LINEAR CONTROL SYSTEMS

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Lecture 22

Controller design in the frequency domain

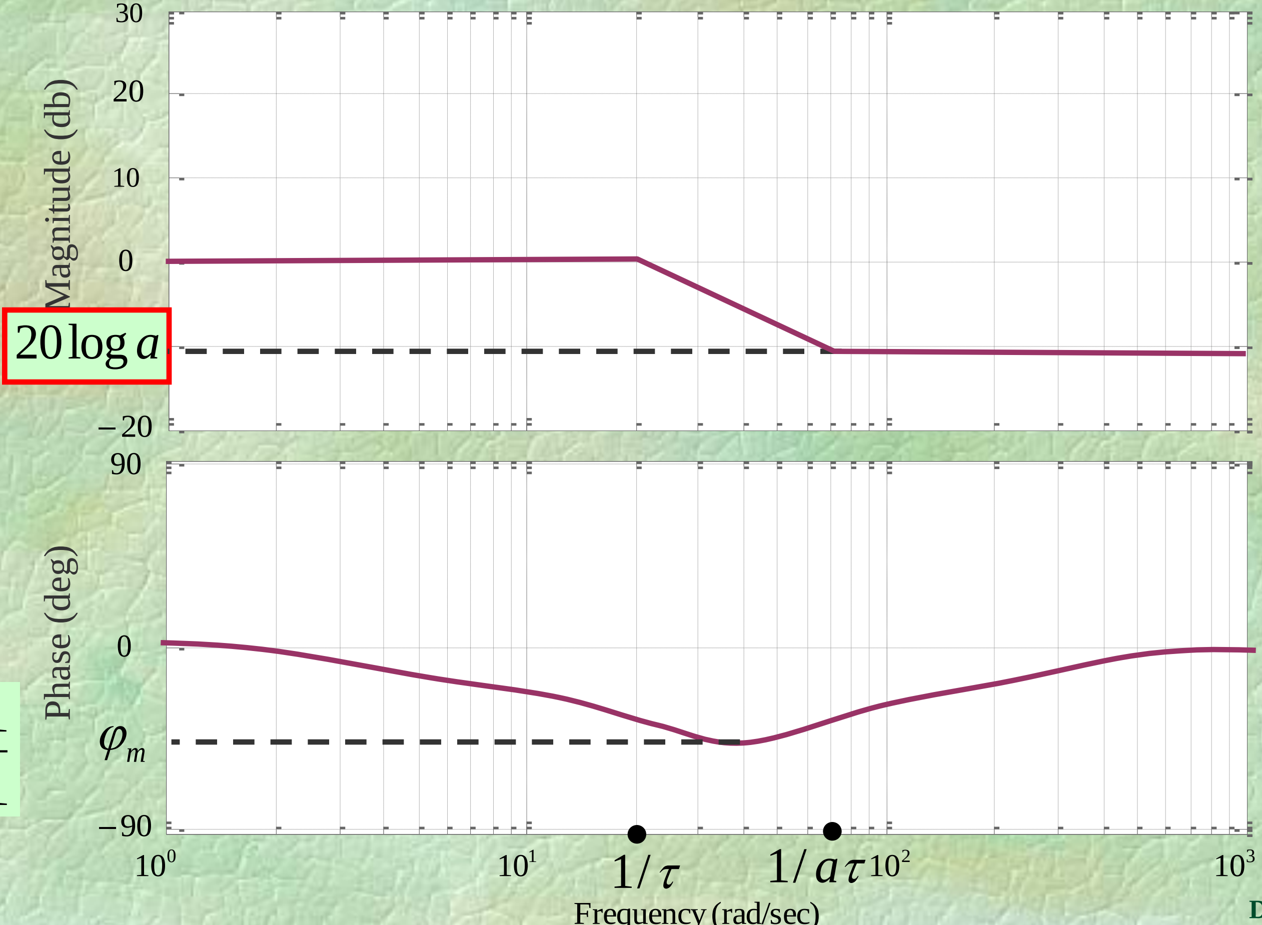
Topics to be covered include:

- Lag controller design

A phase-lag controller

$$G(s) = \frac{a\tau s + 1}{\tau s + 1}$$

$Let \quad a < 1$



$$\sin\varphi_m = \frac{a-1}{a+1}$$

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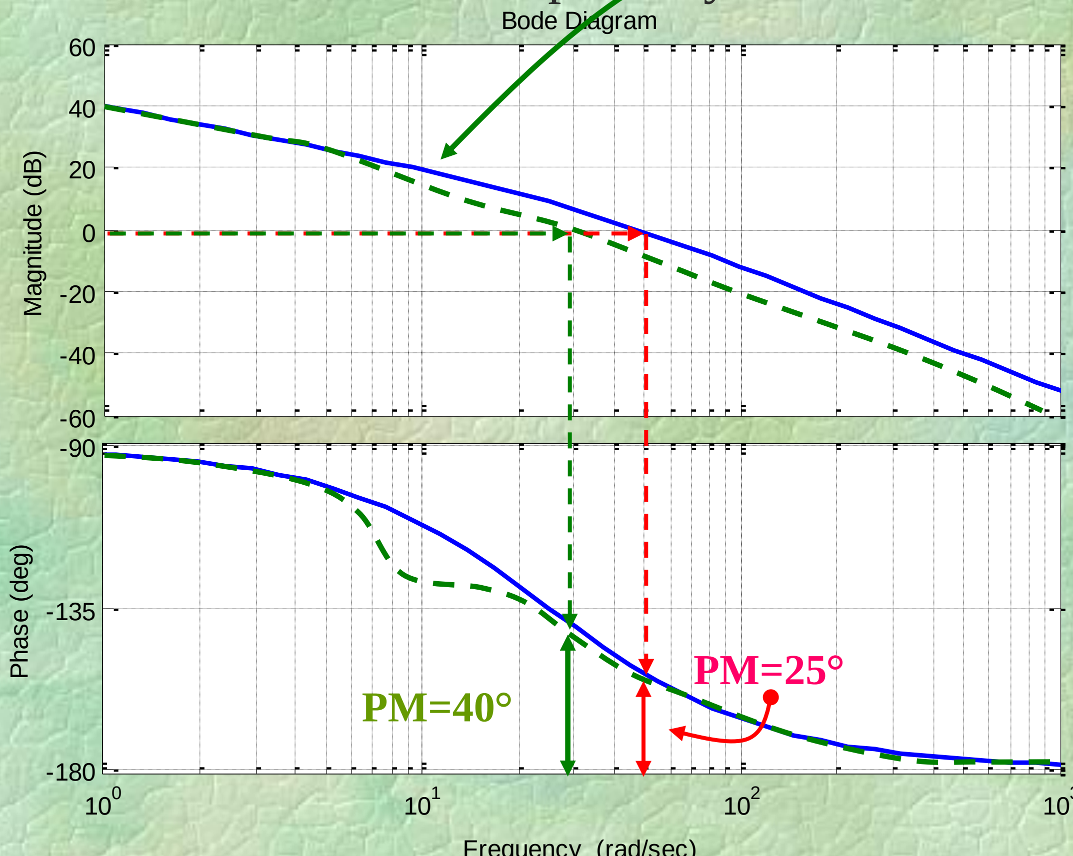
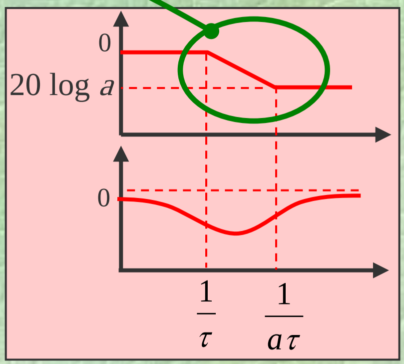
Design fundamental of a lag controller

Analysis Design

Consider a minimum phase system.





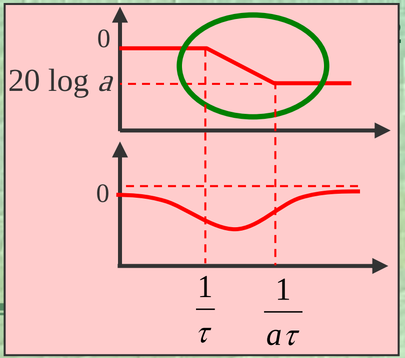
How can the lag controller help us?

Is it different from a Proportional controller?

What is the effect of a lag controller on BW? Speed of system?

What is the effect of a lag controller on noise effect?

Design procedure of a phase-lag controller in the frequency domain



Step 1: Consider $G_c(s) = k\dfrac{a\tau s+1}{\tau s+1}$ with $a<1$ as a phase-lag controller.

Note: If the plant has another gain *k*, let $G_c(s) = \dfrac{a\tau s+1}{\tau s+1}$

Step 2: Try to fix *k* according to the performance request, otherwise let *k*=1

Step 3: Sketch the Bode plot of the system (with the fixed *k*) without controller.

Step 4: According to desired PM (GM) choose the new gain crossover frequency (Phase crossover frequency). (reduce it a little).?

Step 5: Find the required gain by lag controller and derive the parameter a.

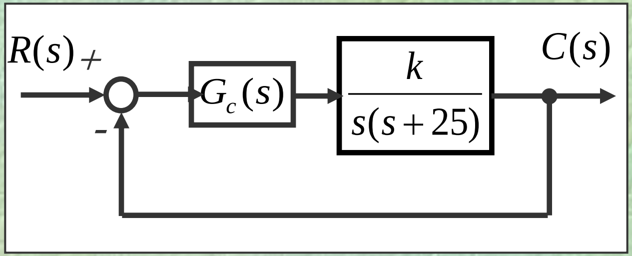
$$\text{Required gain}\,|_{db} + 20\log(a) = 0 \qquad \Rightarrow \qquad a =$$

Step 6: Put the right corner of the controller sufficiently far from crossover frequency.

$$\frac{1}{a\tau} = \frac{\omega_{cross}^{new}}{10} \qquad \Rightarrow \qquad \tau =$$

Step 7: Check the designed controller.

Example 1: Design a lag controller for the following system such that the phase margin be 45° and the ramp error constant be 100. Find the M_p of overall system.



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Note: If the plant has another gain *k*, let $G_c(s) = \dfrac{a\tau s+1}{\tau s+1}$

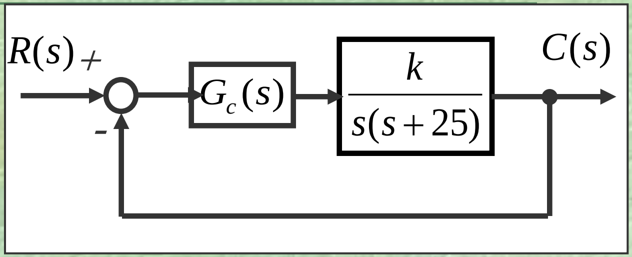
$$G_c(s) = \frac{a\tau s+1}{\tau s+1} \qquad a<1$$

Step 2: Try to fix *k* according to the performance request, otherwise let *k*=1

$$k_v = \lim_{s\to 0} sG_c(s)\frac{k}{s(s+25)} = 100$$

$$k = 2500$$

Example 1: Design a lag controller for the following system such that the phase margin be 45° and the ramp error constant be 100. Find the M_p of overall system.

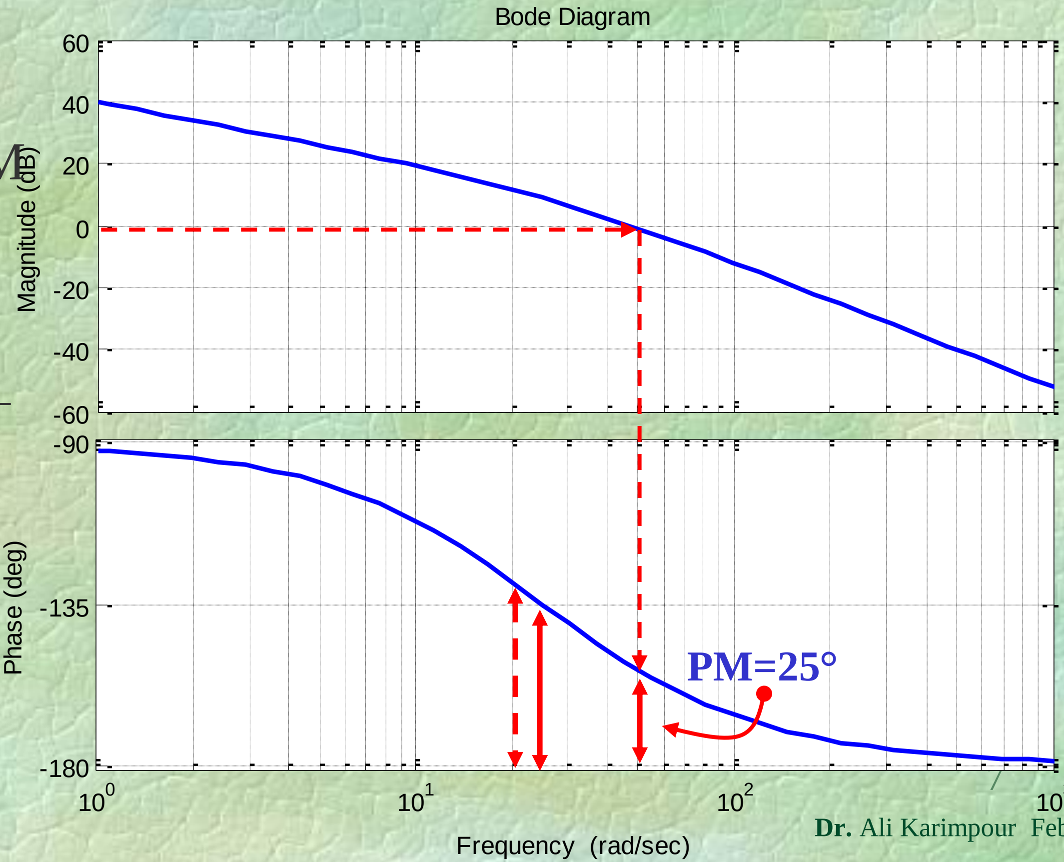


Step 3: Sketch the Bode plot of the system (with the fixed k) without controller.

$$G(s)=\frac{2500}{s(s+25)}=\frac{100}{s(s/25+1)}$$

Step 4: According to desired PM (GM) choose the new gain crossover frequency (Phase crossover frequency). (reduce it a little).

$$\cancel{\omega_{croos}^{new}=25} \qquad \omega_{cross}^{new}=20$$



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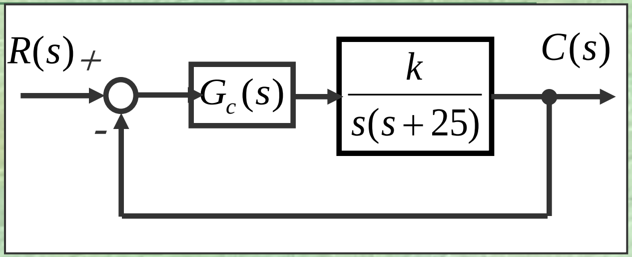


Step 5: Find the required gain by lag controller and derive the parameter a.

$$\text{Required gain } |_{db} + 20\log(a) = 0$$

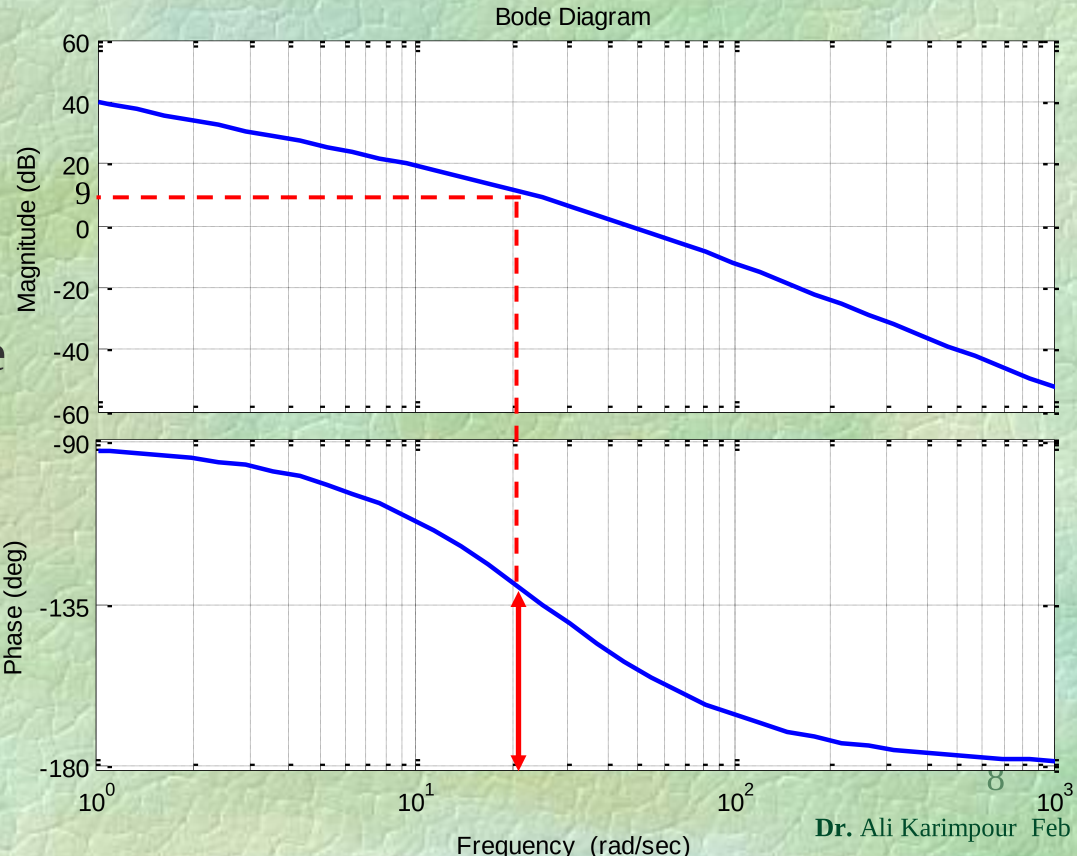
$$9 + 20\log(a) = 0$$

$$a = 10^{-\frac{9}{20}} = 0.35$$

Step 6: Put the right corner of the controller sufficiently far from crossover frequency.

$$\frac{1}{a\tau} = \frac{\omega_{cross}^{new}}{10} = 2 \qquad \tau = 1.41$$

$$G_c(s) = \frac{a\tau s + 1}{\tau s + 1} = \frac{0.5s + 1}{1.41s + 1}$$



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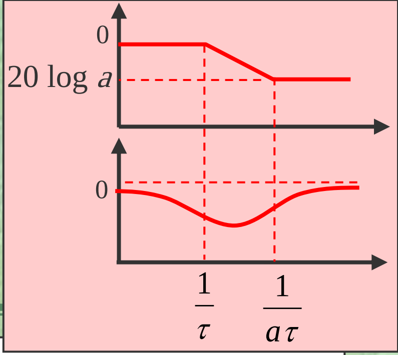
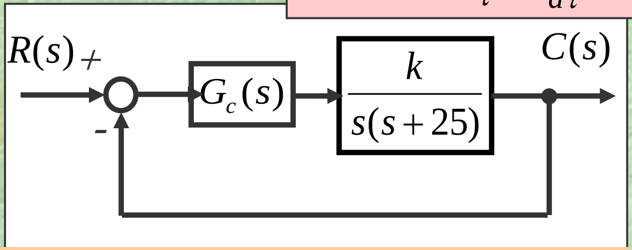
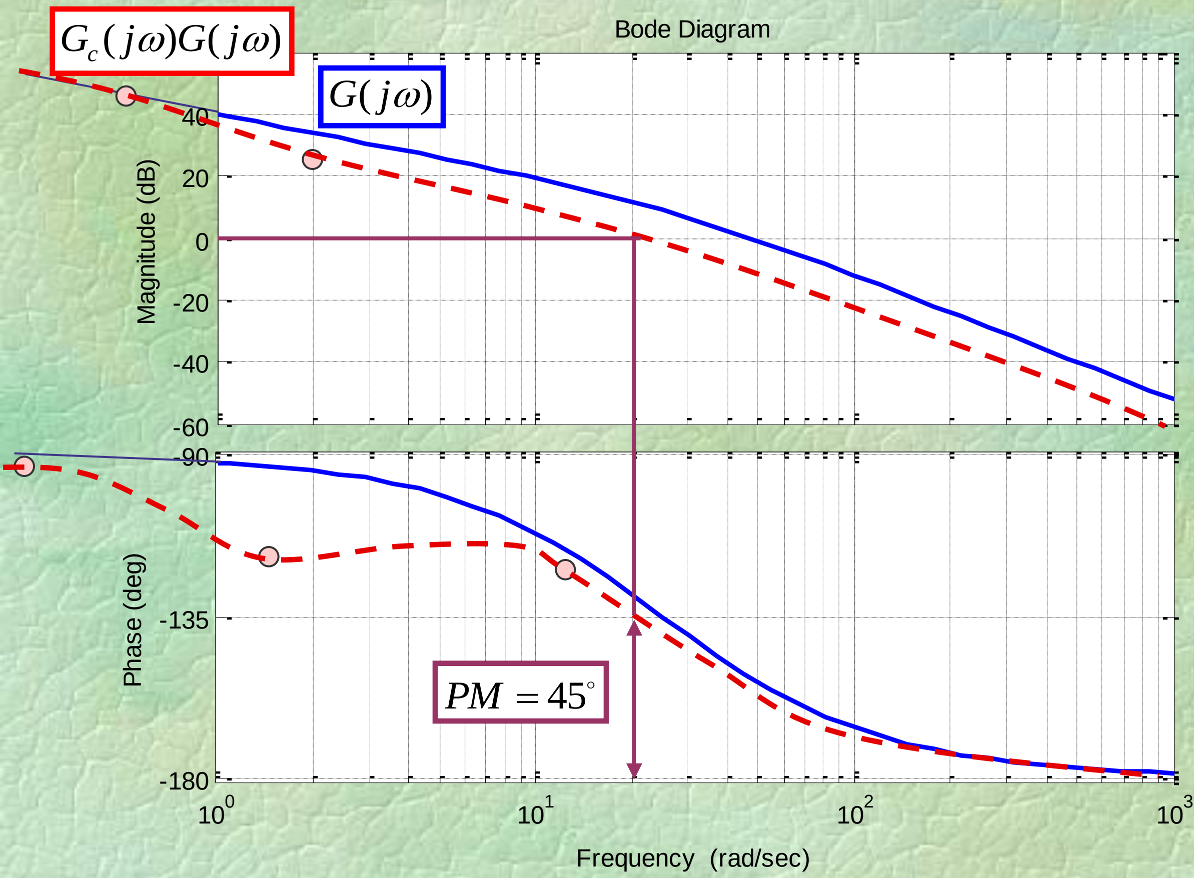




Step 7: Check the designed controller.



$$G_c(s) = \frac{a\tau s + 1}{\tau s + 1} = \frac{1.41 \times 0.35 s + 1}{1.41 s + 1}$$

$$1/\tau = 0.71,\ 1/a\tau = 2$$

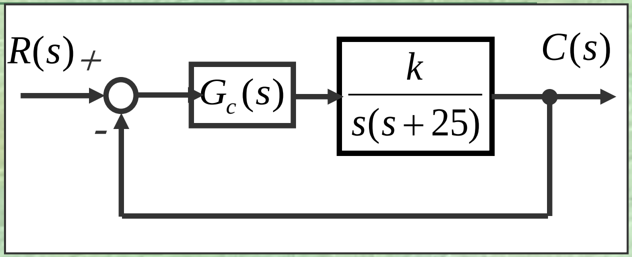
$$20 \log a = -9\, db$$

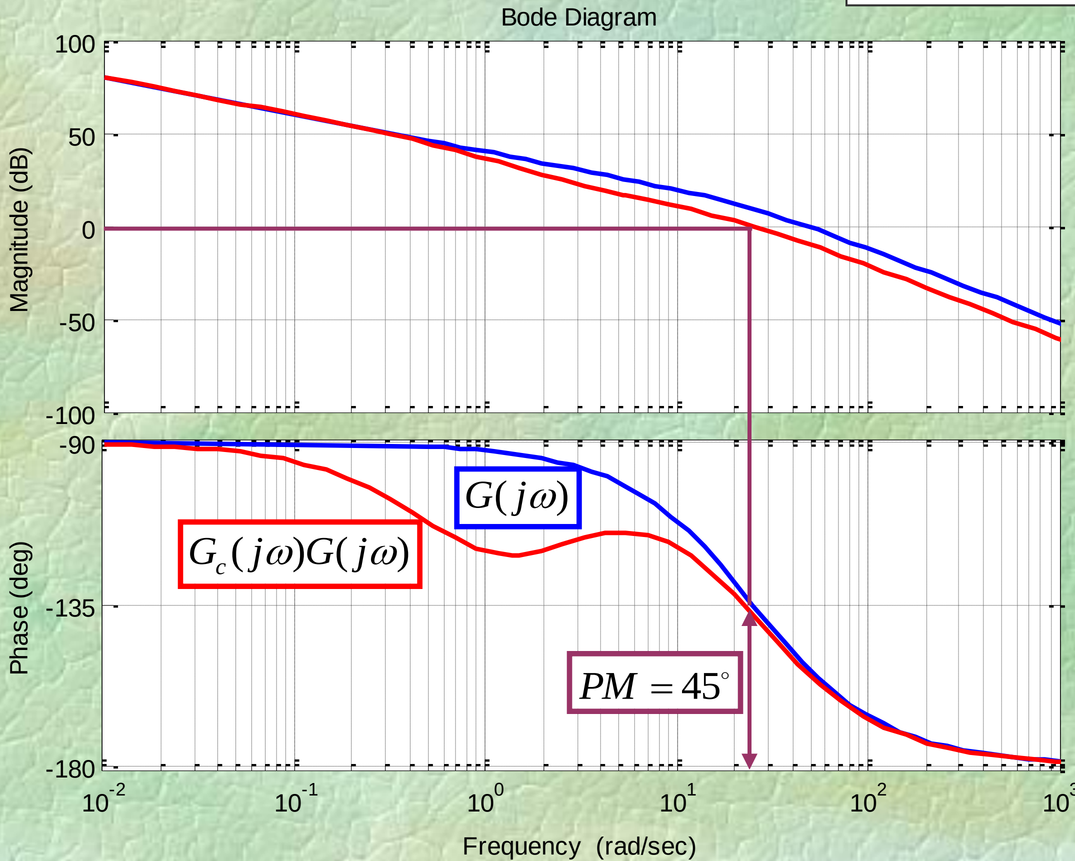
$$1/\sqrt{a\tau} = 1.19$$

$$\sin \varphi_m = \frac{0.354 - 1}{0.354 + 1} \Rightarrow \varphi_m = -28^\circ$$

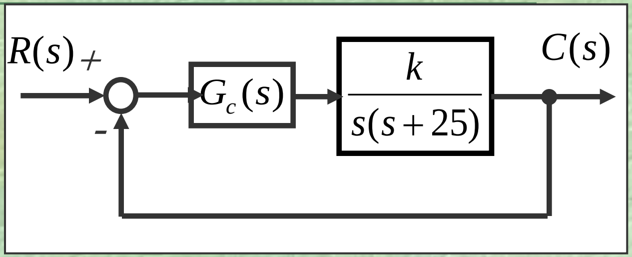
Example 1: Design a lag controller for the following system such that the phase margin be 45° and the ramp error constant be 100. Find the M_p of overall system.

Step 7: Check the designed controller.





Example 1: Design a lag controller for the following system such that the phase margin be 45° and the ramp error constant be 100. Find the M_p of overall system.

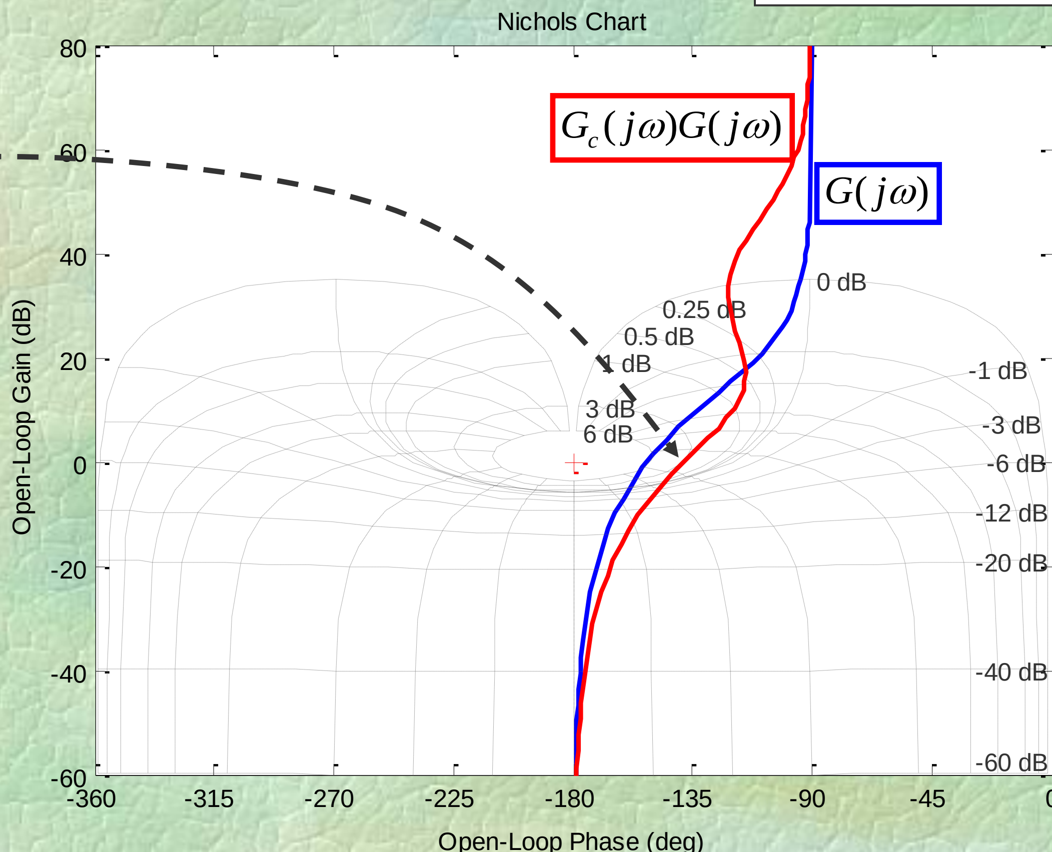


Finding M_p

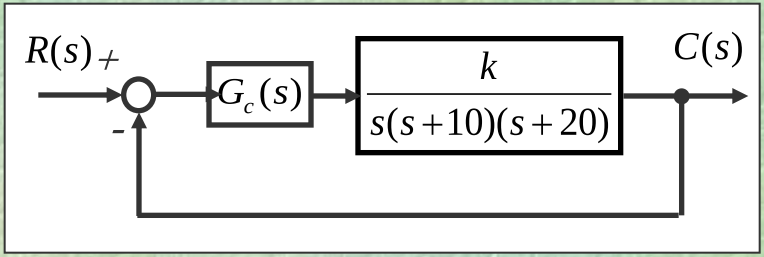
Without controller M_p=6 db

After applying controller

M_p=3 db



Example 2: Design a lag controller for the following system such that the phase margin be 45° and the ramp error constant be 100.



Step 1: Consider $G_c(s) = k\frac{a\tau s+1}{\tau s+1}$ with $a<1$ as a phase-lag controller.

Note: If the plant has another gain *k*, let $G_c(s) = \frac{a\tau s+1}{\tau s+1}$

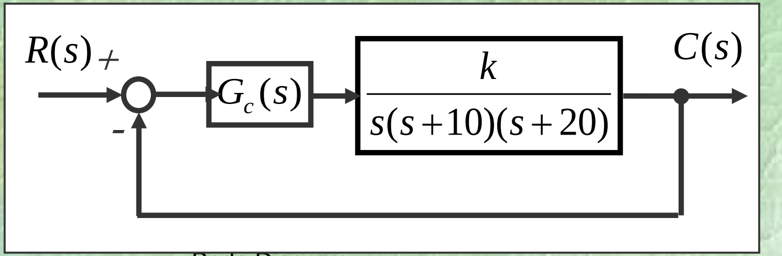
$$G_c(s) = \frac{a\tau s+1}{\tau s+1} \qquad a<1$$

Step 2: Try to fix *k* according to the performance request, otherwise let *k*=1

$$k_v = \lim_{s\to 0} sG_c(s)\frac{k}{s(s+10)(s+20)} = 100$$

$$k = 20000$$

Example 2: Design a lag controller for the following system such that the phase margin be 45° and the ramp error constant be 100.

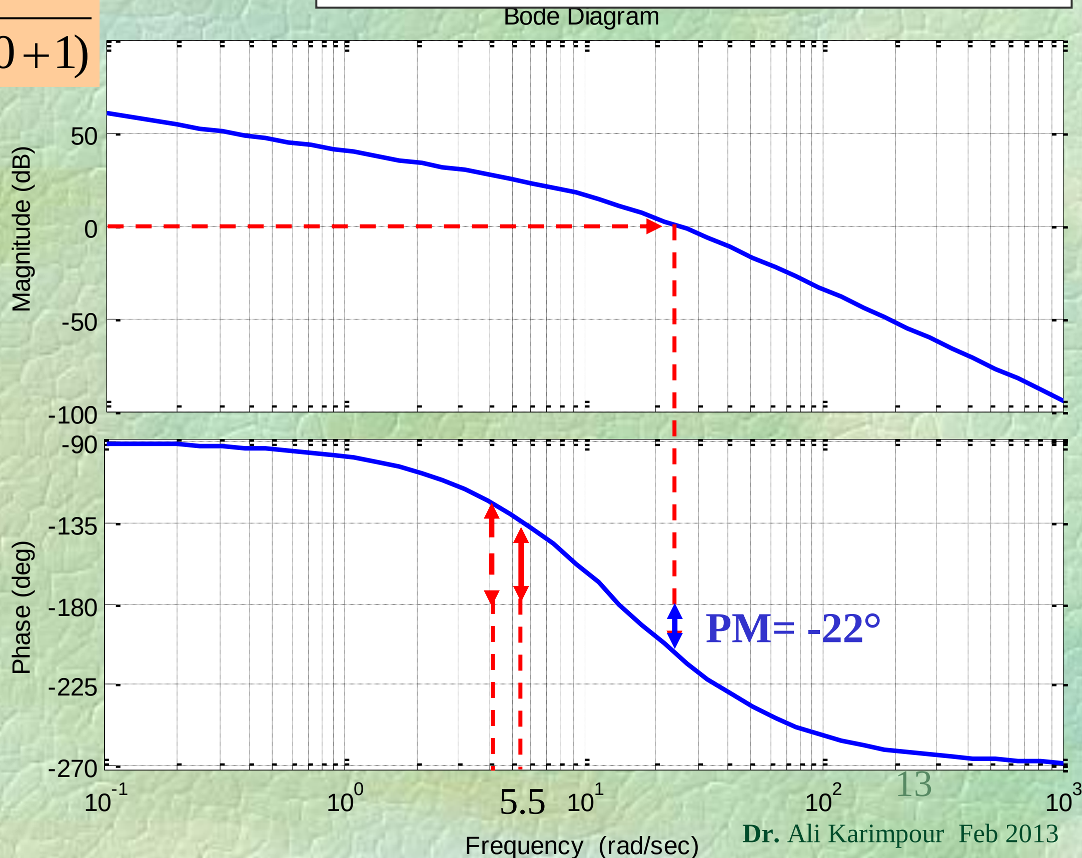


Step 3: Sketch the Bode plot of the system (with the fixed k) without controller.

$$G(s)=\frac{20000}{s(s+10)(s+20)}=\frac{100}{s(s/10+1)(s/20+1)}$$

Step 4: According to desired PM (GM) choose the new gain crossover frequency (Phase crossover frequency). (reduce it a little).

$\cancel{\omega_{cross}^{new}=5.5}$ $\qquad \omega_{cross}^{new}=4$



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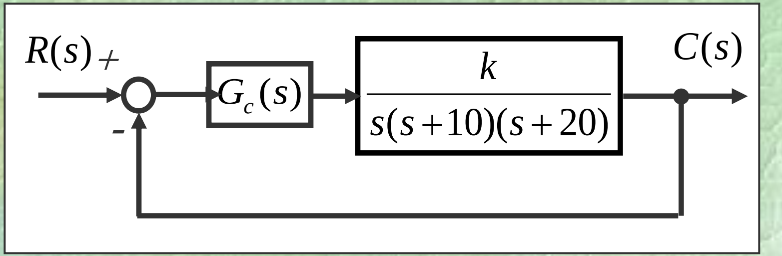
Example 2: Design a lag controller for the following system such that the phase margin be 45° and the ramp error constant be 100.



Step 5: Find the required gain by lag controller and derive the parameter a.

$$\text{Required gain } |_{\text{db}} + 20\log(a) = 0$$

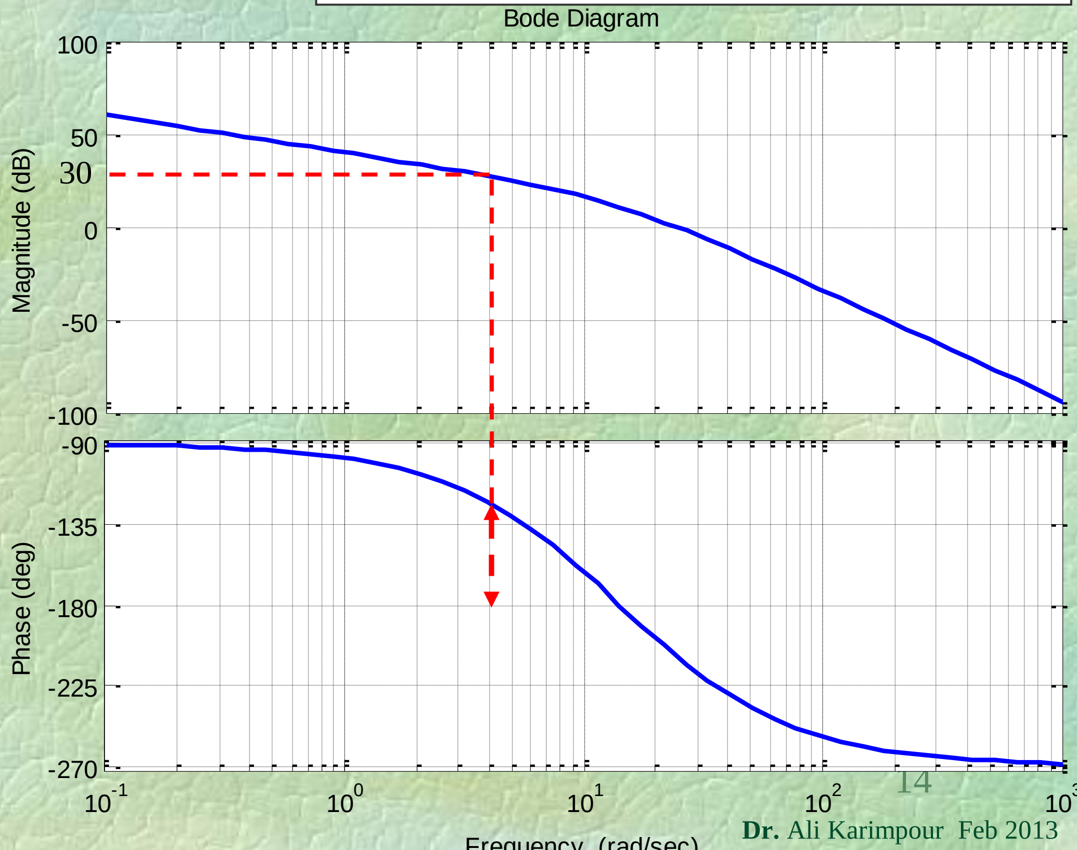
$$30 + 20\log(a) = 0$$

$$a = 10^{-\frac{30}{20}} = 0.0316$$

Step 6: Put the right corner of the controller sufficiently far from crossover frequency.

$$\frac{1}{a\tau} = \frac{\omega_{cross}^{new}}{10} = 0.4 \qquad \tau = 79$$

$$G_c(s) = \frac{a\tau s + 1}{\tau s + 1} = \frac{2.5s + 1}{79s + 1}$$



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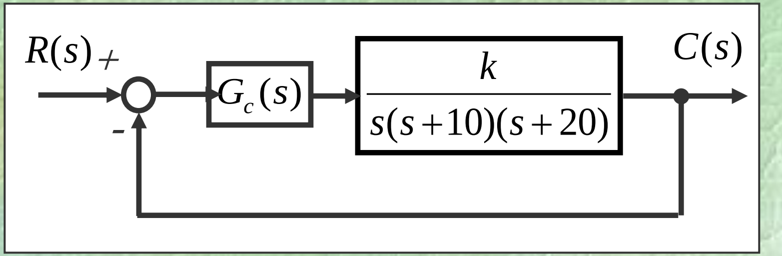
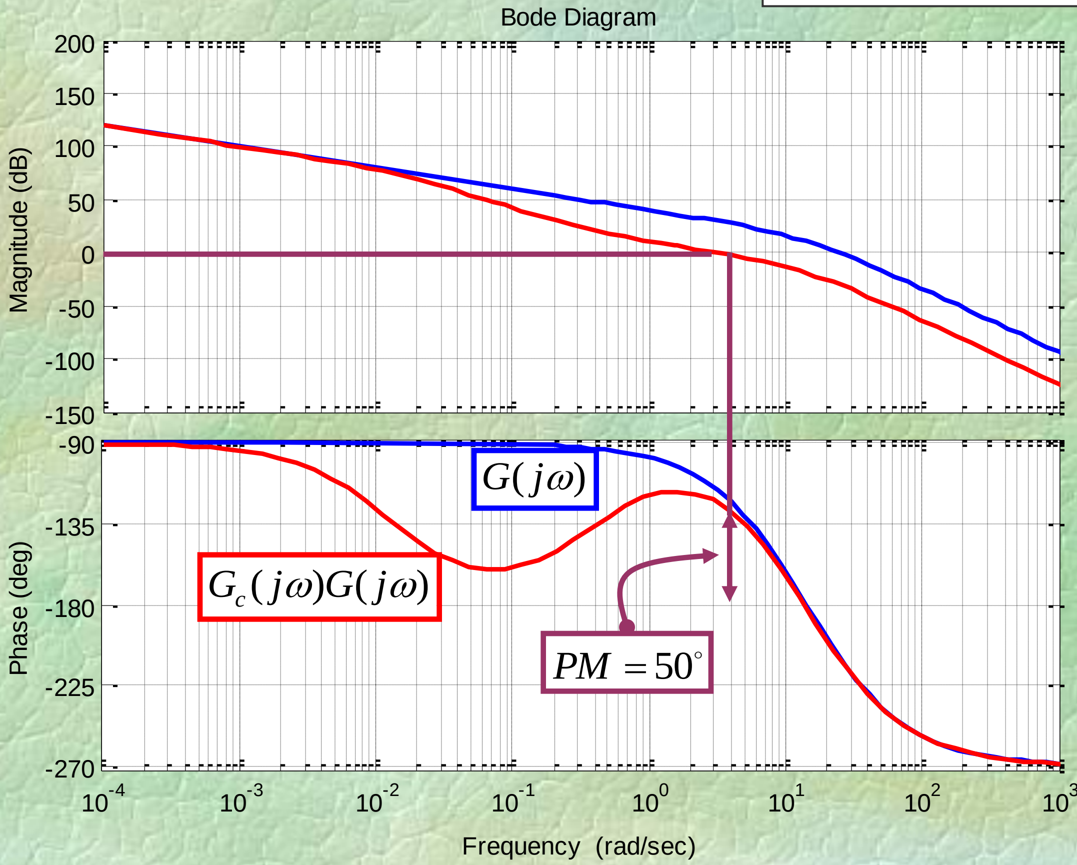


Step 7: Check the designed controller.



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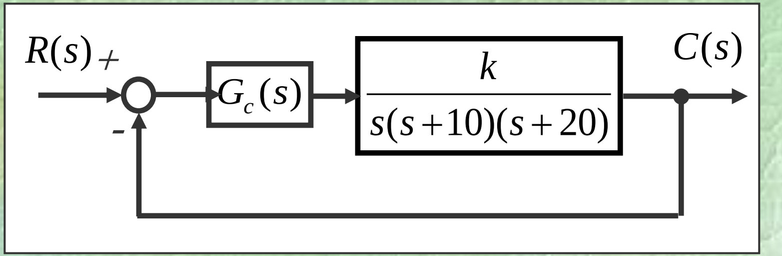

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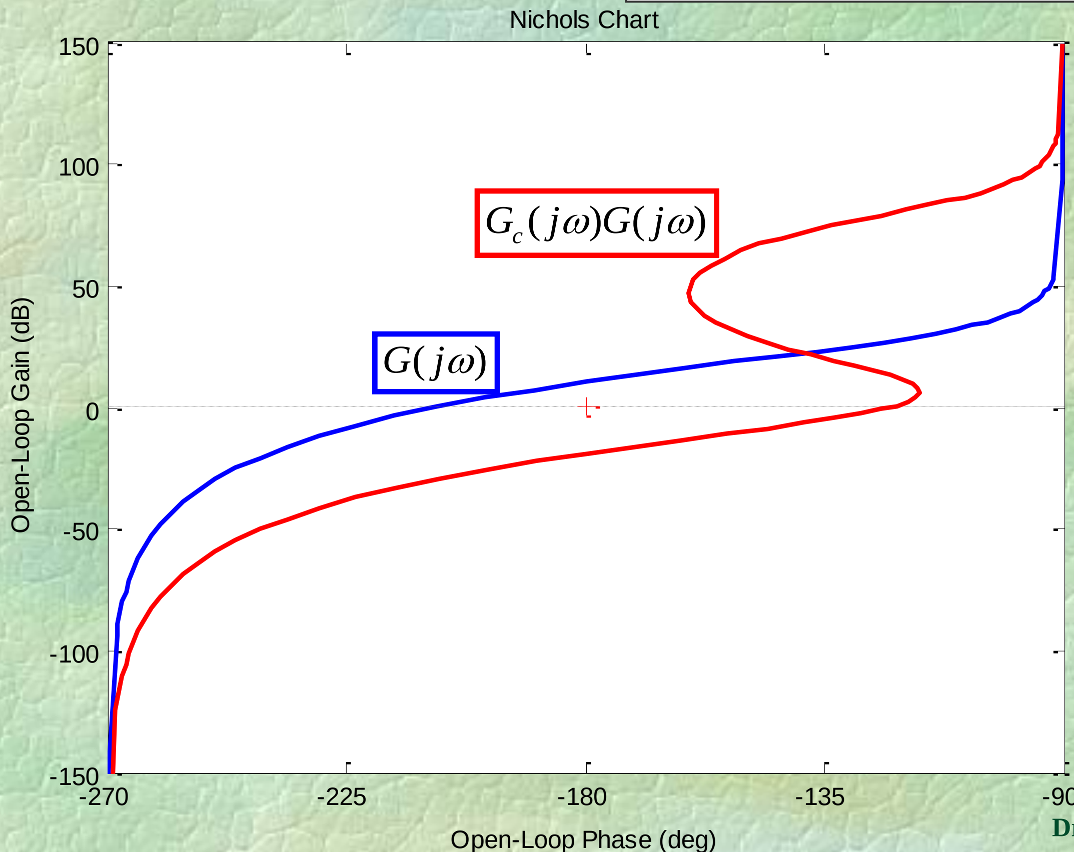
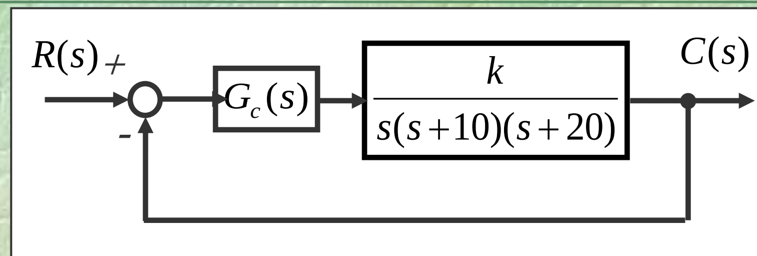


Example 3: Design a lag controller for the following system such that the gain margin be 10 db and the ramp error constant be 100.



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Note: If the plant has another gain *k*, let $G_c(s) = \dfrac{a\tau s+1}{\tau s+1}$

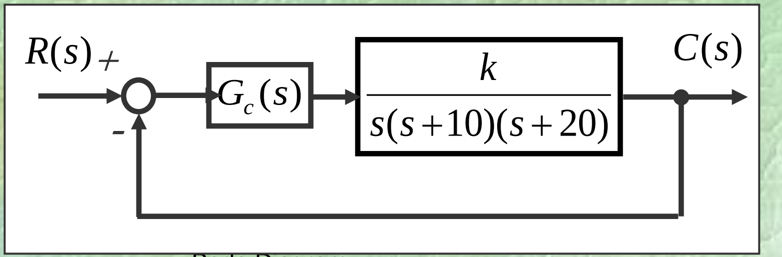
$$G_c(s) = \frac{a\tau s+1}{\tau s+1} \qquad a<1$$

Step 2: Try to fix *k* according to the performance request, otherwise let $k=1$

$$k_v = \lim_{s\to 0} sG_c(s)\frac{k}{s(s+10)(s+20)} = 100$$

$$k = 20000$$

Example 3: Design a lag controller for the following system such that the gain margin be 10 db and the ramp error constant be 100.

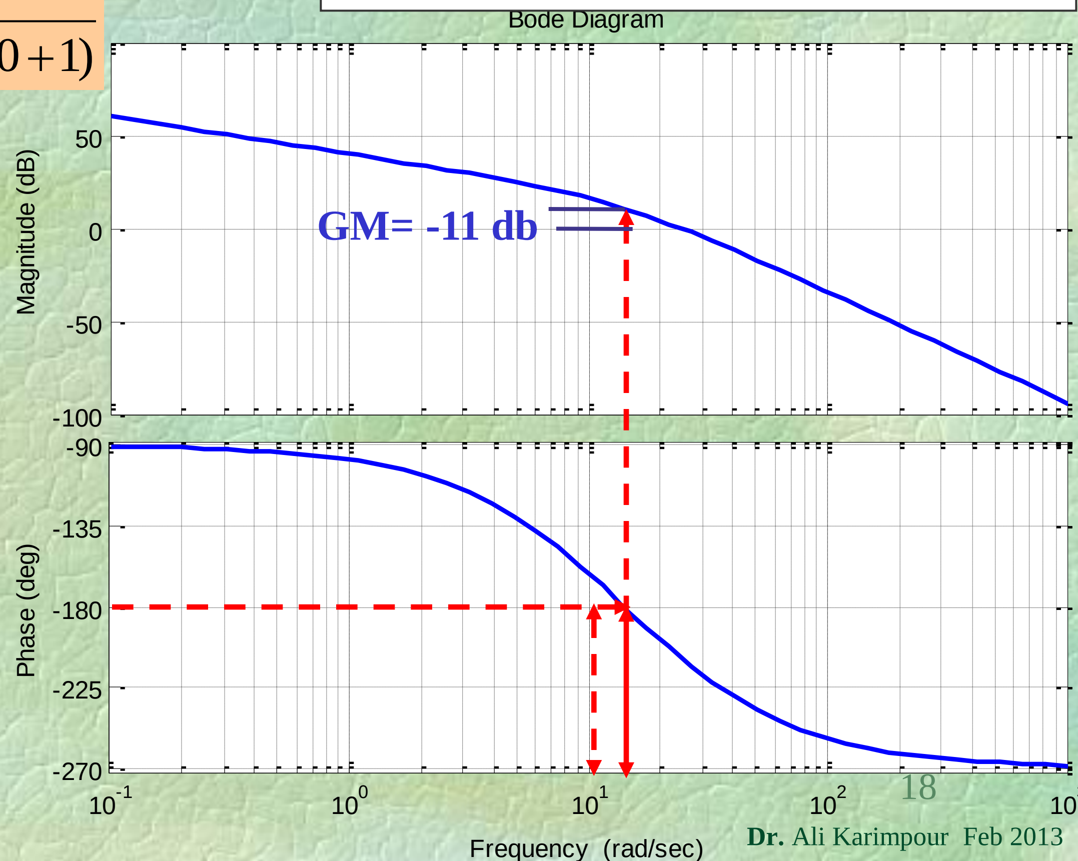


Step 3: Sketch the Bode plot of the system (with the fixed k) without controller.

$$G(s)=\frac{20000}{s(s+10)(s+20)}=\frac{100}{s(s/10+1)(s/20+1)}$$

Step 4: According to desired PM (GM) choose the new gain crossover frequency (Phase crossover frequency). (reduce it a little).

$$\cancel{\omega_{cross}^{new}=15} \qquad \omega_{cross}^{new}=15$$



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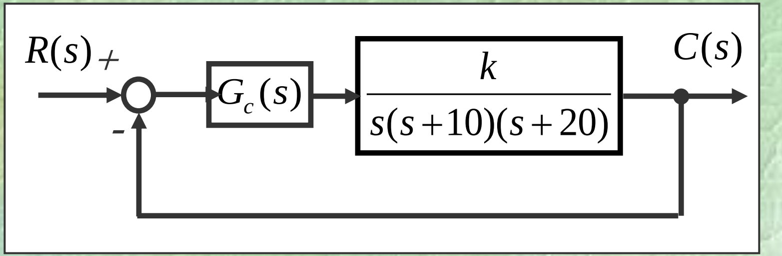


Step 5: Find the required gain by lag controller and derive the parameter a.

$$\text{Required gain}\,\big|_{\text{db}} + 20\log(a) = 0$$

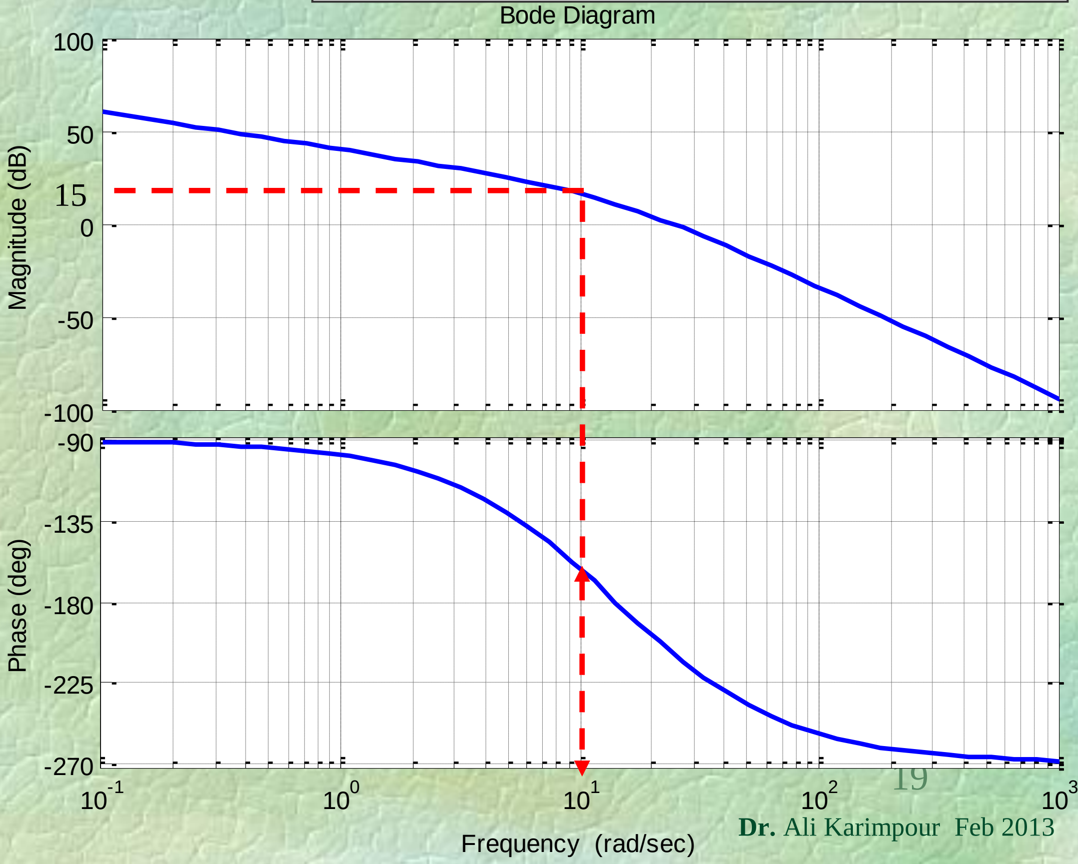
$$25 + 20\log(a) = 0$$

$$a = 10^{-\frac{25}{20}} = 0.056$$

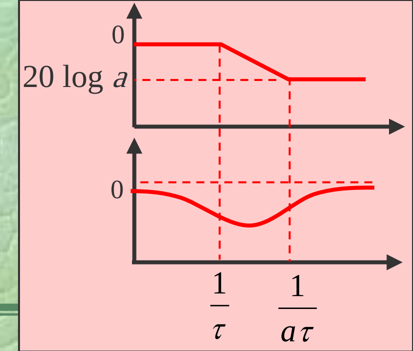
Step 6: Put the right corner of the controller sufficiently far from crossover frequency.

$$\frac{1}{a\tau} = \frac{10}{10} = 1 \qquad \tau = 17.9$$

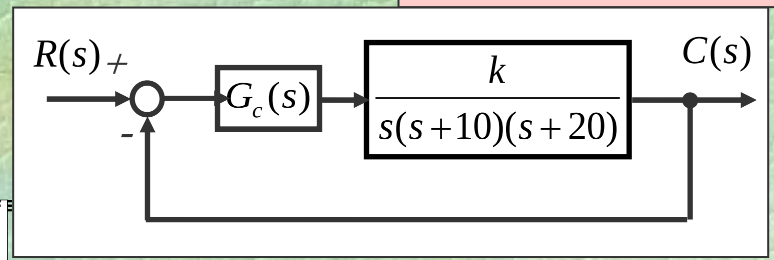
$$G_c(s) = \frac{a\tau s + 1}{\tau s + 1} = \frac{s+1}{17.9s+1}$$

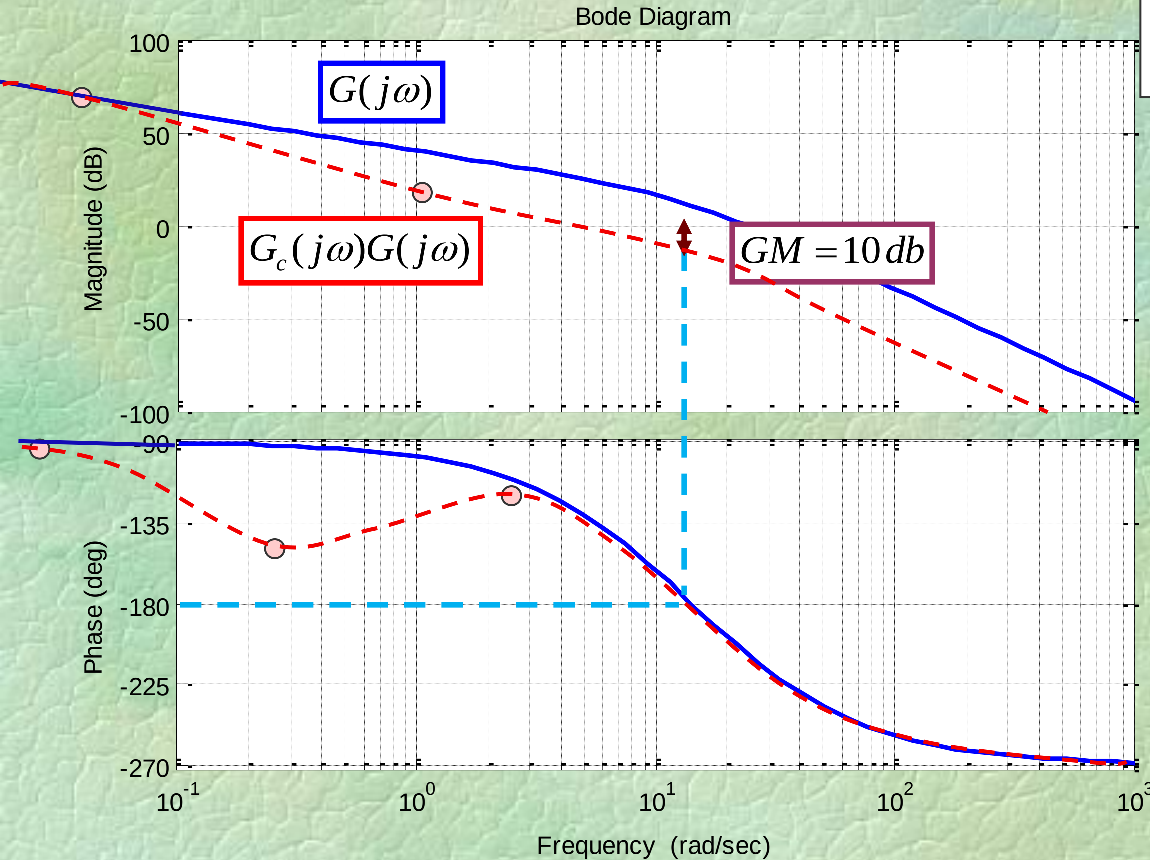


Example 3: Design a lag controller for the following system such that the gain margin be 10 db and the ramp error constant be 100.



Step 7: Check the designed controller.





$$G_c(s)=\frac{a\tau s+1}{\tau s+1}=\frac{0.056\times 17.9s+1}{17.9s+1}$$

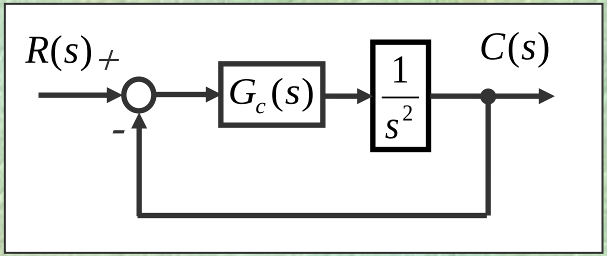
$$1/\tau=0.06,\ 1/a\tau=1$$

$$20\log a=-25db$$

$$1/\sqrt{a\tau}=0.25$$

$$\sin\varphi_m=\frac{0.056-1}{0.056+1}\Rightarrow\varphi_m=-63^\circ$$

Example 4: Design a lag controller for the following system such that the phase margin be 45° and the open loop bandwidth be 10 rad/sec



It is not possible explain why?

Exercises

1 Derive the gain margin of the compensated system in example 1.

2 Derive the phase margin of the compensated system in example 3.

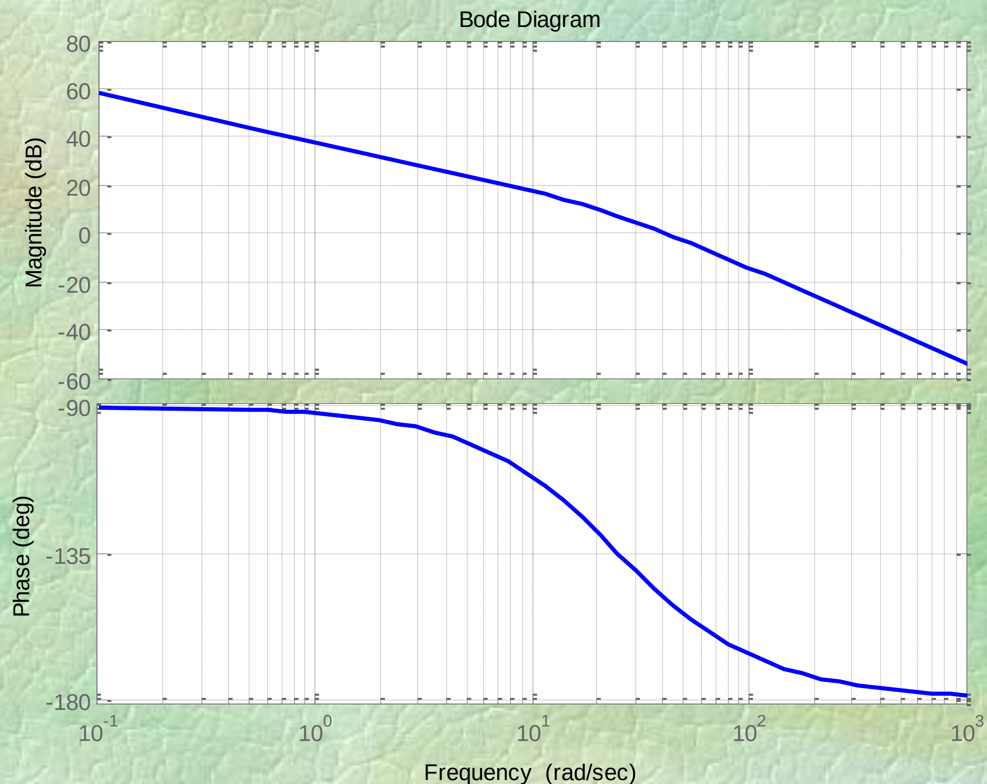
Exercises

3 Following is the open loop transfer function of a system.

a) What is the velocity error constant. (answer 80)

b) Design lag controller such that PM=45°. (answer $G_c(s)=(0.5s+1)/(1.56s+1)$)

c) Design lag controller such that PM=45° and the velocity error constant be 200. (answer $G_c(s)=(0.49s+1)/(3.75s+1)$)



Exercises

4 Following is the open loop transfer function of a system.

a) Design a controller that the GM of system be 50 db.

b) Design lag controller such that PM=45° and the velocity error constant be 30.