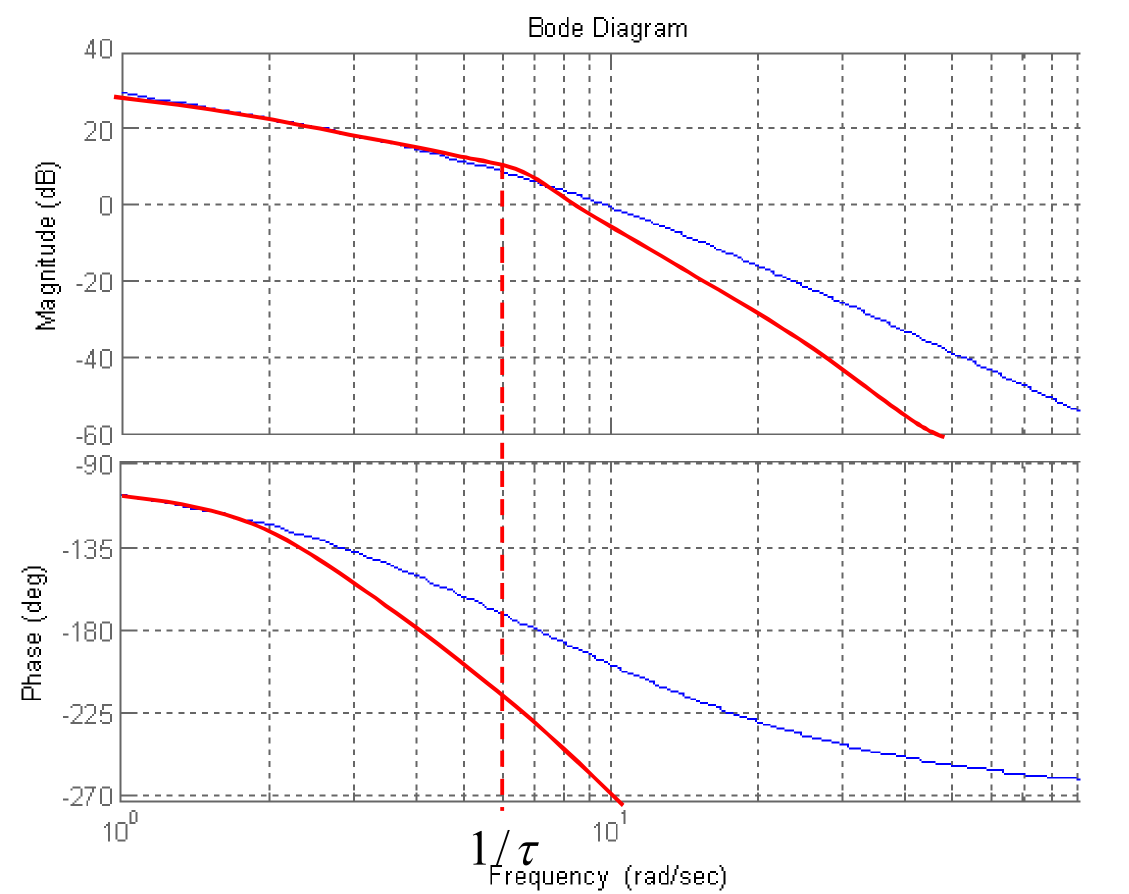
How to derive ω_p?

How to derive type of system?

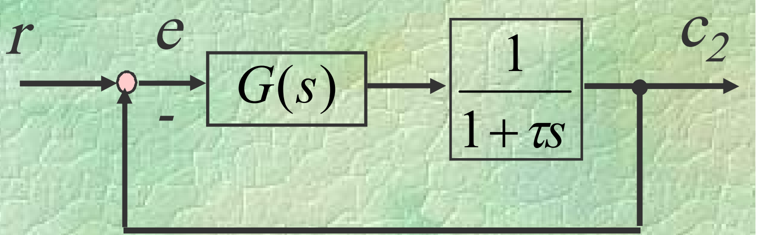
How to derive error coefficient?

Effect of adding poles on Bode plot.





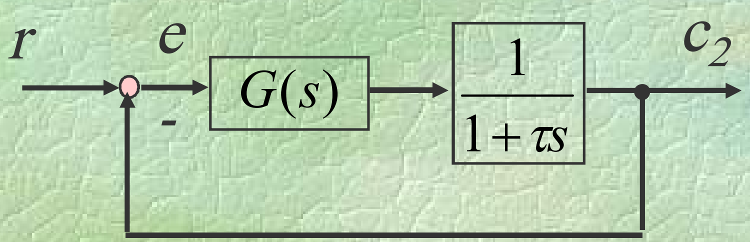
Adding poles



BW ↓ System speed ↓ t_r ↑

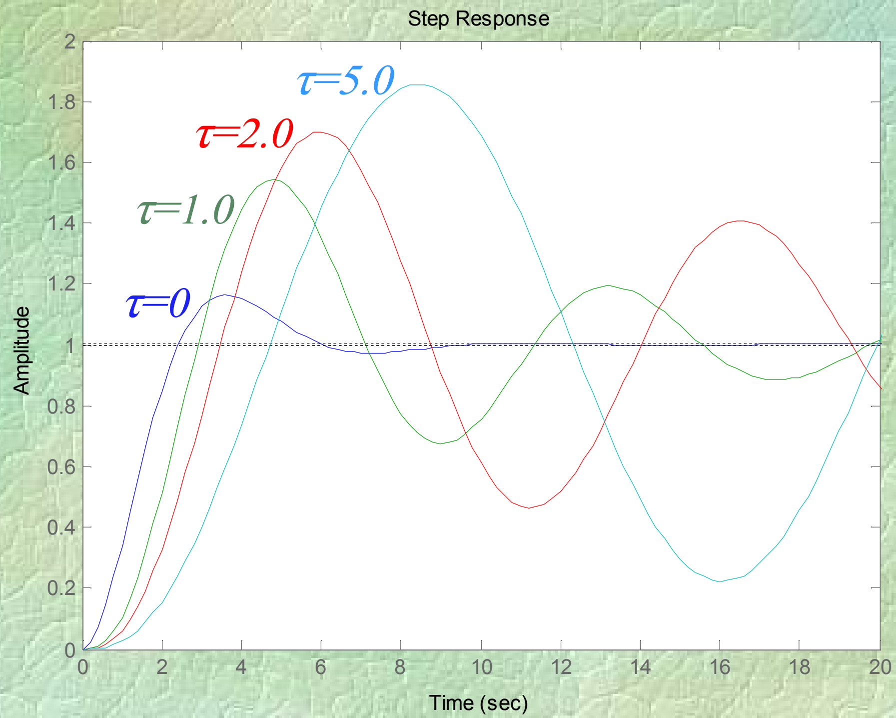
Adding poles to open loop transfer functions

اضافه کردن قطب به تابع انتقال حلقه باز



$$M_2(s)=\frac{C_2(s)}{R(s)}=\frac{\omega_n^2}{\tau s^3+(1+2\zeta\omega_n\tau)s^2+2\zeta\omega_n s+\omega_n^2}$$

$$\omega_n=1 \qquad \zeta=0.5 \qquad \tau=0,1,2,5$$



P.O. ↑

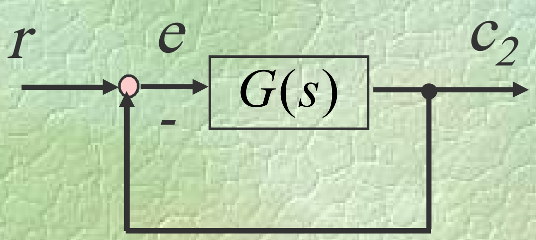
t_r ↑

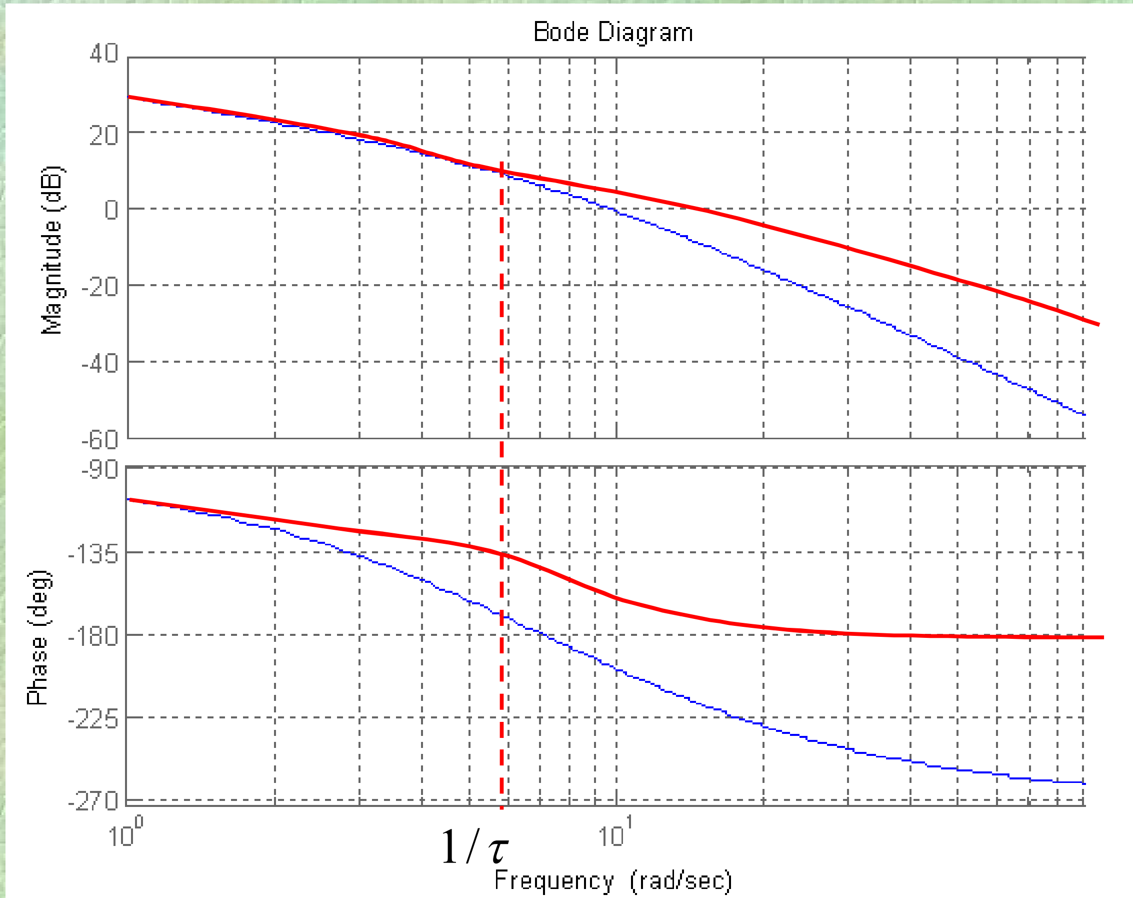
System speed ↓

BW ↓

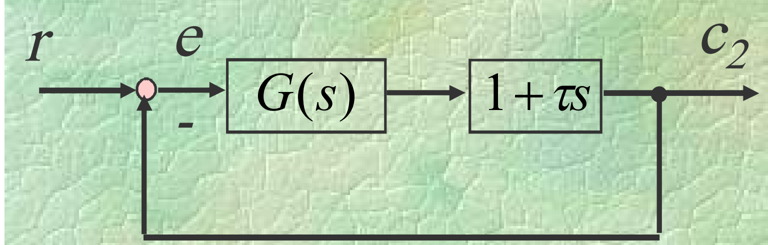
More problem as poles go to ??

Effect of adding zeros on Bode plot.





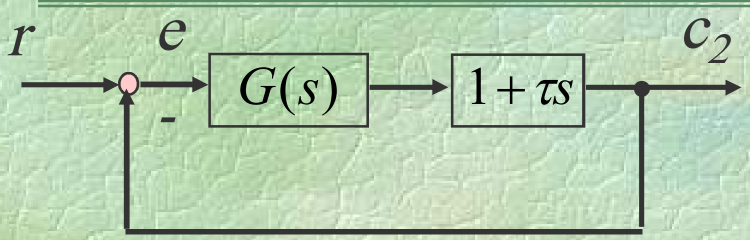
Adding zeros



BW ↑ System speed ↑ t_r ↓

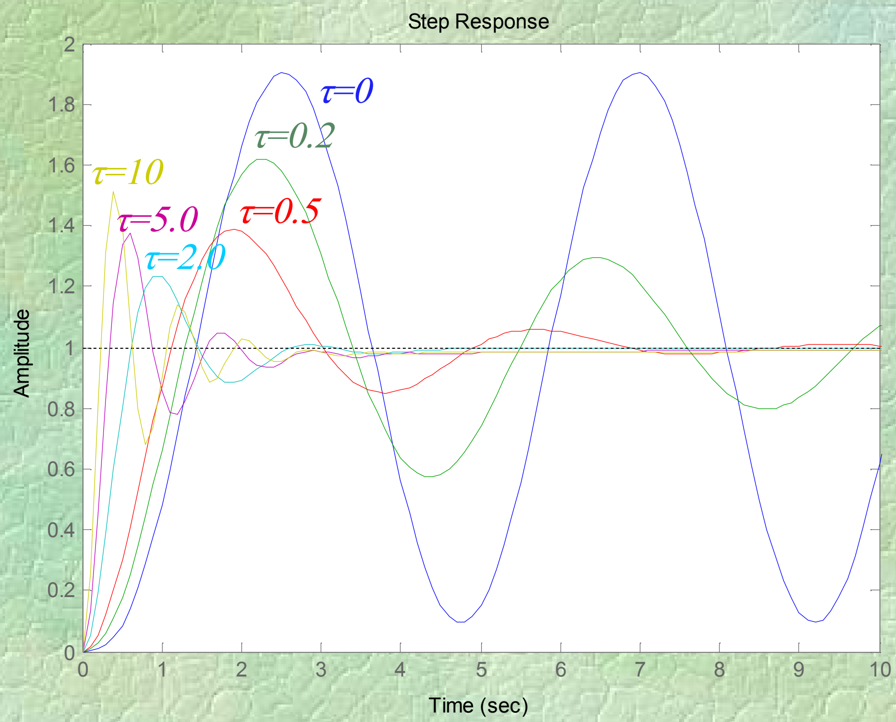
Adding zeros to open loop transfer functions

اضافه کردن صفر به تابع انتقال حلقه باز



$$M_2(s)=\frac{C_2(s)}{R(s)}=\frac{6(1+\tau s)}{s^3+3s^2+(2+6\tau)s+6}$$

$$\tau=0,0.2,0.5,2,5,10$$



P.O. ↓ ↑

t_r ↓

System speed ↑

BW ↑

Note: For τ<0 system is unstable. Why?

Example 1: Derive the Bode plot of following system.

$$G(s)=\frac{a\tau s+1}{\tau s+1}$$

$$Let \quad a>1$$

$$20\log|G(j\omega)|=$$

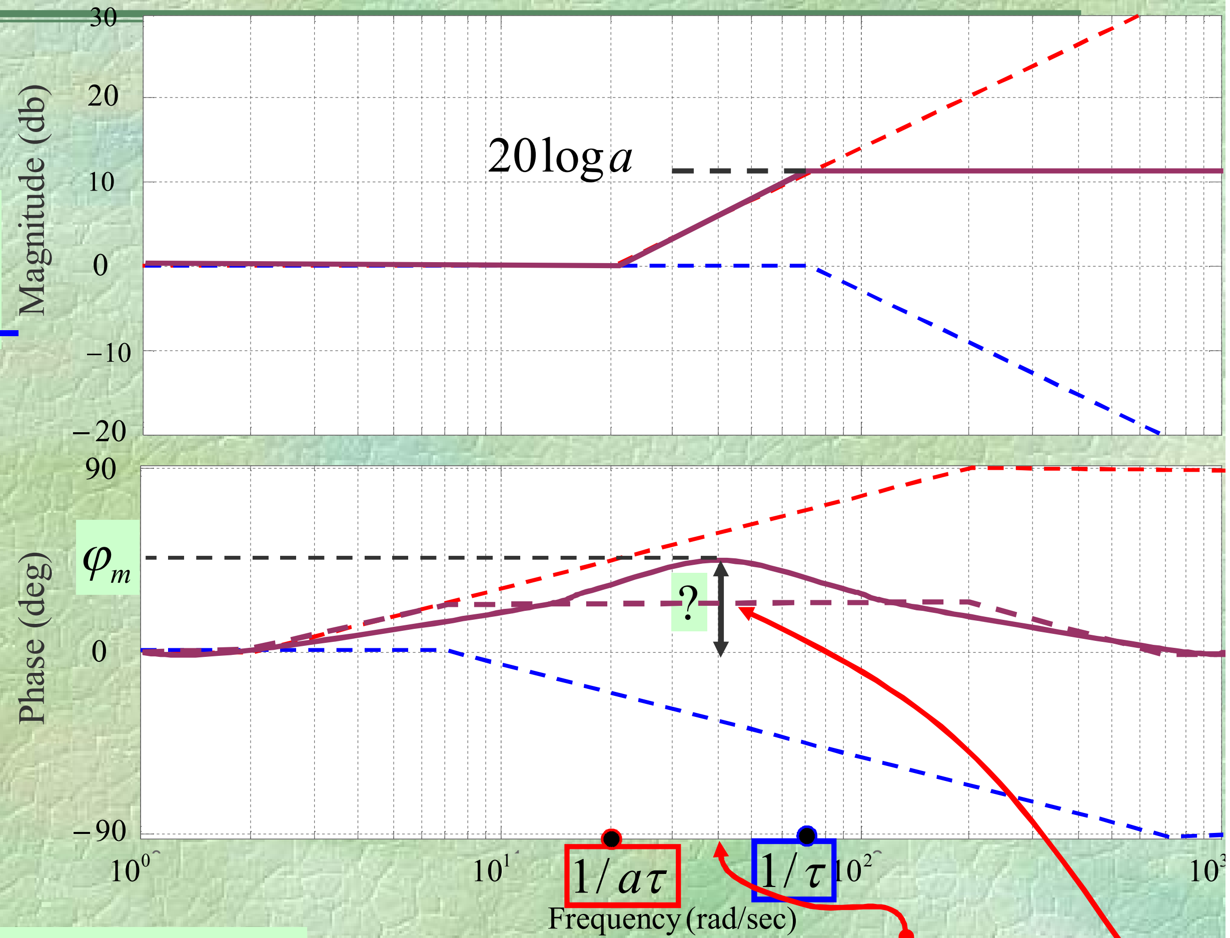
$$=\underline{20\log|a\tau\, j\omega+1|}+\underline{20\log\left|\frac{1}{\tau\, j\omega+1}\right|}$$

$$\angle G(j\omega)=$$

$$=\underline{\angle(a\tau\, j\omega+1)}+\underline{\angle\left(\frac{1}{\tau\, j\omega+1}\right)}$$

$$\varphi_m=\tan^{-1}(a\tau\omega)-\tan^{-1}(\tau\omega)$$

$$\tan(\varphi_m)=\frac{a\tau\omega-\tau\omega}{1+a\tau^2\omega^2}$$



$$\frac{\partial\tan(\varphi_m)}{\partial\omega}=\frac{(a\tau-\tau)(1+a\tau^2\omega^2)-2a\tau^2\omega(a\tau\omega-\tau\omega)}{(1+a\tau^2\omega^2)^2}$$

$$(1+a\tau^2\omega^2)=2a\tau^2\omega^2$$

$$\omega=\frac{1}{\tau\sqrt{a}}$$

$$\sin\varphi_m=\frac{a-1}{a+1}$$

Example 1: Derive the Bode plot of the following system.

$$G(s)=\frac{a\tau s+1}{\tau s+1}$$

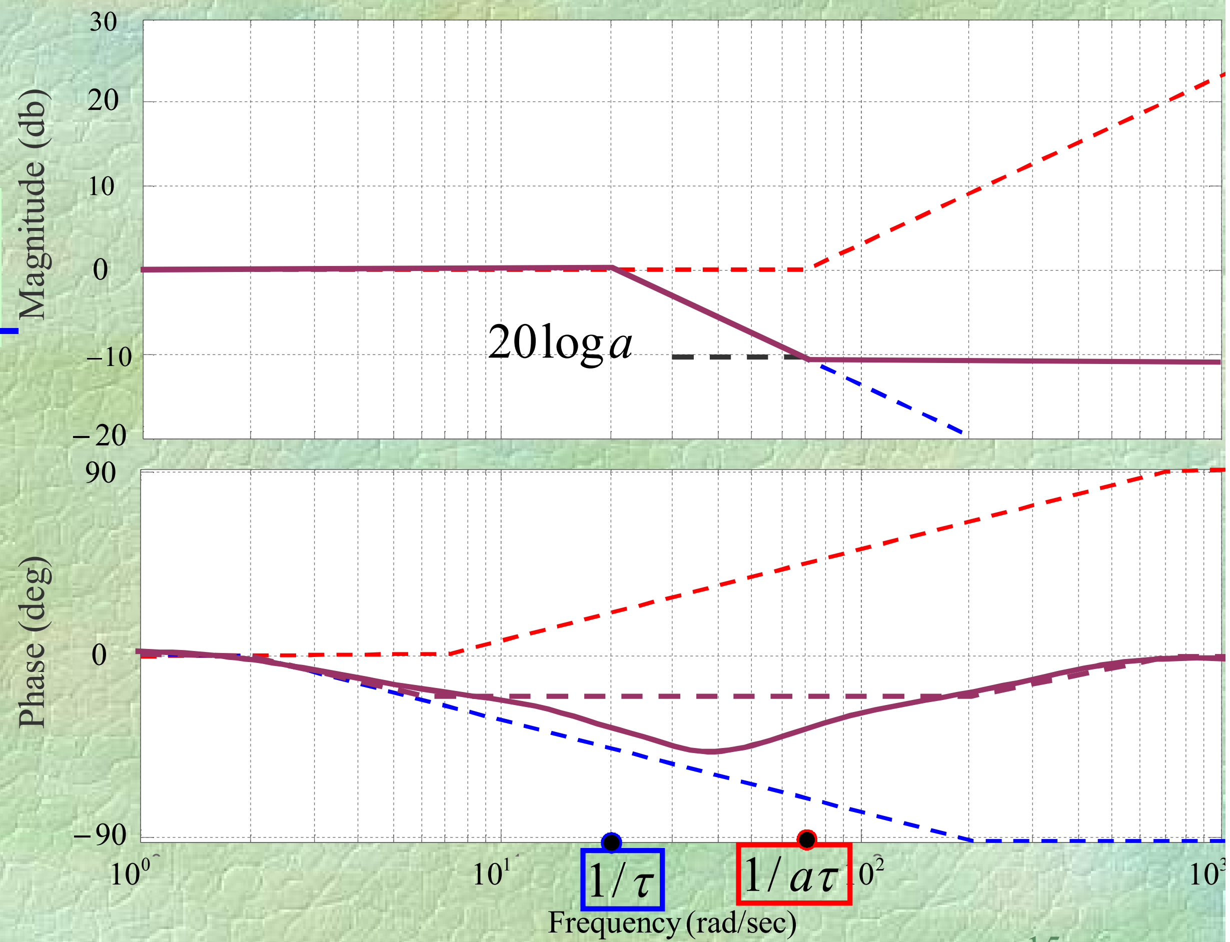
$$Let \quad a<1$$

$$20\log\left|G(j\omega)\right|=$$

$$=\underline{20\log\left|a\tau j\omega+1\right|}+\underline{20\log\left|\frac{1}{\tau j\omega+1}\right|}$$

$$\angle G(j\omega)=$$

$$=\underline{\angle(a\tau j\omega+1)}+\underline{\angle(\frac{1}{\tau j\omega+1})}$$



Dr.Ali Karimpour Apr 2013

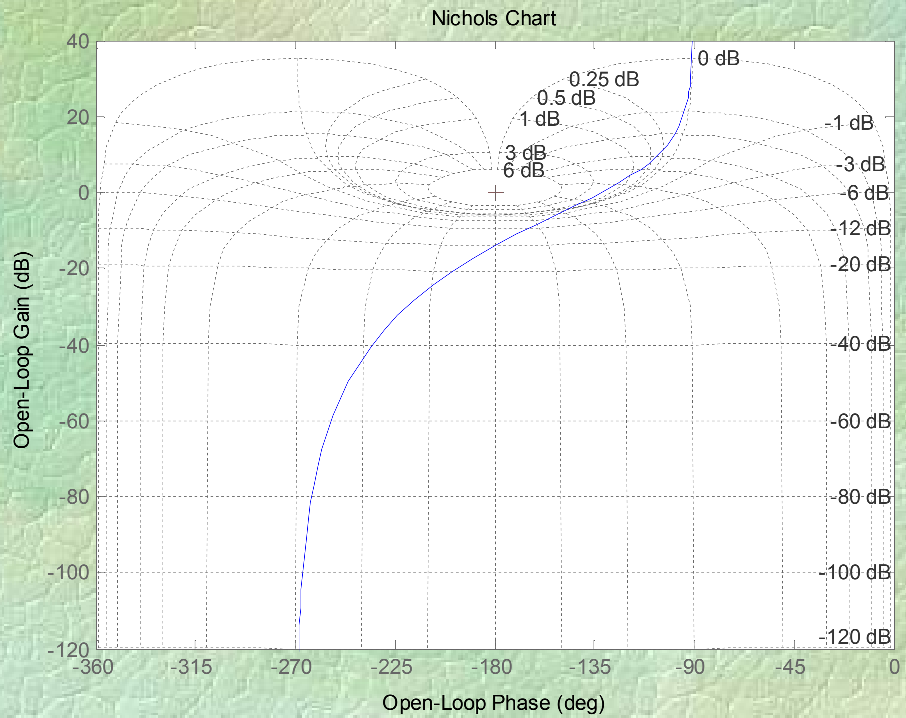
Exercises

تمرینها

1 The Nichols chart of an open loop system with negative unit feedback is shown.

a) Find the GM and PM.

b) Find M_P.



$$answer\ a: GM = 14\,db, PM = 45^\circ \quad b: M_p = 1.8\,db$$

Exercises

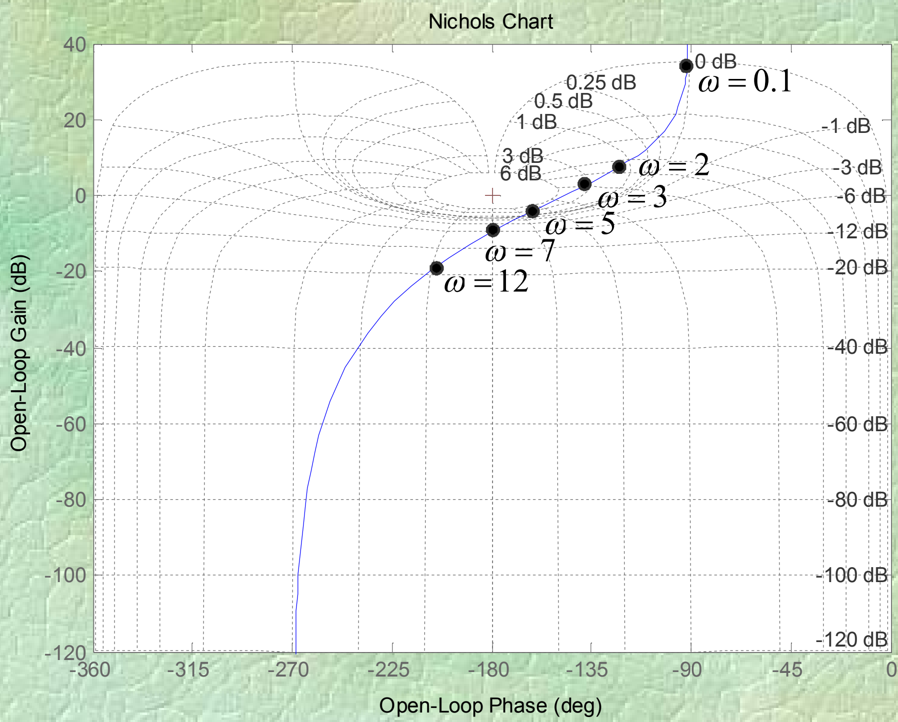
تمرينها

2 The Nichols chart of a open loop system with negative unit feedback is shown.

a) Find the error constants

b) Find the GM and PM and gain crossover frequency and phase crossover frequency.

c) Find M_P, open loop bandwidth and closed loop bandwidth.



$answer\ a: k_p = \infty, k_v = 5, k_a = 0$

$b: GM = 10\,db, PM = 32^\circ, \omega_c = 3.75\,rad/\sec, \omega_{180} = 7\,rad/\sec$

$c: M_p = 5.3\,db, BW_{openloop} = 4.7\,rad/\sec, BW_{closedloop} = 6.3\,rad/\sec$