LINEAR CONTROL SYSTEMS

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Lecture 23

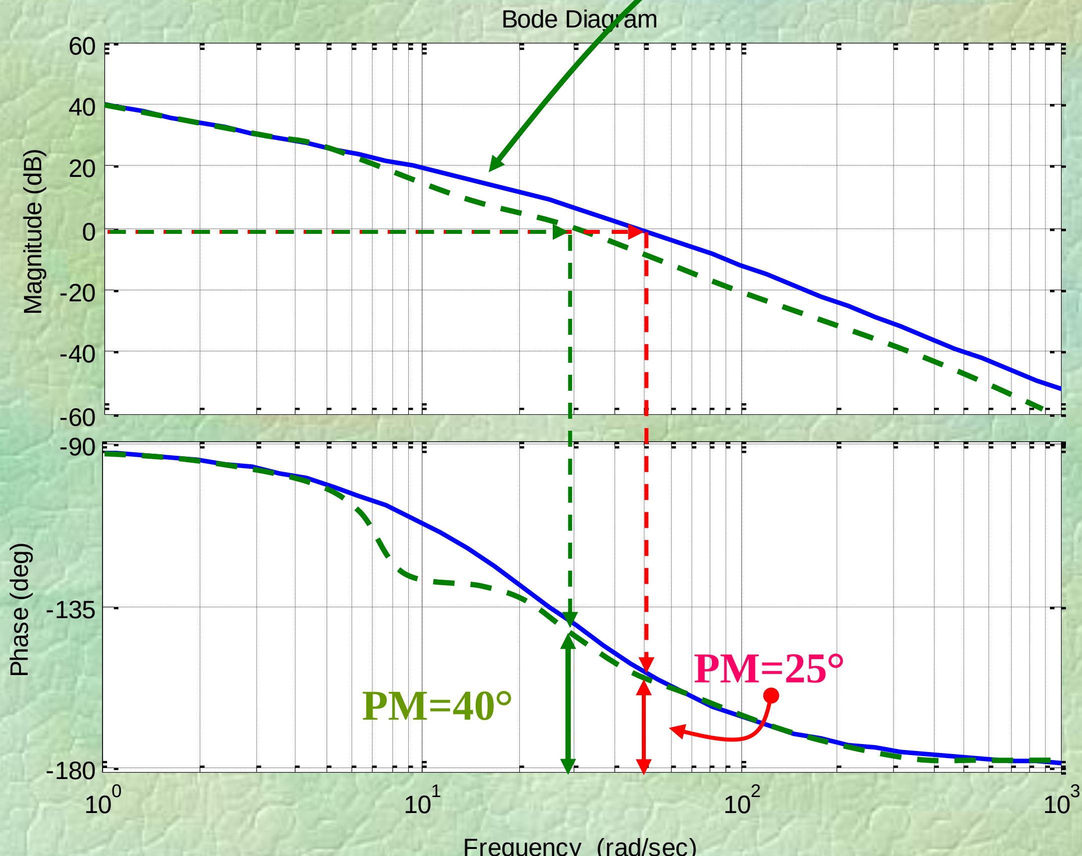
Controller design in the frequency domain

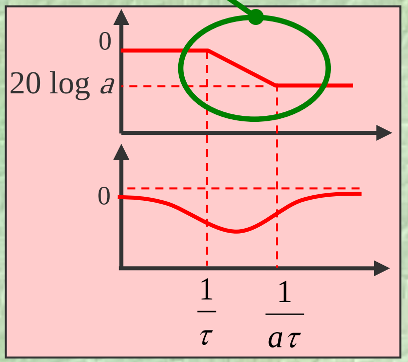
Topics to be covered include:

- Lead controller design

Design fundamental of a lag controller (remember)

Consider a minimum phase system.





BW reduced.

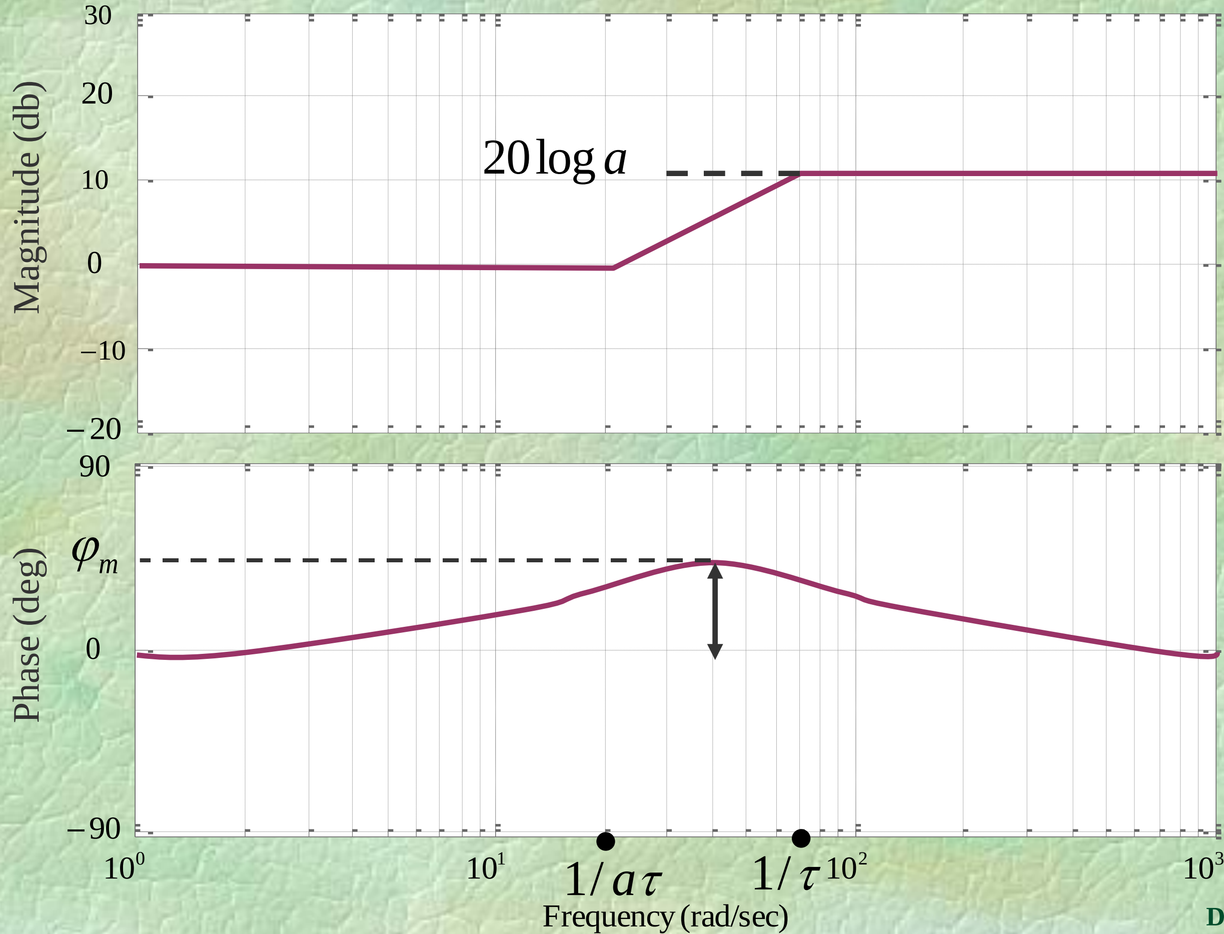
Speed of system reduced.

Effect of the noise is reduced.

A phase-lead controller

$$G(s) = \frac{a\tau s + 1}{\tau s + 1}$$

$Let \quad a > 1$

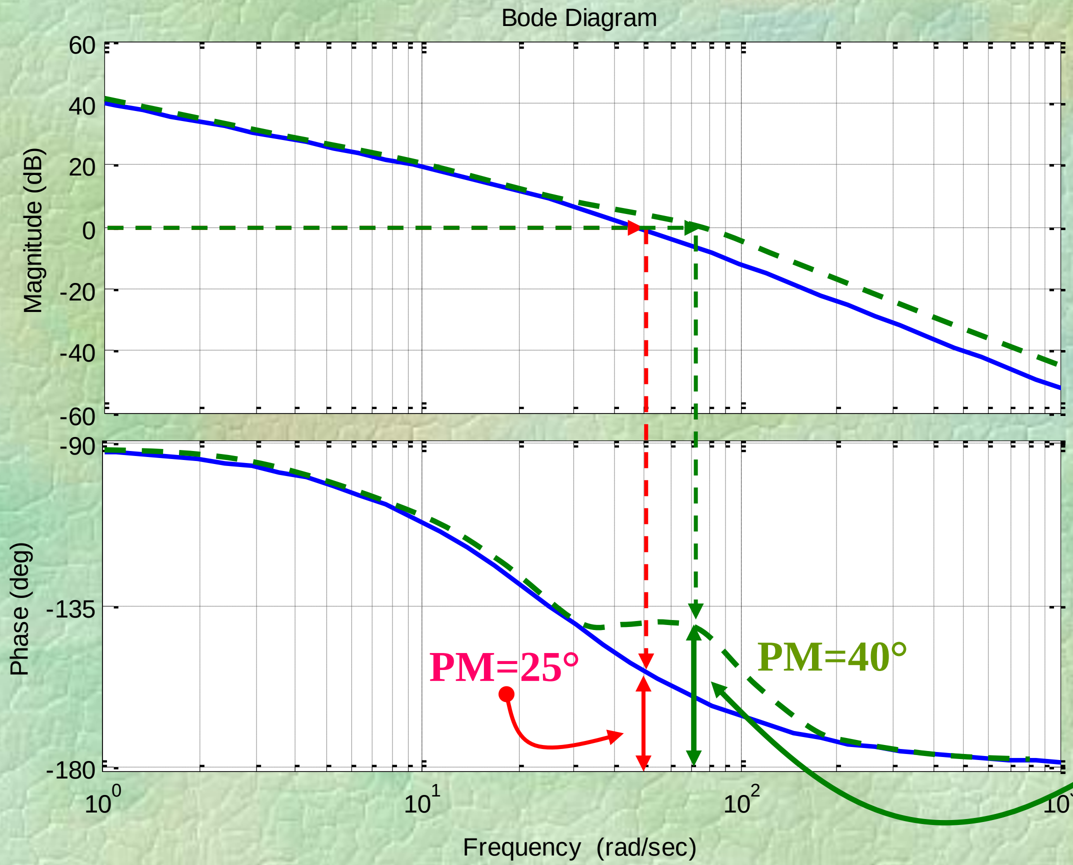


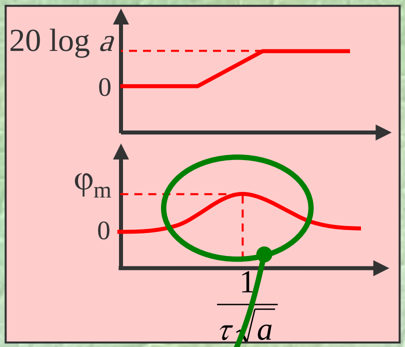
$$\sin\varphi_m = \frac{a-1}{a+1}$$

Design fundamental of the lead controller

Analysis Design

Consider a minimum phase system.



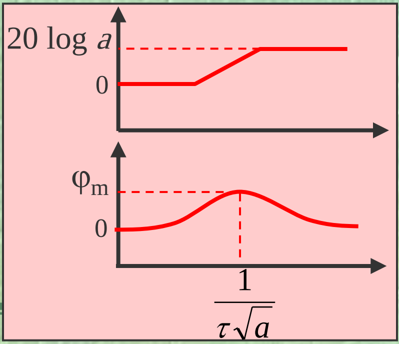


How can the lead controller help us?

What is the effect of a lead controller on BW? Speed of system?

What is the effect of a lead controller on noise effect?

Design procedure of the phase-lead controller in the frequency domain



Step 1: Consider $G_c(s) = k\dfrac{a\tau s+1}{\tau s+1}$ with $a>1$ as a phase lead controller.

Note: If the plant has another gain *k*, let $G_c(s) = \dfrac{a\tau s+1}{\tau s+1}$

Step 2: Try to fix *k* according to the performance request, otherwise let *k*=1

Step 3: Sketch the Bode plot of the system (with the fixed *k*) without controller.

Step 4: Find the system PM and if it is not sufficient choose the required phase by:

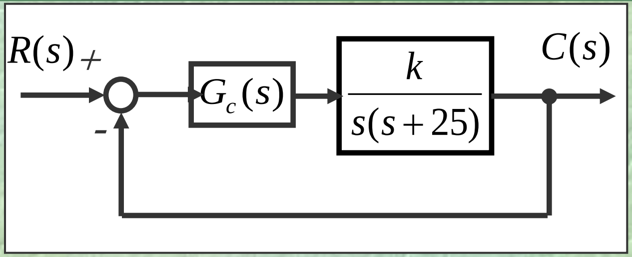
$$\varphi_m = \text{Desired PM - Existed PM} + \Delta \quad ? \qquad \sin\varphi_m = \frac{a-1}{a+1} \quad \Rightarrow \quad a =$$

Step 5: Put the center of the controller in the new gain crossover frequency:

$$20\log\left|G(j\omega)\right|_{\omega=\omega_c^{new}} + 10\log(a) = 0 \qquad \omega_c^{new} = \qquad \omega_c^{new} = \frac{1}{\tau\sqrt{a}} \quad \Rightarrow \quad \tau =$$

Step 6: Check the controller.

Example 1: Design a lead controller for the following system such that the phase margin be 45° and the ramp error constant be 100. Find the M_p of overall system.



Step 1: Consider $G_c(s) = k\dfrac{a\tau s+1}{\tau s+1}$ with $a>1$ as a phase-lead controller.

Note: If the plant has another gain *k*, let $G_c(s) = \dfrac{a\tau s+1}{\tau s+1}$

$$G_c(s) = \frac{a\tau s+1}{\tau s+1} \qquad a>1$$

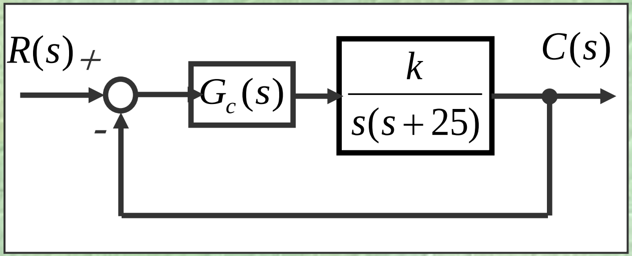
Step 2: Try to fix *k* according to the performance request, otherwise let $k=1$

$$k_v = \lim_{s\to 0} sG_c(s)\frac{k}{s(s+25)} = 100$$

$$k = 2500$$

Example 1: Design a lead controller for the following system such that the phase margin be 45° and the ramp error constant be 100. Find the M_p of overall system.

Step 3: Sketch the Bode plot of the system (with the fixed k) without controller.



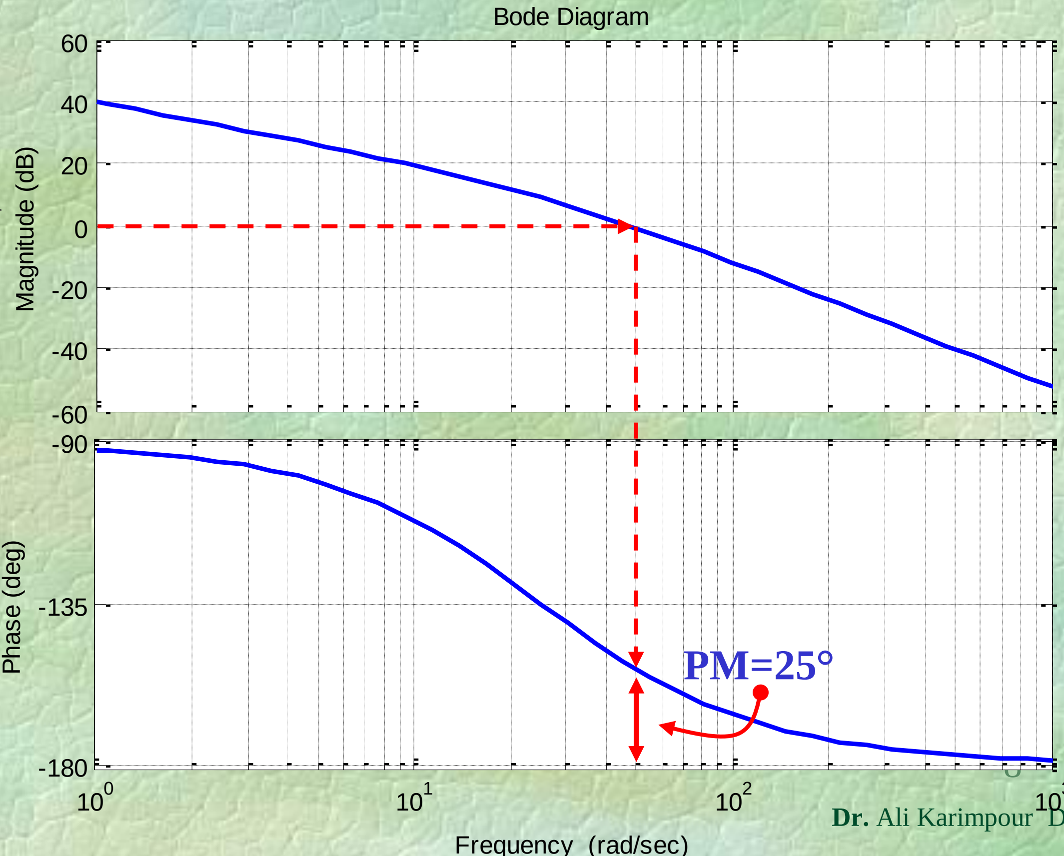
$$G(s)=\frac{2500}{s(s+25)}=\frac{100}{s(s/25+1)}$$

Step 4: Find the system PM and if it is not sufficient choose the required phase by:

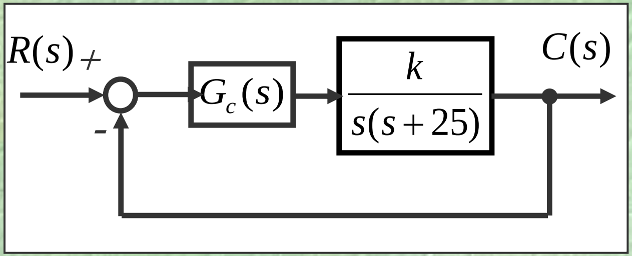
$$\varphi_m = \text{Desired PM - Existed PM} + \Delta$$

$$\varphi_m = 45^\circ - 25^\circ + 5^\circ = 25^\circ$$

$$\sin 25 = \frac{a-1}{a+1} \quad \Rightarrow \quad a = 2.46$$



Example 1: Design a lead controller for the following system such that the phase margin be 45° and the ramp error constant be 100. Find the M_p of overall system.



Step 5: Put the center of the controller in the new gain crossover frequency:

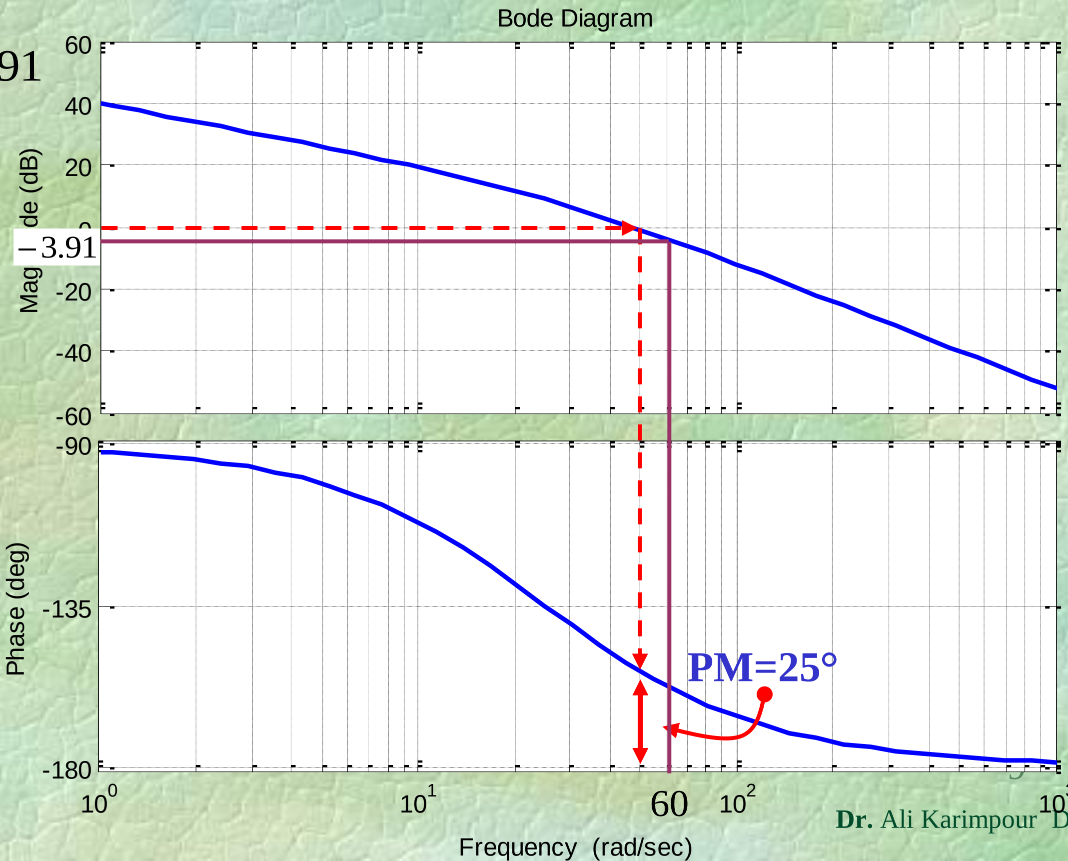
$$20\log|G(j\omega)|_{\omega=\omega_c^{new}} + 10\log(a) = 0$$

$$20\log|G(j\omega)|_{\omega=\omega_c^{new}} = -10\log(a) = -3.91$$

$$\omega_c^{new} = 60$$

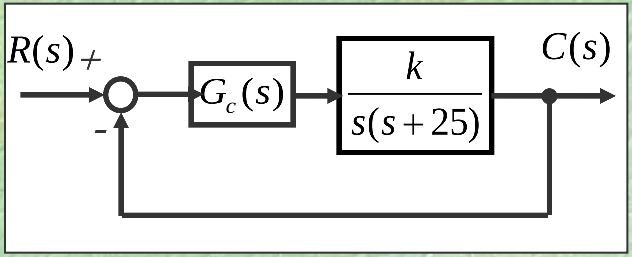
$$\omega_c^{new} = \frac{1}{\tau\sqrt{a}} \Rightarrow \quad \tau = 0.0106$$

$$G_c(s) = \frac{a\tau s+1}{\tau s+1} = \frac{0.0261s+1}{0.0106s+1}$$



Example 1: Design a lead controller for the following system such that the phase margin be 45° and the ramp error constant be 100. Find the M_p of overall system.

Step 6: Check the controller.



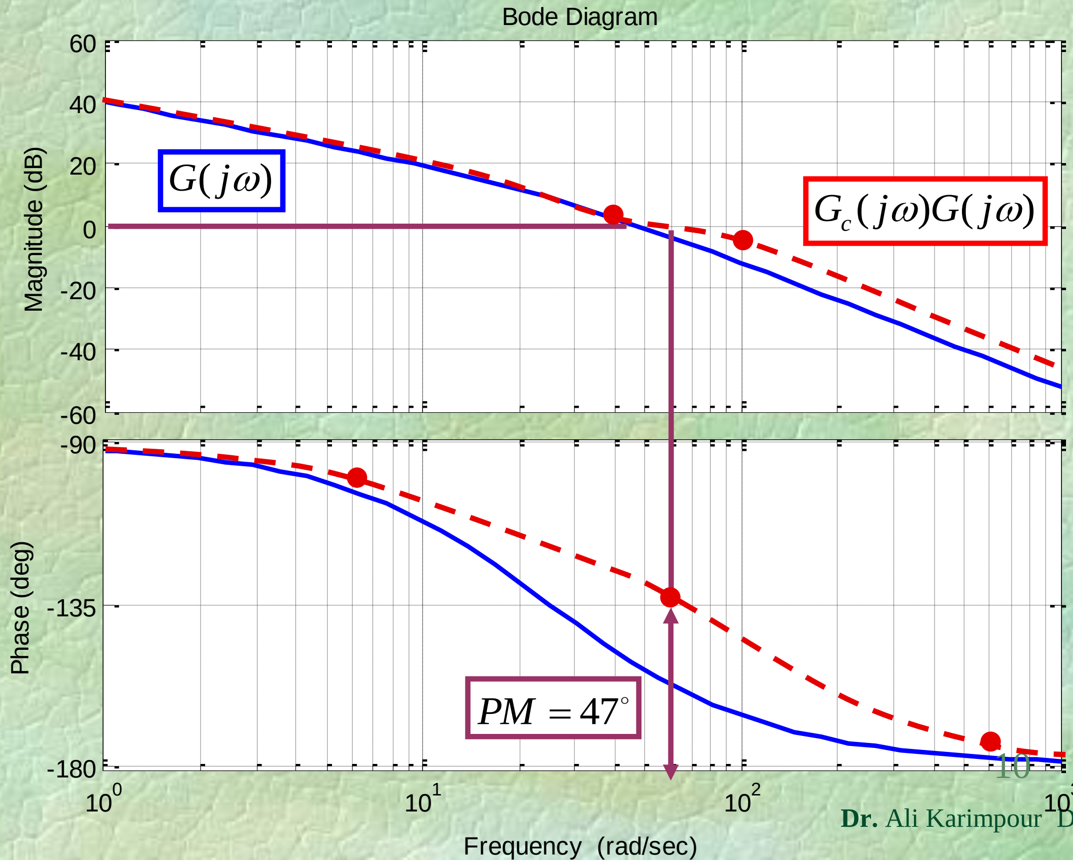
$$G_c(s) = \frac{0.0261s + 1}{0.0106s + 1}$$

$$1/\tau = 94.33,\ 1/a\tau = 38.3$$

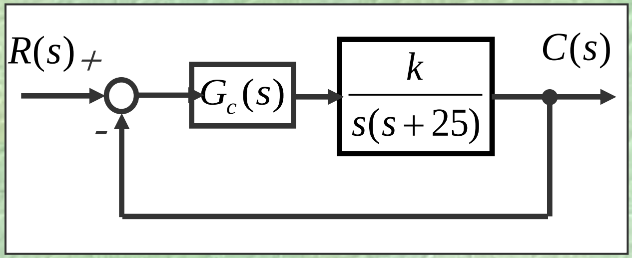
$$20\log a = 7.8$$

$$1/\sqrt{a}\tau = 60$$

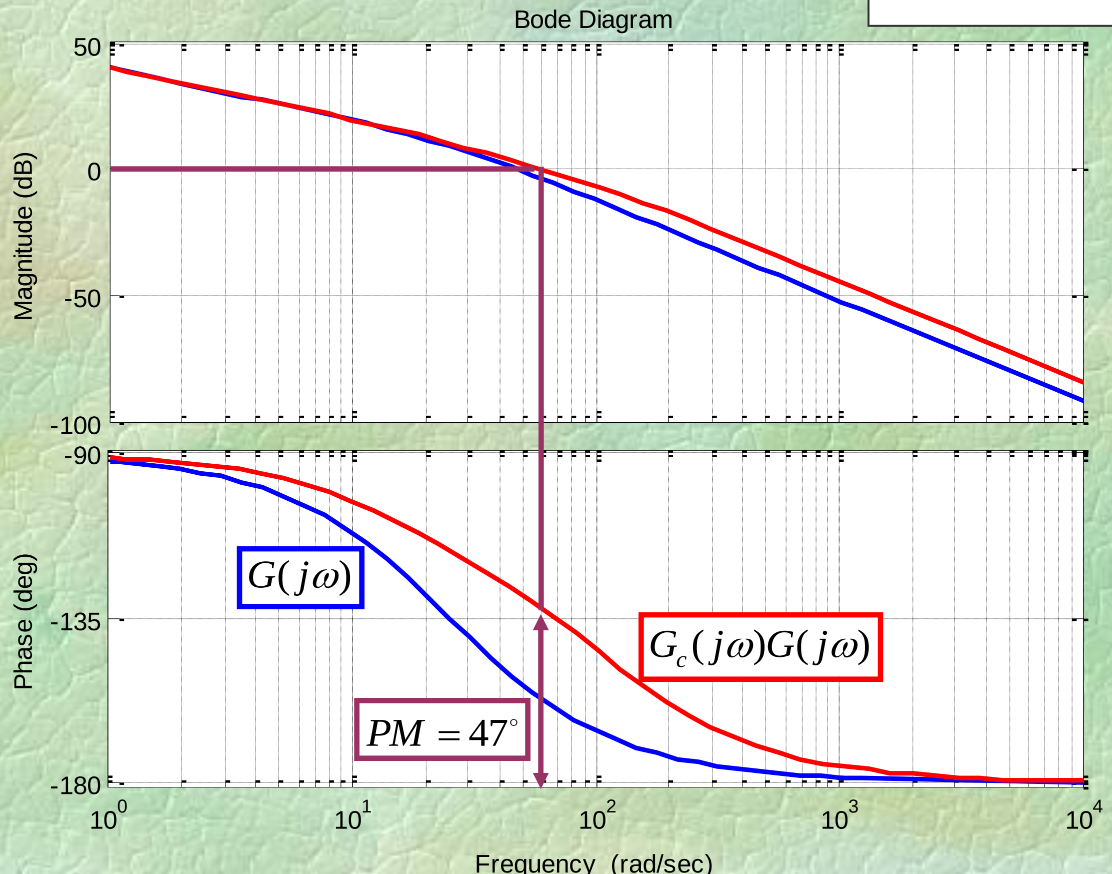
$$\varphi_m = 25^\circ$$



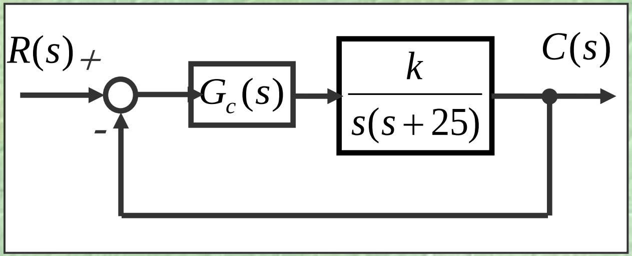
Example 1: Design a lead controller for the following system such that the phase margin be 45° and the ramp error constant be 100. Find the M_p of overall system.



Step 6: Check the controller.

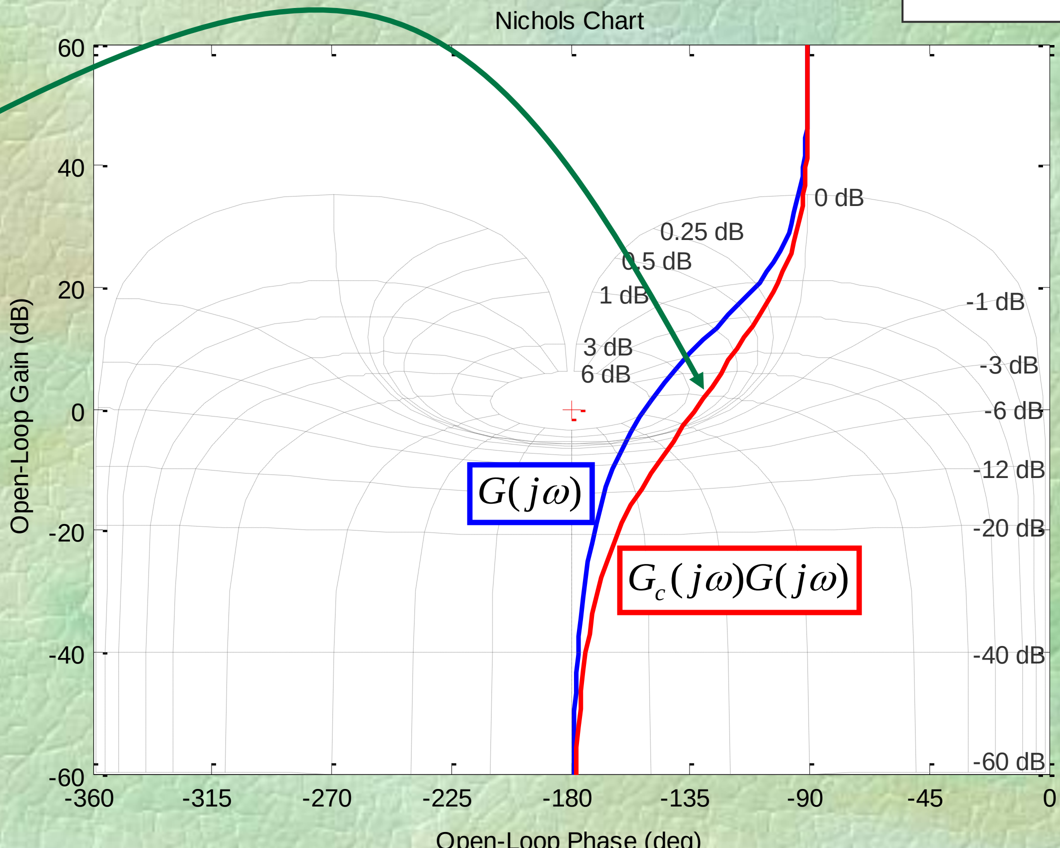


Example 1: Design a lead controller for the following system such that the phase margin be 45° and the ramp error constant be 100. Find the M_p of overall system.



Find M_p

M_p= 2 db



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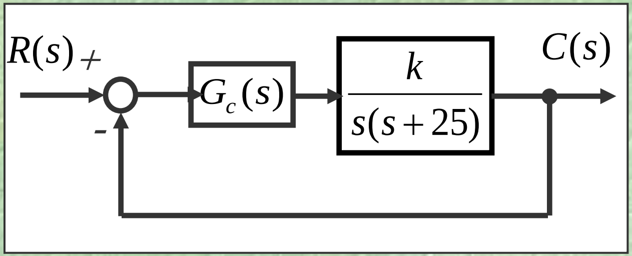
Example 2: Find the step response of example 1 and compare it with lag design in pervious lectures.



$$G_{clead}(s) = \frac{0.0261s+1}{0.0106s+1}$$ Lead controller of example 1

$$G_{clag}(s) = \frac{0.5s+1}{1.4092s+1}$$ Lag controller of example 1 lecture 29

$$M_1(s) = \frac{G(s)}{1+G(s)} = \frac{2500}{s^2+25s+2500}$$ Closed loop transfer function without controller

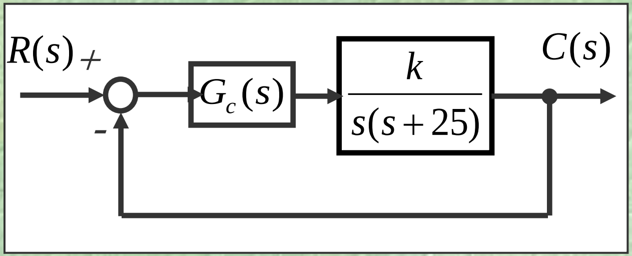
$$M_2(s) = \frac{G_{clead}(s).G(s)}{1+G_{clead}(s).G(s)} = \frac{6156s+235800}{s^3+119.3s^2+8514s+235800}$$ Closed loop transfer function with a phase-lead controller

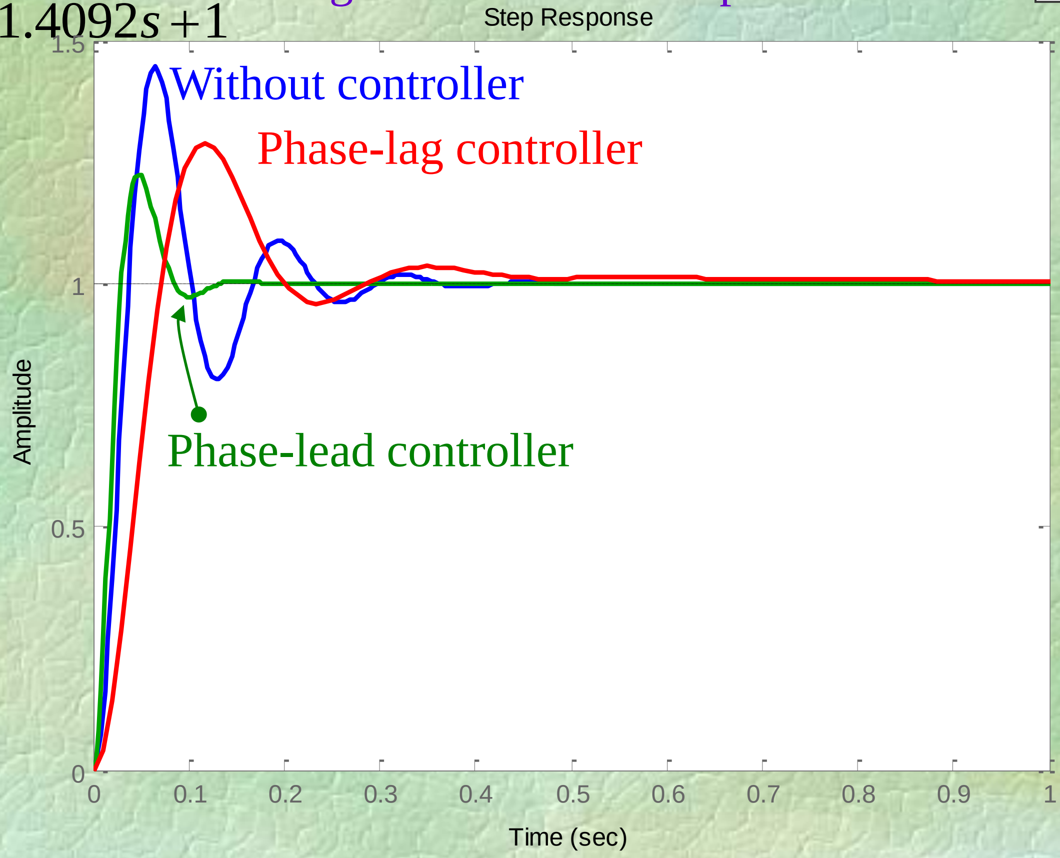
$$M_3(s) = \frac{G_{clag}(s).G(s)}{1+G_{clag}(s).G(s)} = \frac{887s+1774}{s^3+25.71s^2+904.8s+1774}$$ Closed loop transfer function with a phase-lag controller

Example 2: Find the step response of example 1 and compare it with lag design in pervious lectures.

$$G_{clead}(s) = \frac{0.0261s+1}{0.0106s+1}$$ Lead controller of example 1

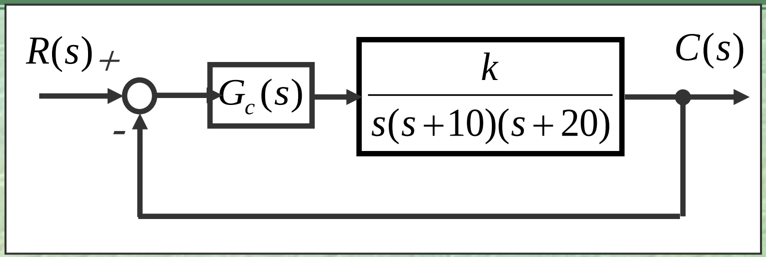
$$G_{clag}(s) = \frac{0.5s+1}{1.4092s+1}$$ Lag controller of example 1





Check the speed of different controllers and compare it with the controller properties in slide #4 and #6

Example 3: Design a lead controller for the following system such that the phase margin be 45° and the ramp error constant be 100.



Step 1: Consider $G_c(s) = k\dfrac{a\tau s+1}{\tau s+1}$ with $a>1$ as a phase-lead controller.

Note: If the plant has another gain *k*, let $G_c(s) = \dfrac{a\tau s+1}{\tau s+1}$

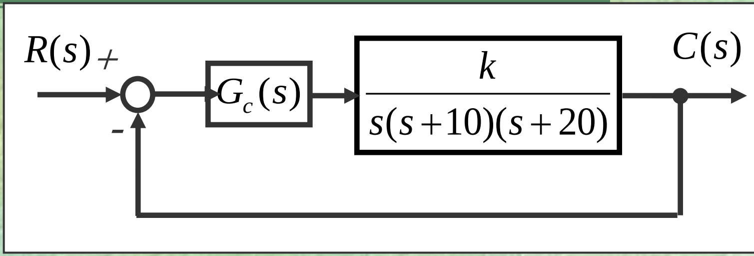
$$G_c(s) = \frac{a\tau s+1}{\tau s+1} \qquad a>1$$

Step 2: Try to fix *k* according to performance request, otherwise let *k*=1

$$k_v = \lim_{s\to 0} sG_c(s)\frac{k}{s(s+10)(s+20)} = 100$$

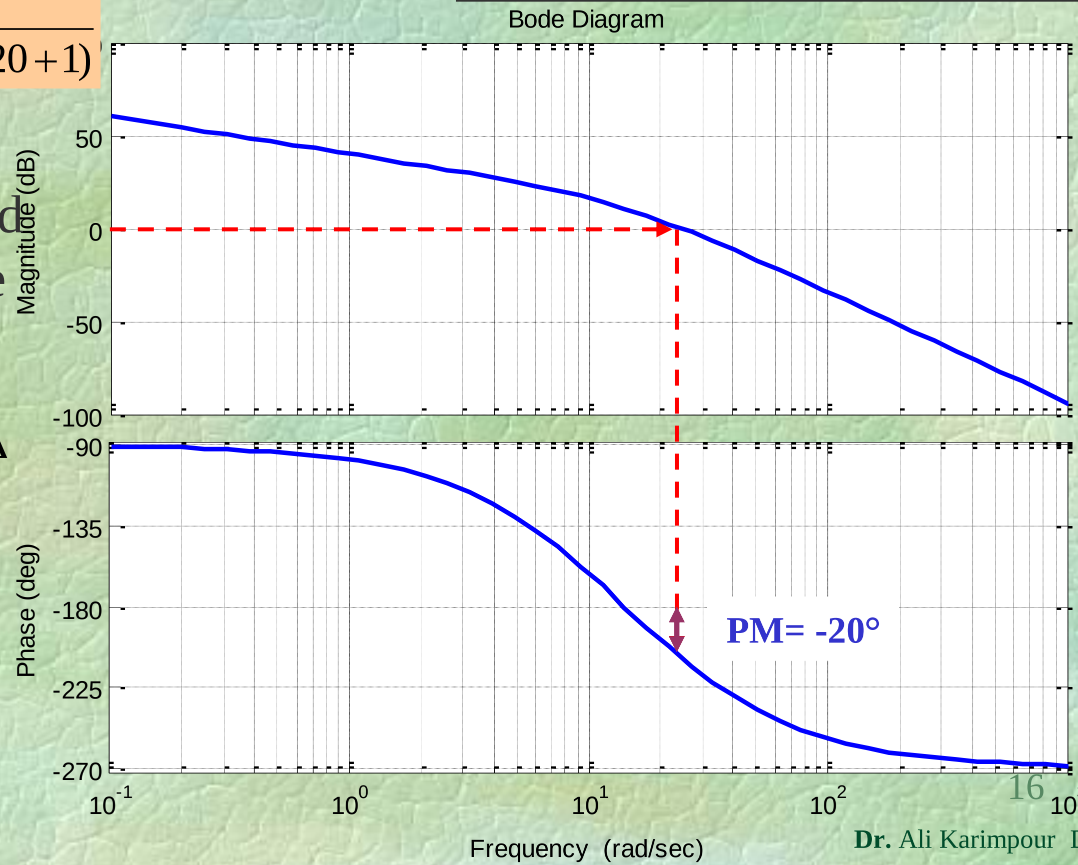
$$k = 20000$$

Example 3: Design a lead controller for the following system such that the phase margin be 45° and the ramp error constant be 100.



Step 3: Sketch the Bode plot of system (with the fixed k) without controller.

$$G(s)=\frac{20000}{s(s+10)(s+20)}=\frac{100}{s(s/10+1)(s/20+1)}$$



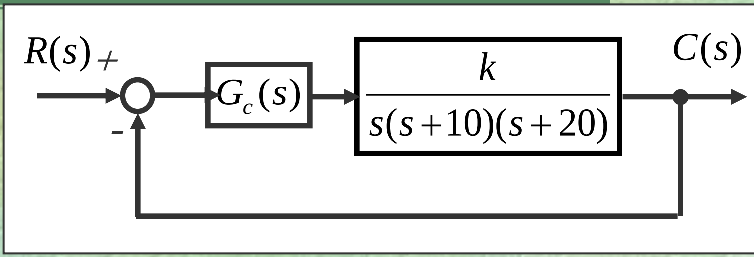
Step 4: Find the system PM and if it is not sufficient choose the required phase by:

$$\varphi_m = \text{Desired PM - Existed PM} + \Delta$$

$$\varphi_m = 45^\circ - (-20) + 5^\circ = 70^\circ$$

$$\sin 70 = \frac{a-1}{a+1} \quad \Rightarrow \quad a = 32$$

Example 3: Design a lead controller for the following system such that the phase margin be 45° and the ramp error constant be 100.



Step 5: Put the center of controller in the new gain crossover frequency:

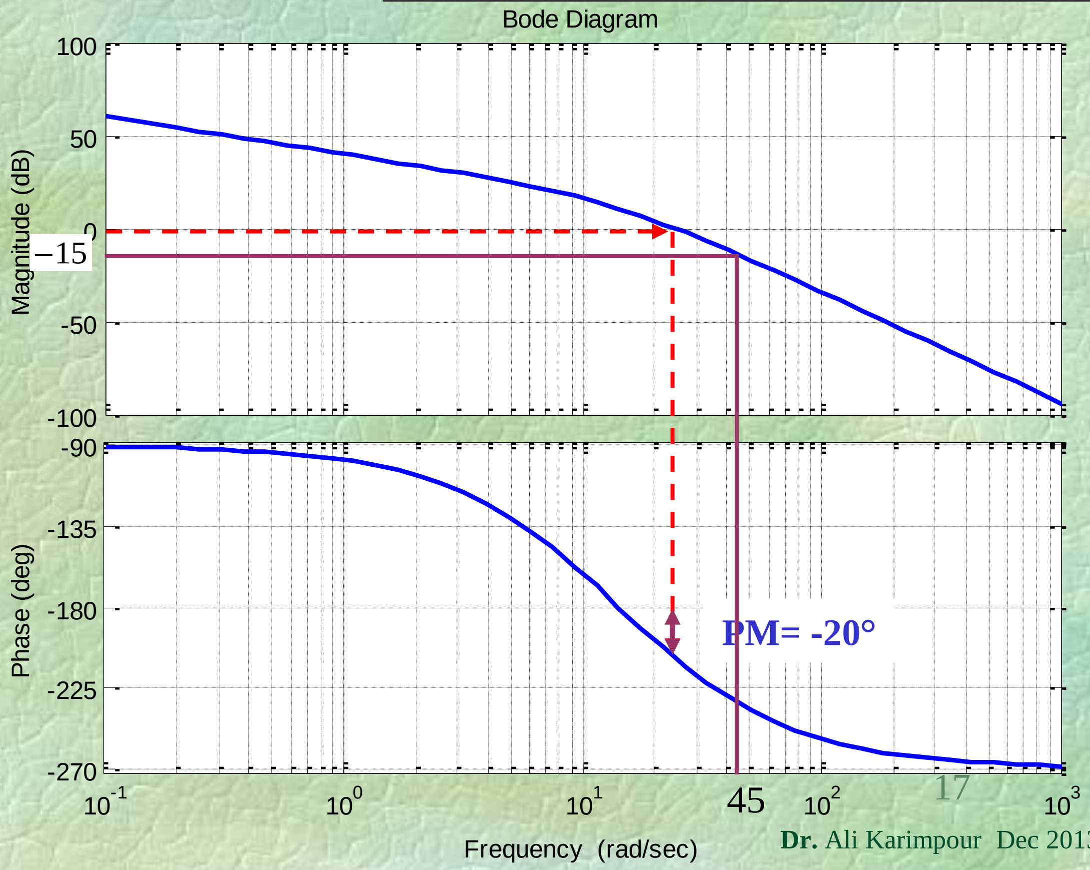
$$20\log\left|G(j\omega)\right|_{\omega=\omega_c^{new}} + 10\log(a) = 0$$

$$20\log\left|G(j\omega)\right|_{\omega=\omega_c^{new}} = -10\log(a) = -15$$

$$\omega_c^{new} = 45$$

$$\omega_c^{new} = \frac{1}{\tau\sqrt{a}} \quad \Rightarrow \quad \tau = 0.0039$$

$$G_c(s) = \frac{a\tau s+1}{\tau s+1} = \frac{0.1248s+1}{0.0039s+1}$$



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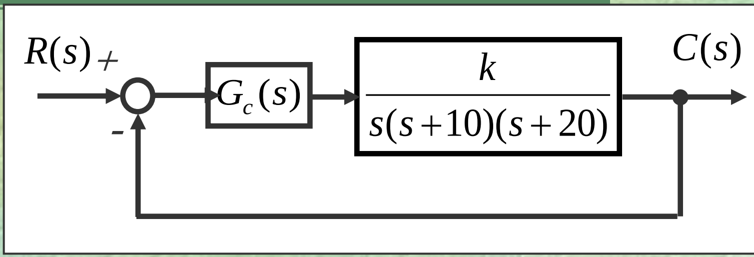
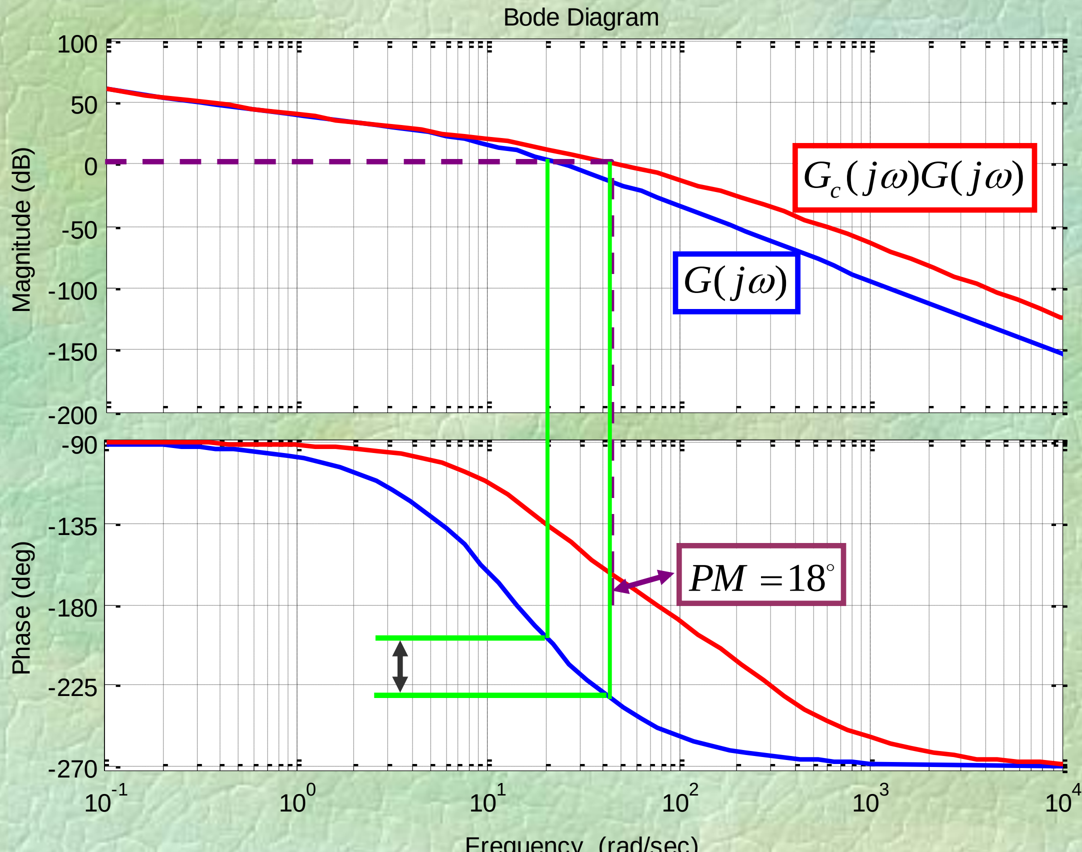
Example 3: Design a lead controller for the following system such that the phase margin be 45° and the ramp error constant be 100.



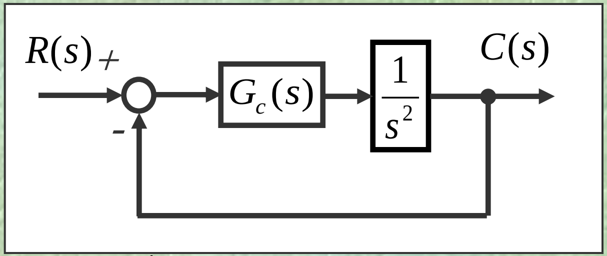
Step 6: Check the controller.



Note: Design is not possible!

Why?

Example 4: Design a lead controller for the following system such that the phase margin be 45° and the open loop bandwidth be 10 rad/sec



Step 1: Consider $G_c(s) = k\dfrac{a\tau s+1}{\tau s+1}$ with $a>1$ as a phase-lag controller.

Note: If the plant has another gain k, let $G_c(s) = \dfrac{a\tau s+1}{\tau s+1}$

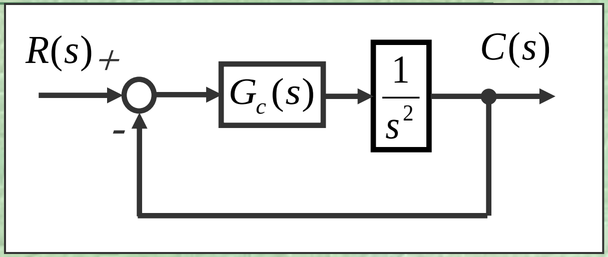
$$\Rightarrow \quad G_c(s) = k\frac{a\tau s+1}{\tau s+1} \qquad a>1$$

Step 2: Try to fix k according to performance request, otherwise let k=1

$$20\log\left|\frac{k}{\omega^2}\right|_{\omega=5} = -3 \quad \Rightarrow \quad k = 17.7$$

Why?

Example 4: Design a lead controller for the following system such that the phase margin be 45° and the open loop bandwidth be 10 rad/sec



Step 3: Sketch the Bode plot of system (with the fixed k) without controller.

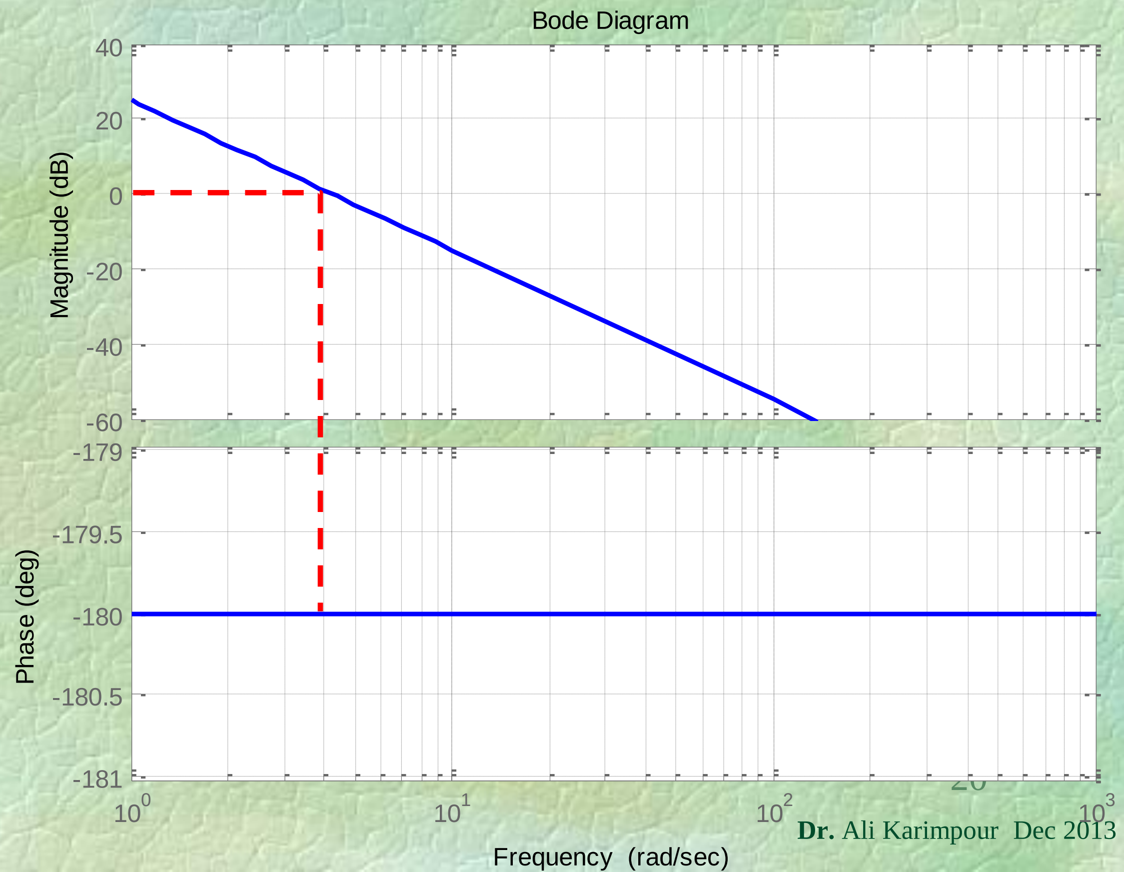
$$G(s) = \frac{17.7}{s^2}$$

Step 4: Find the system PM and if it is not sufficient choose the required phase by:

$$\varphi_m = \text{Desired PM - Existed PM} + \Delta$$

$$\varphi_m = 45^\circ - (0) + 0^\circ \quad = 45^\circ$$

$$\sin 45^\circ = \frac{a-1}{a+1} \quad \Rightarrow \quad a = 5.8$$



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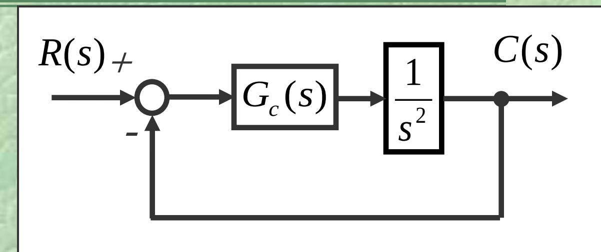
Example 4: Design a lead controller for the following system such that the phase margin be 45° and the open loop bandwidth be 10 rad/sec

Step 5: Put the center of controller in the new gain crossover frequency:



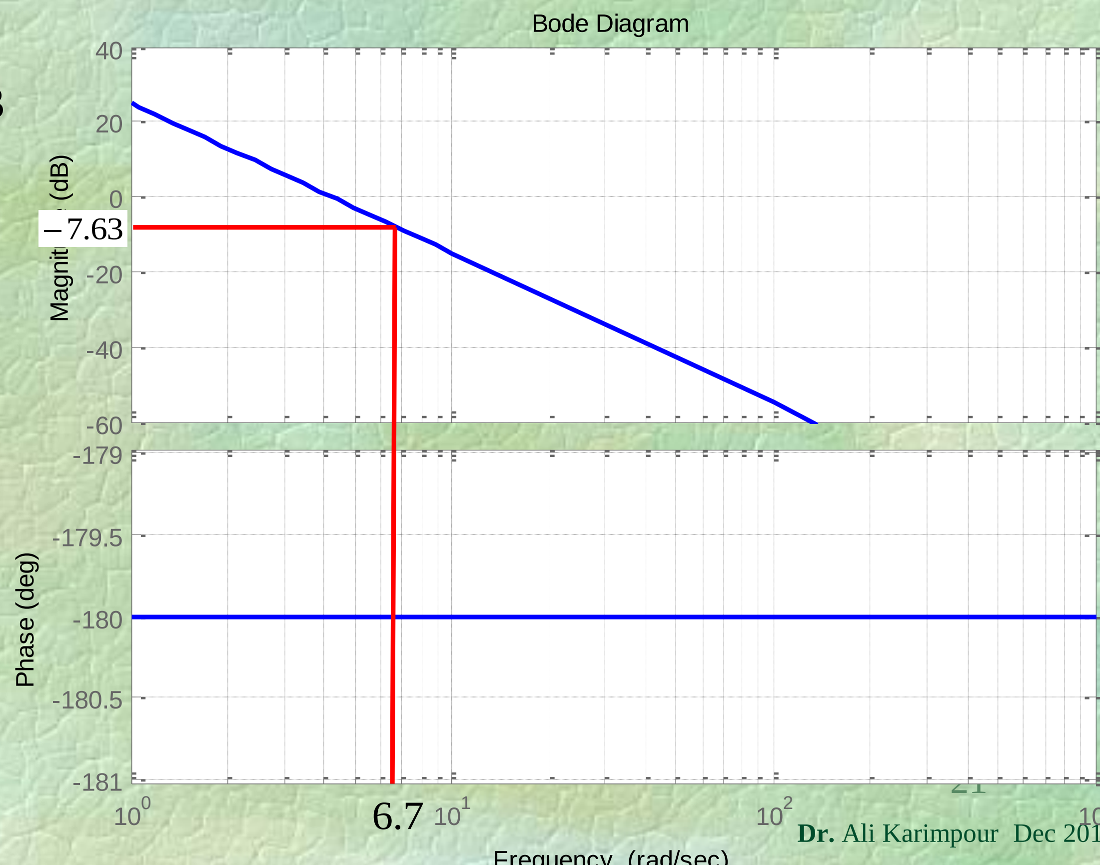
$$20\log\left|G(j\omega)\right|_{\omega=\omega_c^{new}} + 10\log(a) = 0$$

$$20\log\left|G(j\omega)\right|_{\omega=\omega_c^{new}} = -10\log(a) = -7.63$$

$$\omega_c^{new} = 6.7$$

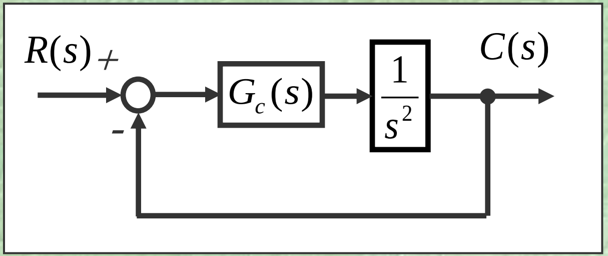
$$\omega_c^{new} = \frac{1}{\tau\sqrt{a}} \quad \Rightarrow \quad \tau = 0.062$$

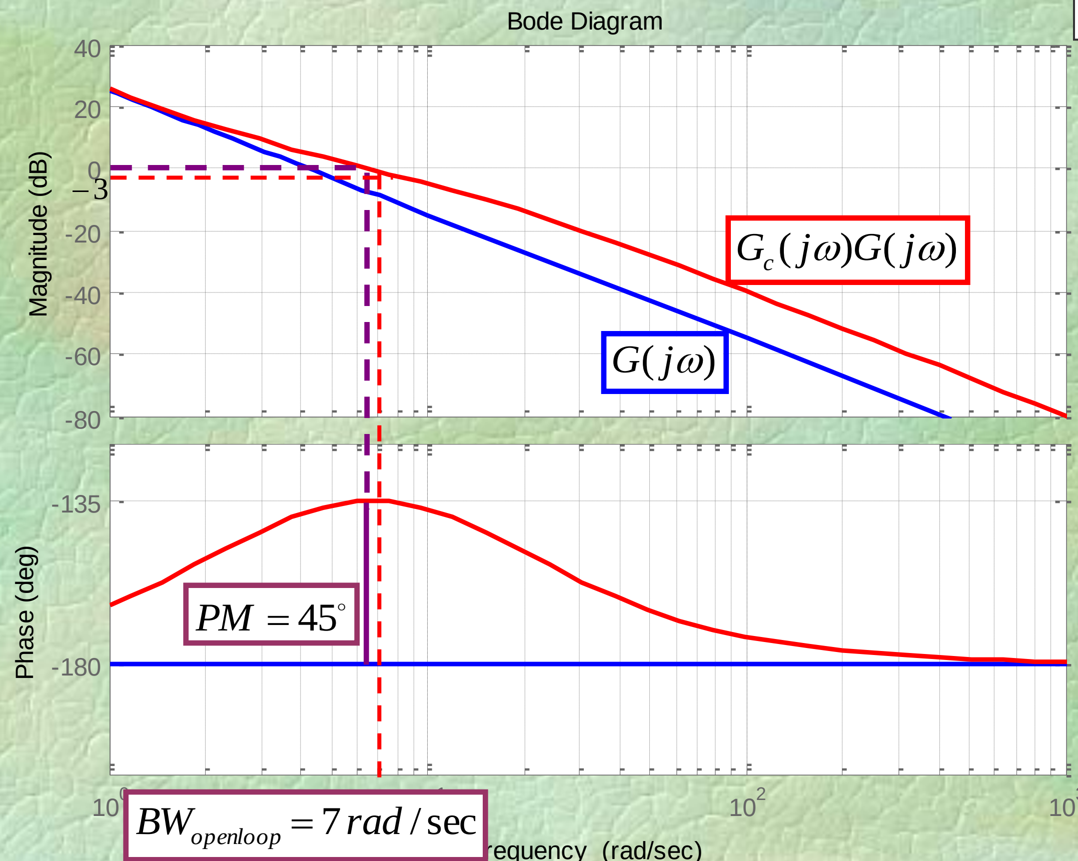
$$G_c(s) = \frac{a\tau s+1}{\tau s+1} = 17.7\frac{0.3596s+1}{0.062s+1}$$



Example 4: Design a lead controller for the following system such that the phase margin be 45° and the open loop bandwidth be 10 rad/sec

Step 6: Check the controller.

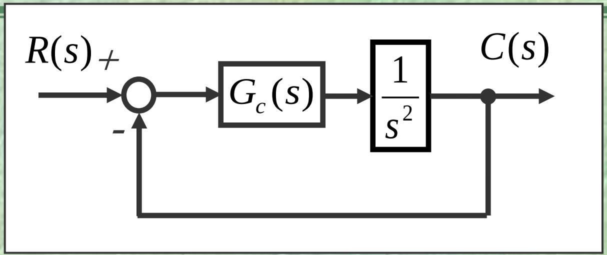




Controller is not ok

Try again

Example 4: Design a lead controller for the following system such that the phase margin be 45° and the open loop bandwidth be 10 rad/sec



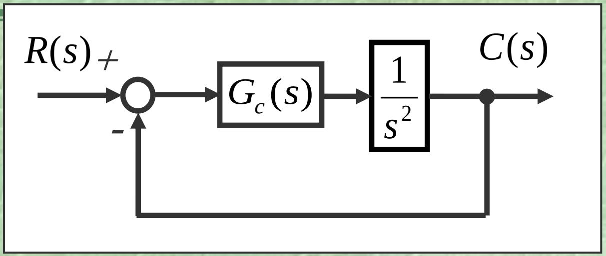
Step 2: Try to fix k according to performance request, otherwise let k=1

Open loop bandwidth is near to gain crossover frequency so:

$$20\log\left|\frac{k}{\omega^2}\right|_{\omega=5}=-3 \quad\Rightarrow\quad k=17.7$$

$$20\log\left|\frac{k}{\omega^2}\right|_{\omega=7}=-3 \quad\Rightarrow\quad k=35.7$$

Example 4: Design a lead controller for the following system such that the phase margin be 45° and the open loop bandwidth be 10 rad/sec



Step 3: Sketch the Bode plot of system (with the fixed k) without controller.

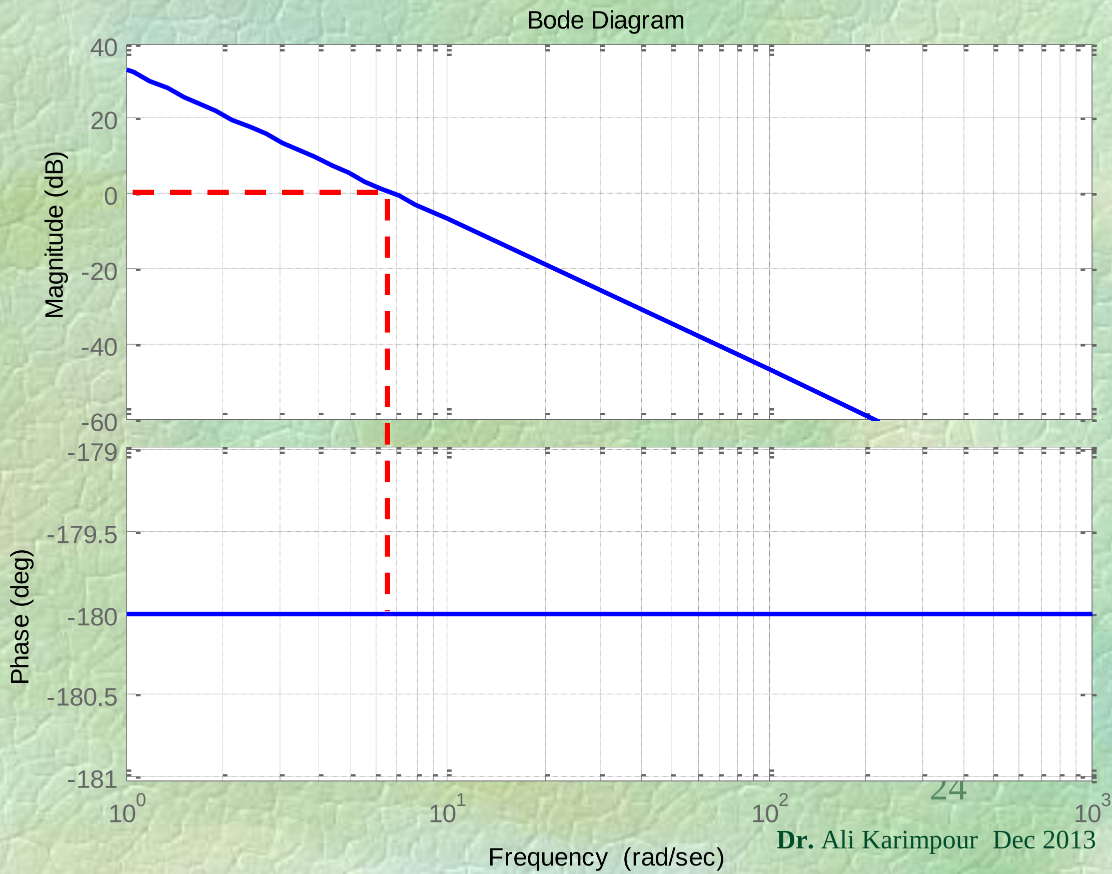
$$G(s) = \frac{35.7}{s^2}$$

Step 4: Find the system PM and if it is not sufficient choose the required phase by:

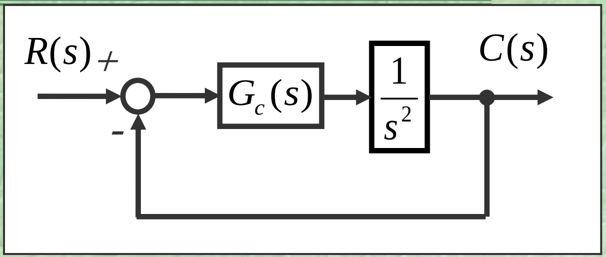
$$\varphi_m = \text{Desired PM - Existed PM} + \Delta$$

$$\varphi_m = 45^\circ - (0) + 0^\circ \quad = 45^\circ$$

$$\sin 45^\circ = \frac{a-1}{a+1} \quad \Rightarrow \quad a = 5.8$$



Example 4: Design a lead controller for the following system such that the phase margin be 45° and the open loop bandwidth be 10 rad/sec



Step 5: Put the center of controller in the new gain crossover frequency:

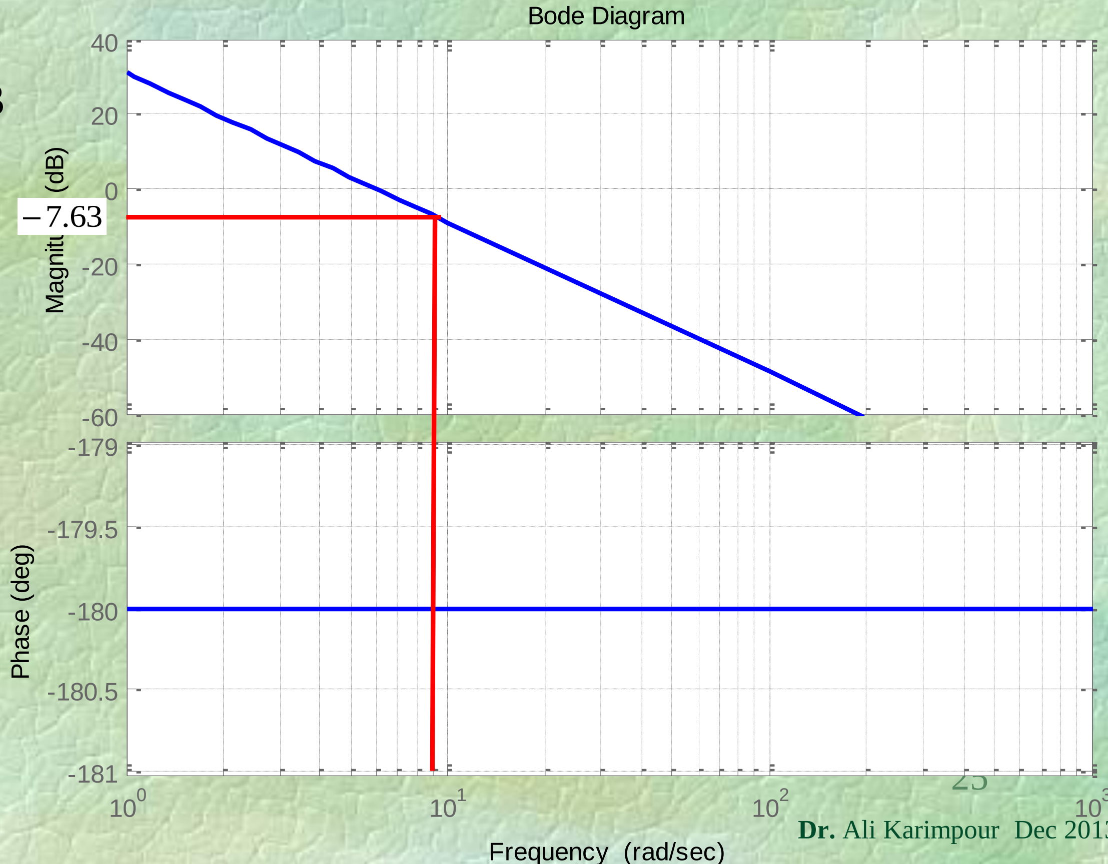
$$20\log\left|G(j\omega)\right|_{\omega=\omega_c^{new}} + 10\log(a) = 0$$

$$20\log\left|G(j\omega)\right|_{\omega=\omega_c^{new}} = -10\log(a) = -7.63$$

$$\omega_c^{new} = 9$$

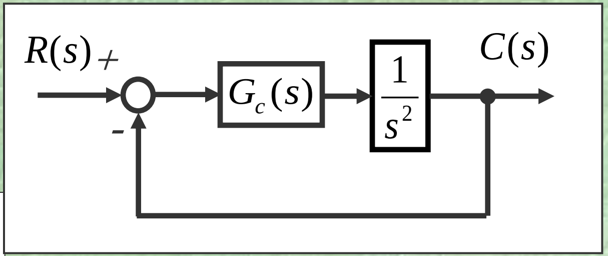
$$\omega_c^{new} = \frac{1}{\tau\sqrt{a}} \quad \Rightarrow \quad \tau = 0.0461$$

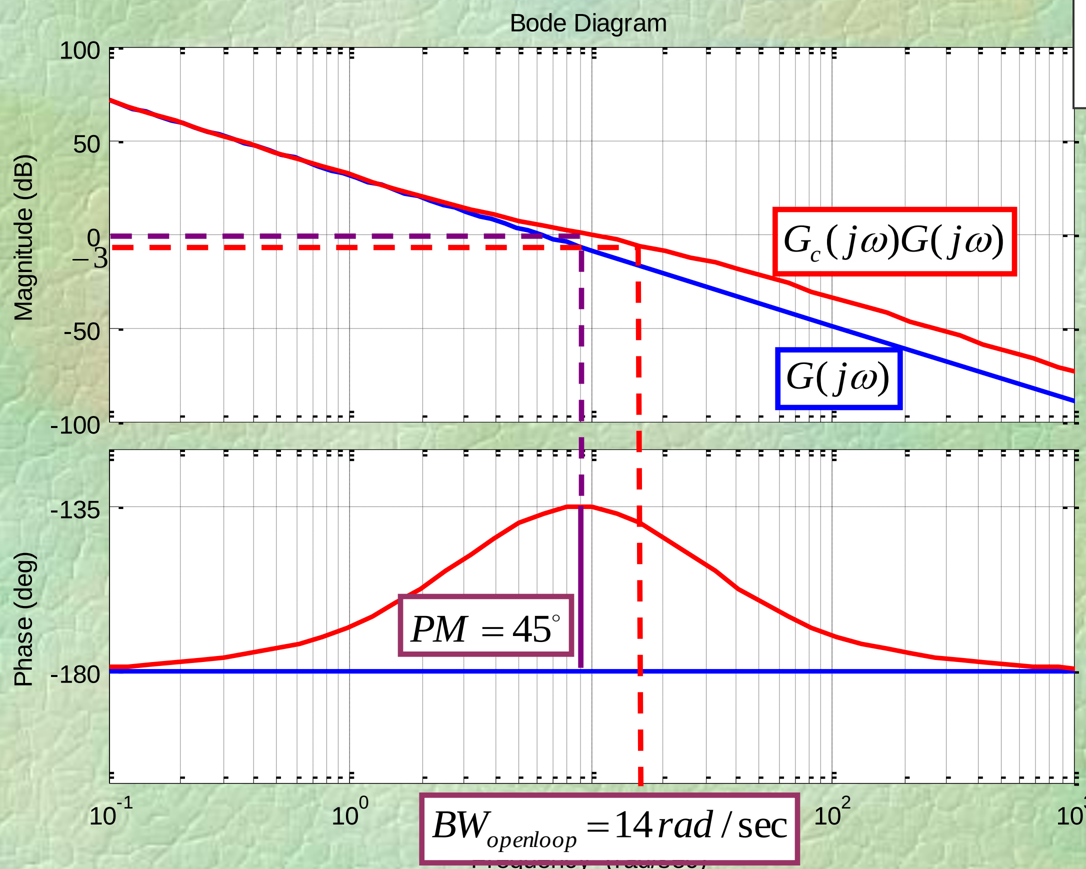
$$G_c(s) = \frac{a\tau s+1}{\tau s+1} = 35.7\,\frac{0.2674s+1}{0.0461s+1}$$



Example 4: Design a lead controller for the following system such that the phase margin be 45° and the open loop bandwidth be 10 rad/sec

Step 6: Check the controller.

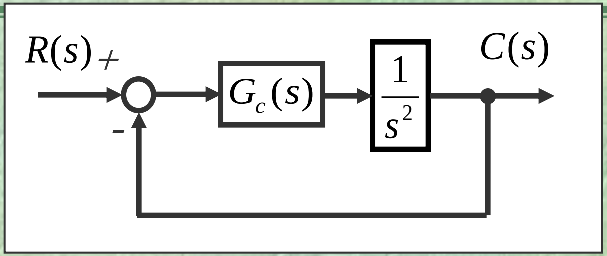




Controller is not ok

Try again

Example 4: Design a lead controller for the following system such that the phase margin be 45° and the open loop bandwidth be 10 rad/sec



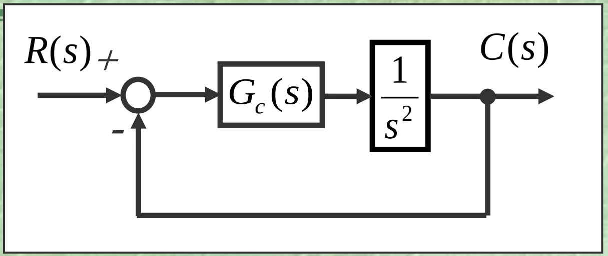
Step 2: Try to fix k according to performance request, otherwise let k=1

Open loop bandwidth is near to gain crossover frequency so:

$$\cancel{20\log\left|\frac{k}{\omega^2}\right|_{\omega=7} = -3} \quad \Rightarrow \quad \cancel{k = 35.7}$$

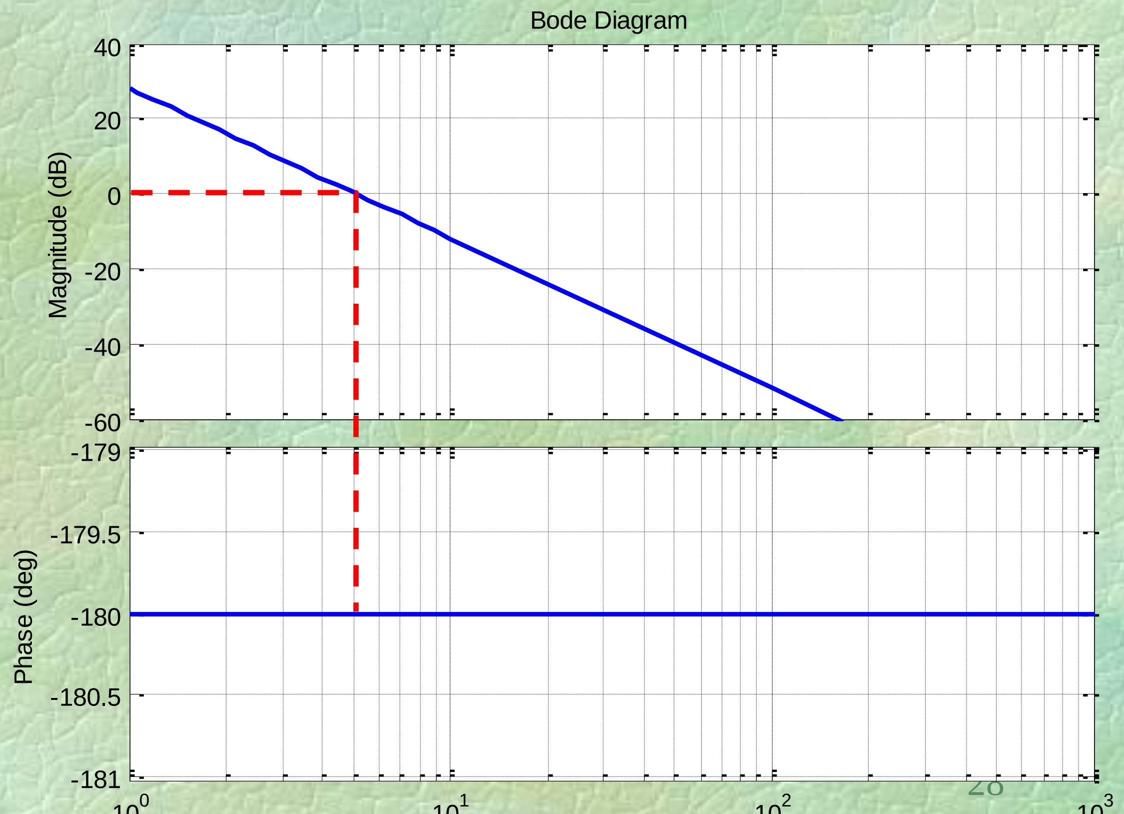
$$20\log\left|\frac{k}{\omega^2}\right|_{\omega=6} = -3 \quad \Rightarrow \quad k = 25.5$$

Example 4: Design a lead controller for the following system such that the phase margin be 45° and the open loop bandwidth be 10 rad/sec



Step 3: Sketch the Bode plot of system (with the fixed k) without controller.

$$G(s) = \frac{25.5}{s^2}$$



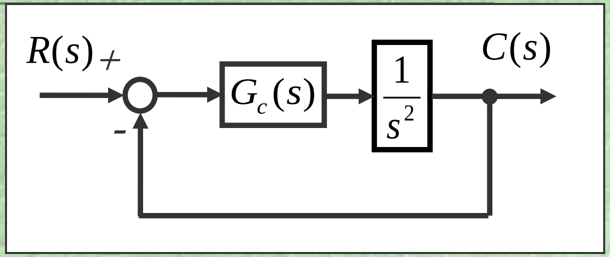
Step 4: Find the system PM and if it is not sufficient choose the required phase by:

$$\varphi_m = \text{Desired PM} - \text{Existed PM} + \Delta$$

$$\varphi_m = 45^\circ - (0) + 0^\circ = 45^\circ$$

$$\sin 45^\circ = \frac{a-1}{a+1} \quad \Rightarrow \quad a = 5.8$$

Example 4: Design a lead controller for the following system such that the phase margin be 45° and the open loop bandwidth be 10 rad/sec



Step 5: Put the center of controller in the new gain crossover frequency:

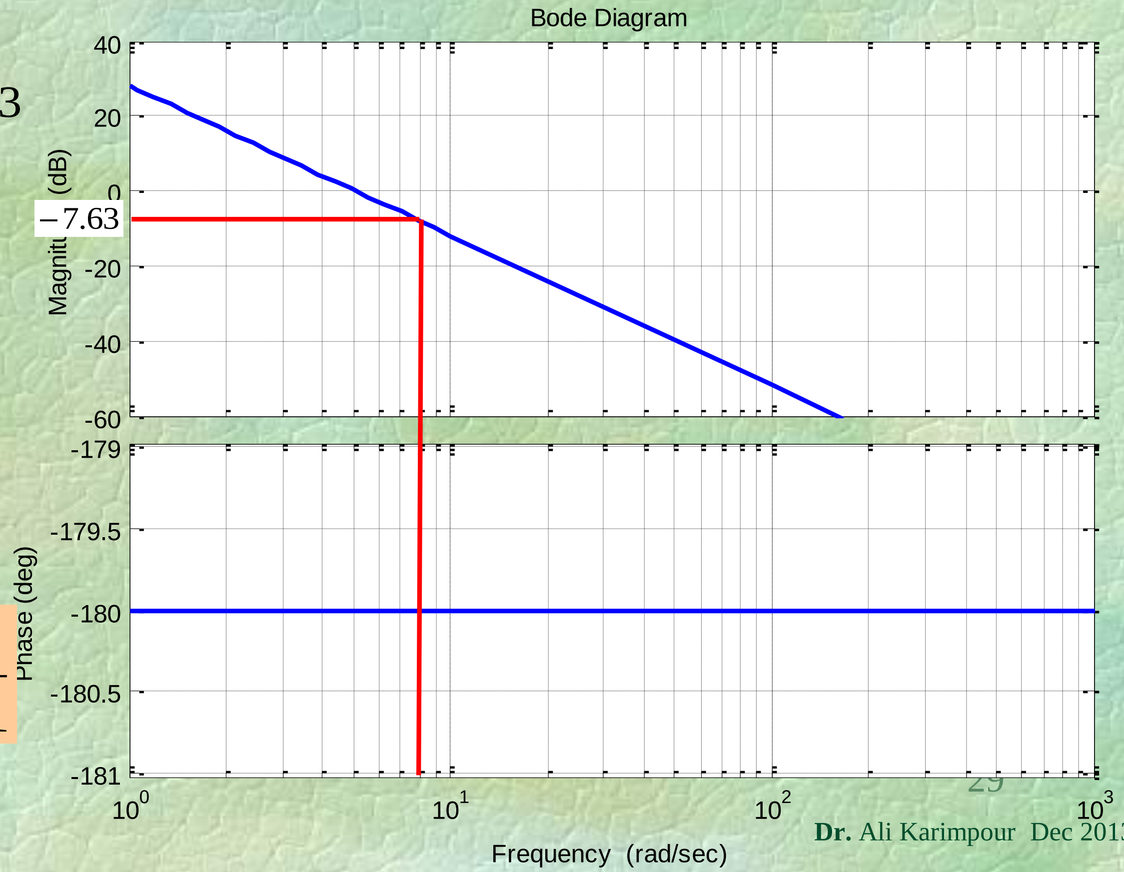
$$20\log\left|G(j\omega)\right|_{\omega=\omega_c^{new}} + 10\log(a) = 0$$

$$20\log\left|G(j\omega)\right|_{\omega=\omega_c^{new}} = -10\log(a) = -7.63$$

$$\omega_c^{new} = 8$$

$$\omega_c^{new} = \frac{1}{\tau\sqrt{a}} \quad \Rightarrow \quad \tau = 0.0519$$

$$G_c(s) = \frac{a\tau s+1}{\tau s+1} = 25.5\frac{0.301s+1}{0.0519s+1}$$



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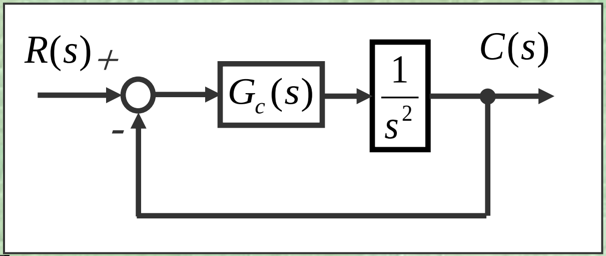
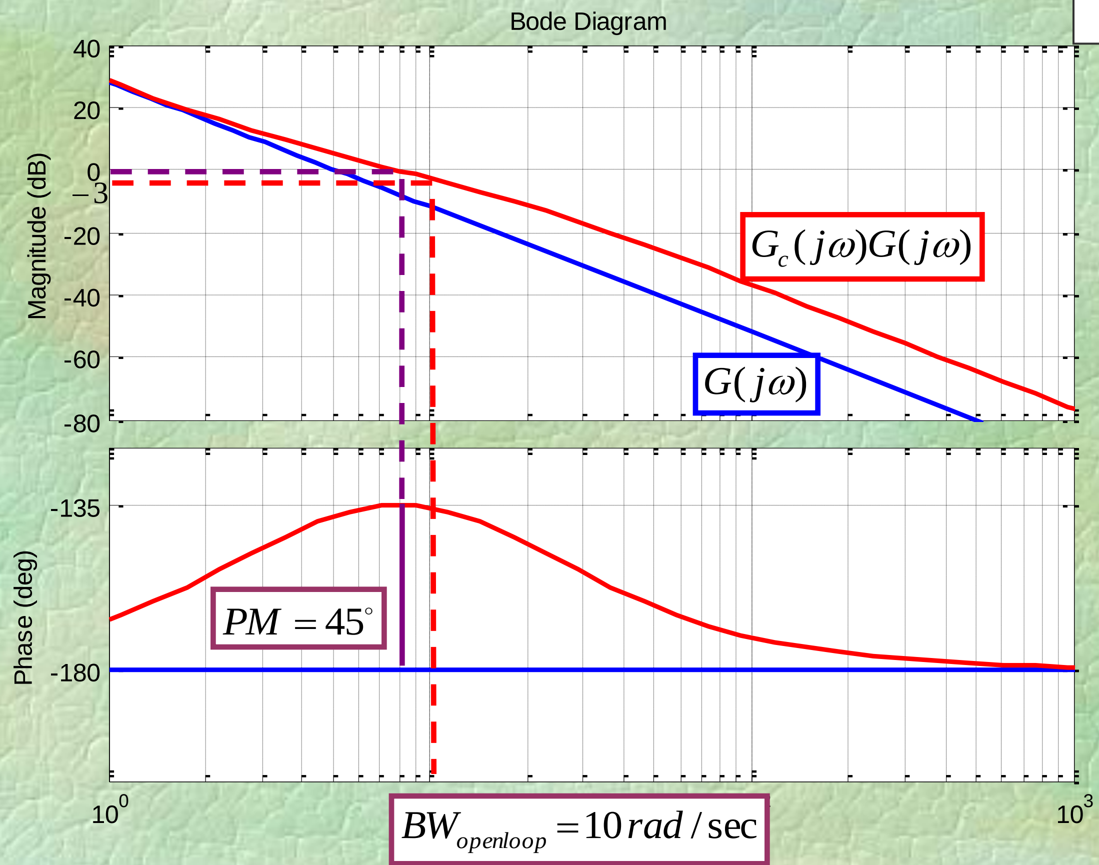
Example 4: Design a lead controller for the following system such that the phase margin be 45° and the open loop bandwidth be 10 rad/sec

Step 6: Check the controller.



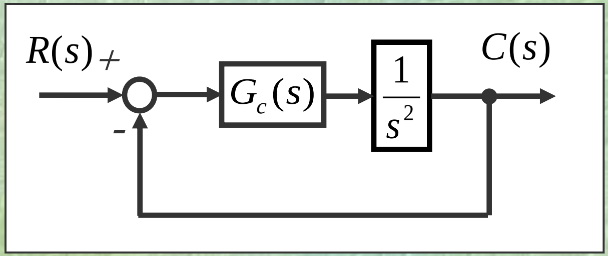


Controller is ok

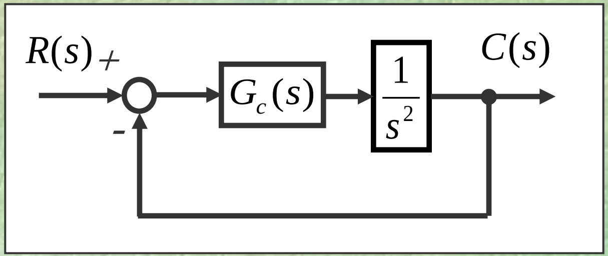
It can be solved by a very better method without trial and error. Try to find it.

Exercises

1 Design a lead controller for the following system such that the phase margin be 40° and the open loop bandwidth be 23 rad/sec



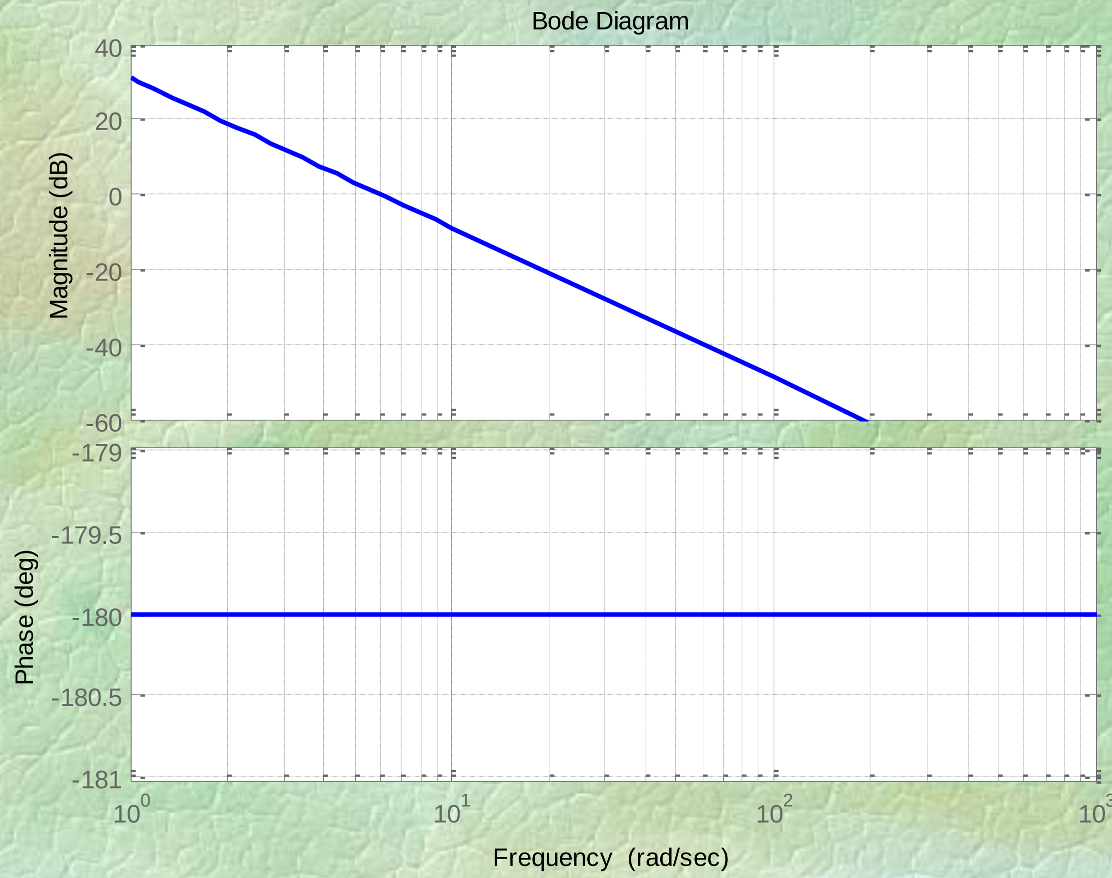
2 Design a lead controller for the following system such that the phase margin be 40° and the acceleration constant be 25.



$$\text{Answer}: G_c(s) = k\frac{0.301s+1}{0.0519s+1}$$

Exercises

3 Following is the open loop transfer function of a system. Design a lead controller such that PM=45°. Draw the bode plot of compensated system.



$$\text{Answer}: G_c(s) = k\frac{0.2674s+1}{0.0461s+1}$$

Exercises

4 Following is the open loop transfer function of a system.

a) What is the velocity error constant. (answer 80)

b) Design a lead controller such that PM=50°. (answer $G_c(s)=(0.0315s+1)/(0.0126s+1)$)

c) Design a lead controller such that PM=50° and the velocity error constant be 200. (answer $G_c(s)=2.5(0.0192s+1)/(0.0052s+1)$)