LINEAR CONTROL SYSTEMS

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Lecture 2

Modeling of Control Systems

Topics to be covered include:

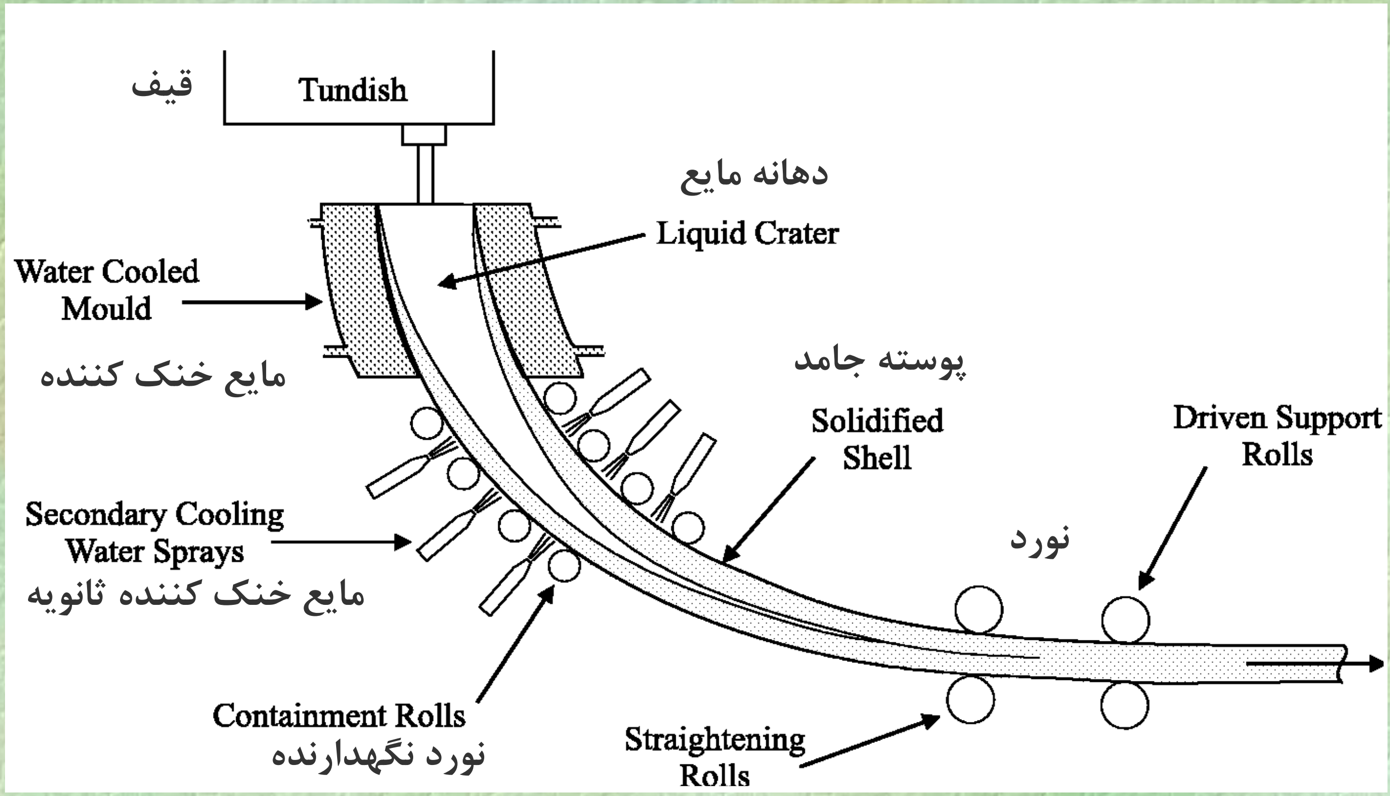
- An industrial example
- Modeling of systems
- Systems with time delay.

Example 1: An industrial plant

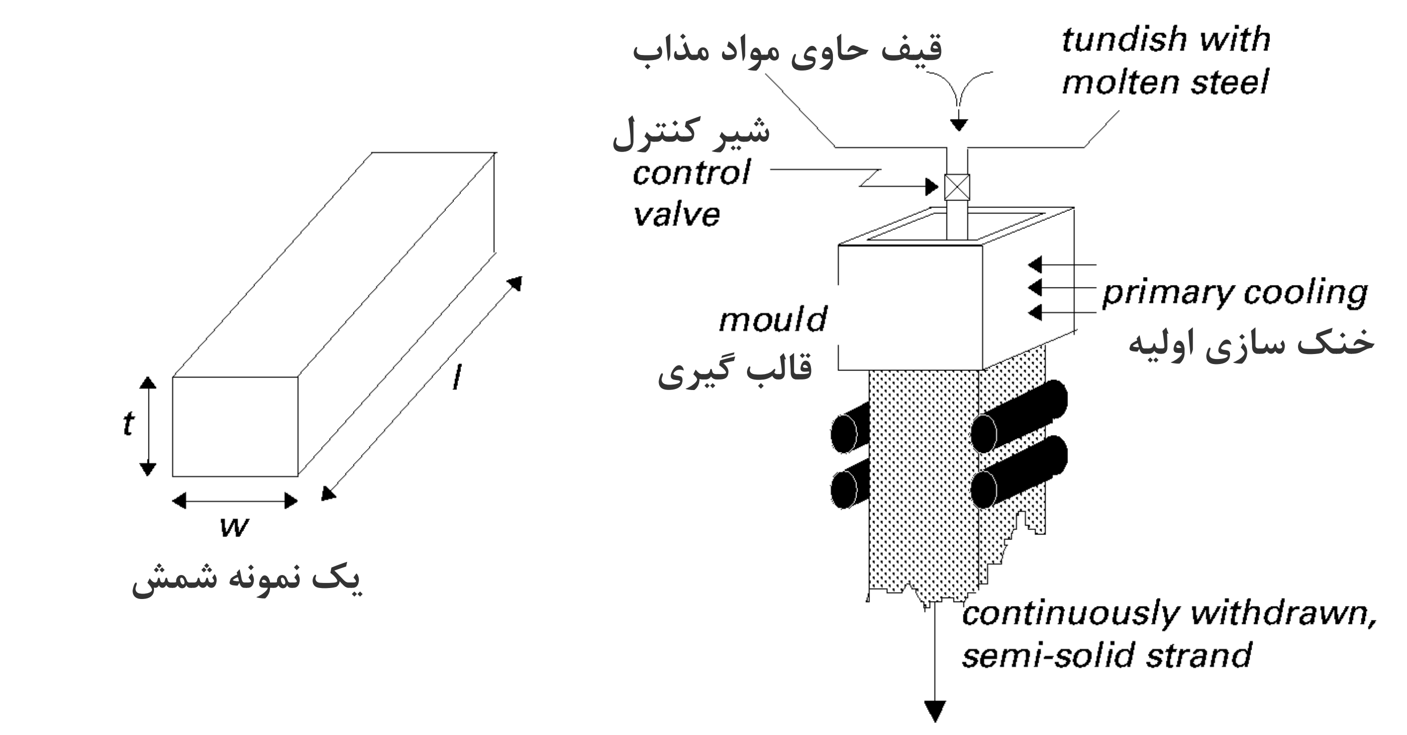
مثال ۱: یک سیستم صنعتی

The example, taken from the **steel industry**, is of a particular nature, however the principal elements of specifying a desired behaviour, modeling and the necessity for trade-off decisions are generic.

Process schematic of a steel industry نمای شماتیک صنعت فولاد

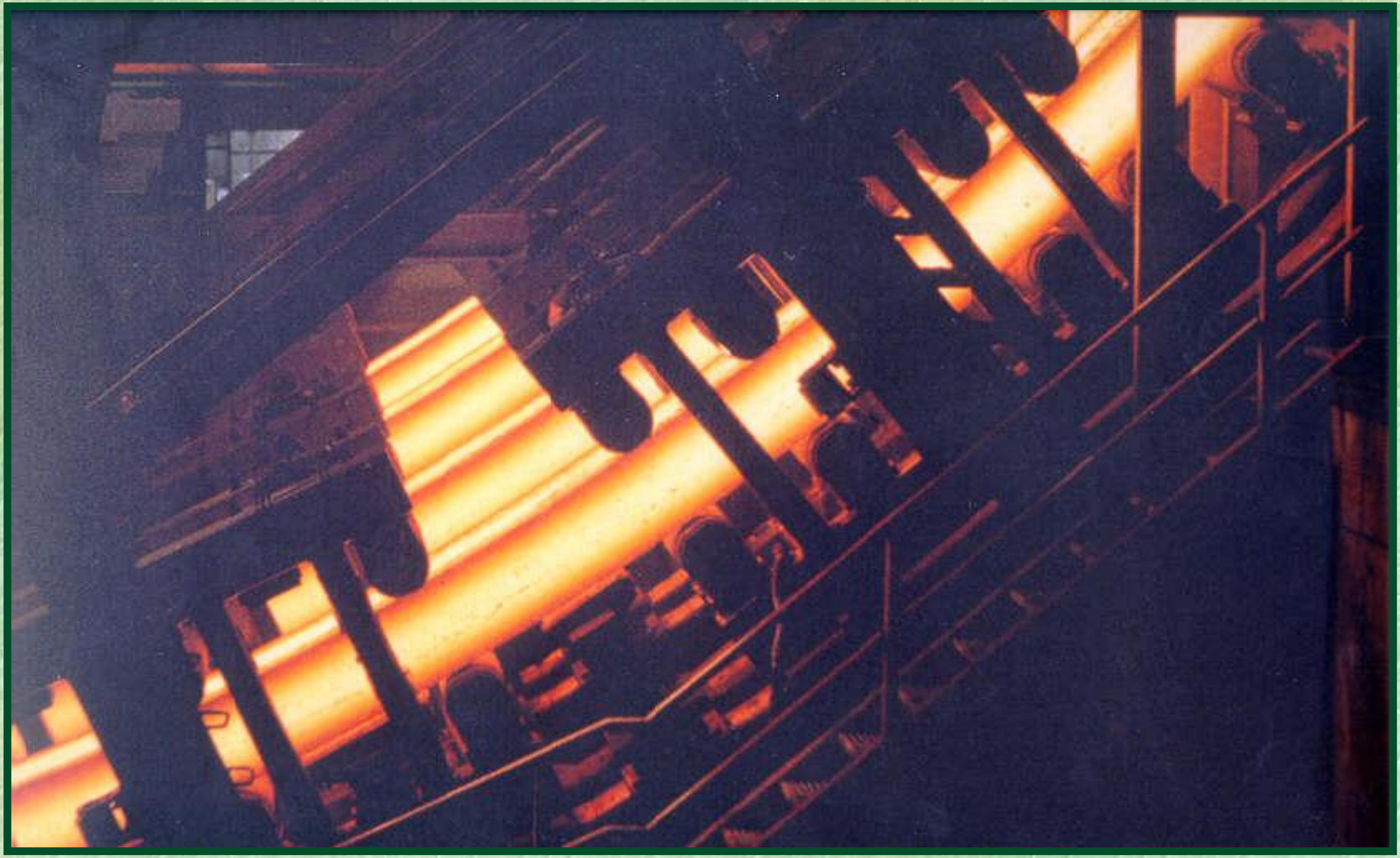


Process schematic of a steel industry (continue)



The cast strip in the secondary cooling chamber

نوار ریخته گری در دومین بخش خنک سازی



Performance specifications

مشخصه های عملکردی

The key performance goals for this problem are:

- *Safety*: Clearly, the mould level must never be in danger of overflowing or emptying as either case would result in molten metal spilling with disastrous consequences.
- *Profitability*: Aspects which contribute to this requirement include:
  - Product quality
  - Maintenance
  - Throughput

Definition of the control problem

Abstracting from the above particular problem, we can introduce:

Definition 2.1:

The central problem in control is to find a technically feasible way to act on a given process so that the process behaves, as closely as possible, to some desired behaviour. Furthermore, this approximate behaviour should be achieved in the face of uncertainty of the process and in the presence of uncontrollable external disturbances acting on the process.

Models

Model: Relationship among observed signals.

Model types

- **1- Mental models**
- **2- Graphical models**
- **3- Mathematical (analytical) models**
- **4- Software models**

Building models

- **1- Modeling**
  - **Split up system into subsystems,**
  - **Joined subsystems mathematically,**
  - **Does not necessarily involve any experimentation on the actual system.**
- **2- System identification**
  - **It is directly based on experimentation.**
  - **Input and output signals from the system are recorded.**
- **3- Combined**

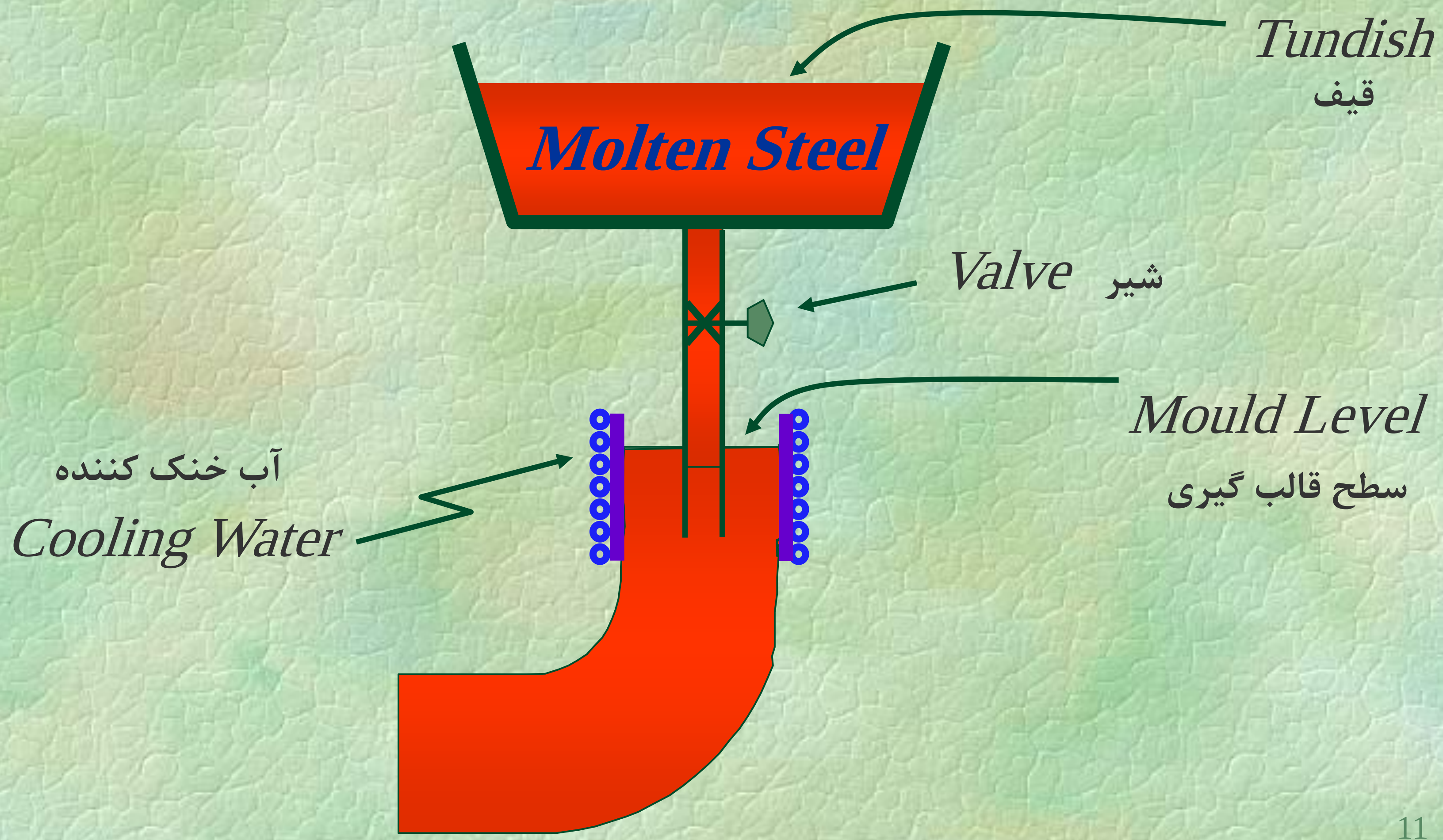
Modeling

مدلسازی

h^*	:	commanded level of steel in mould
$h(t)$	:	actual level of steel in mould
$v(t)$	:	valve position
$\sigma(t)$	:	casting speed
$q_{in}(t)$	:	inflow of matter into the mould
$q_{out}(t)$	:	outflow of matter from the mould

Modelling a simple tank

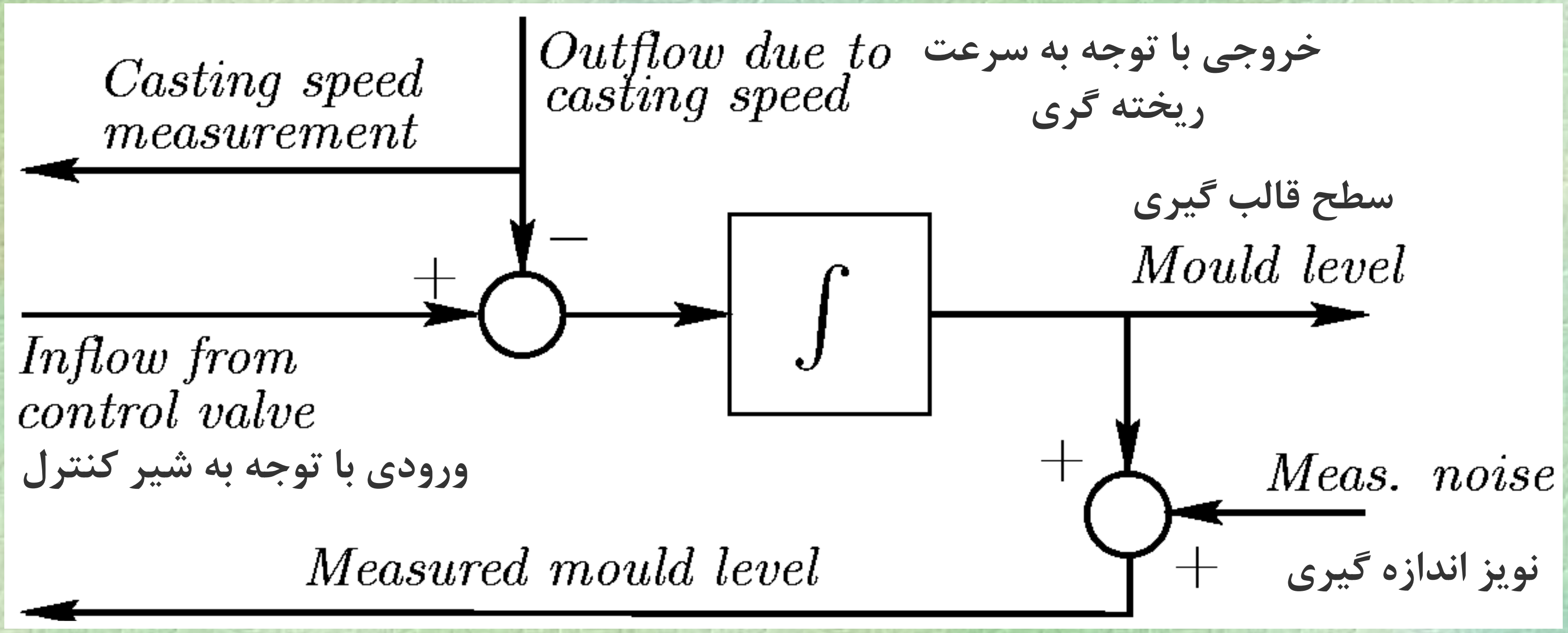
مدلسازی یک منبع ساده



Simplified block diagram for modeling

بلوک دیاگرام ساده شده برای مدلسازی

Block diagram of the simplified mould level dynamics, sensors and actuators



سطح قالب گیری اندازه گیری شده

Level of compromise

سطح مصالحه

We may ask if these trade-offs are unavoidable or whether we could improve on the situation by such measures as:

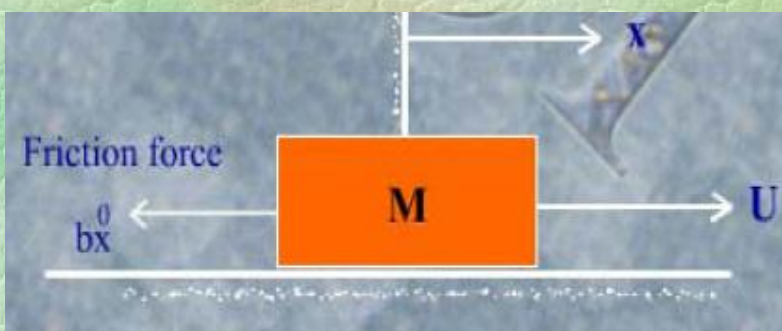
- better modelling
- more sophisticated control system design

(*Aside*: Actually the trade-off is fundamental as we shall see presently).

Example 2: Dynamics of a mechanical system

مثال ۲: دینامیک یک سیستم مکانیکی

خروجی

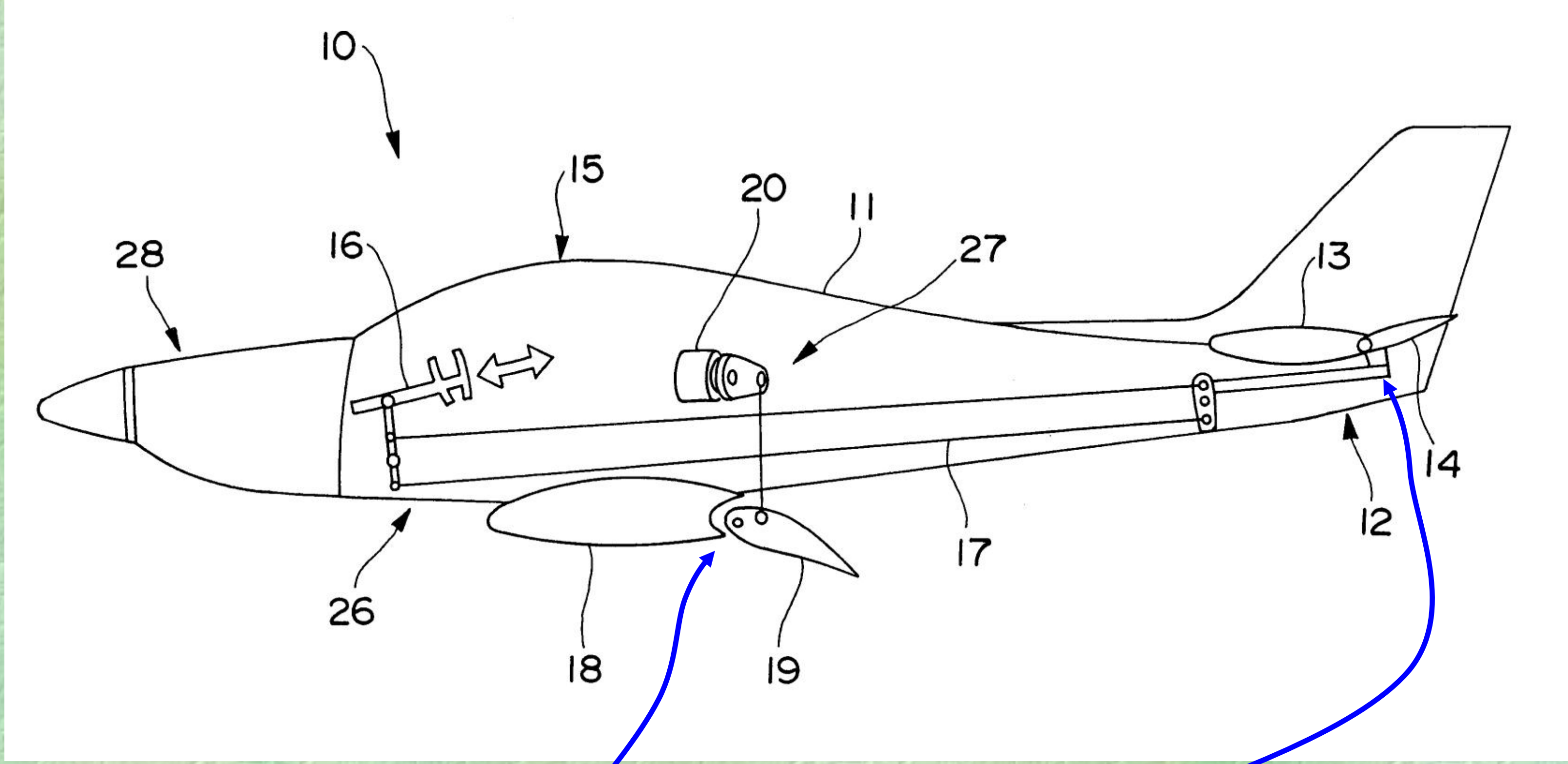


ورودی

$$u - b\dot{x} = M\ddot{x} \;\rightarrow\; \ddot{x} + \frac{b}{M}\dot{x} = \frac{1}{M}u \quad , \; with \; a = \ddot{x}$$

A simplified aeroplane

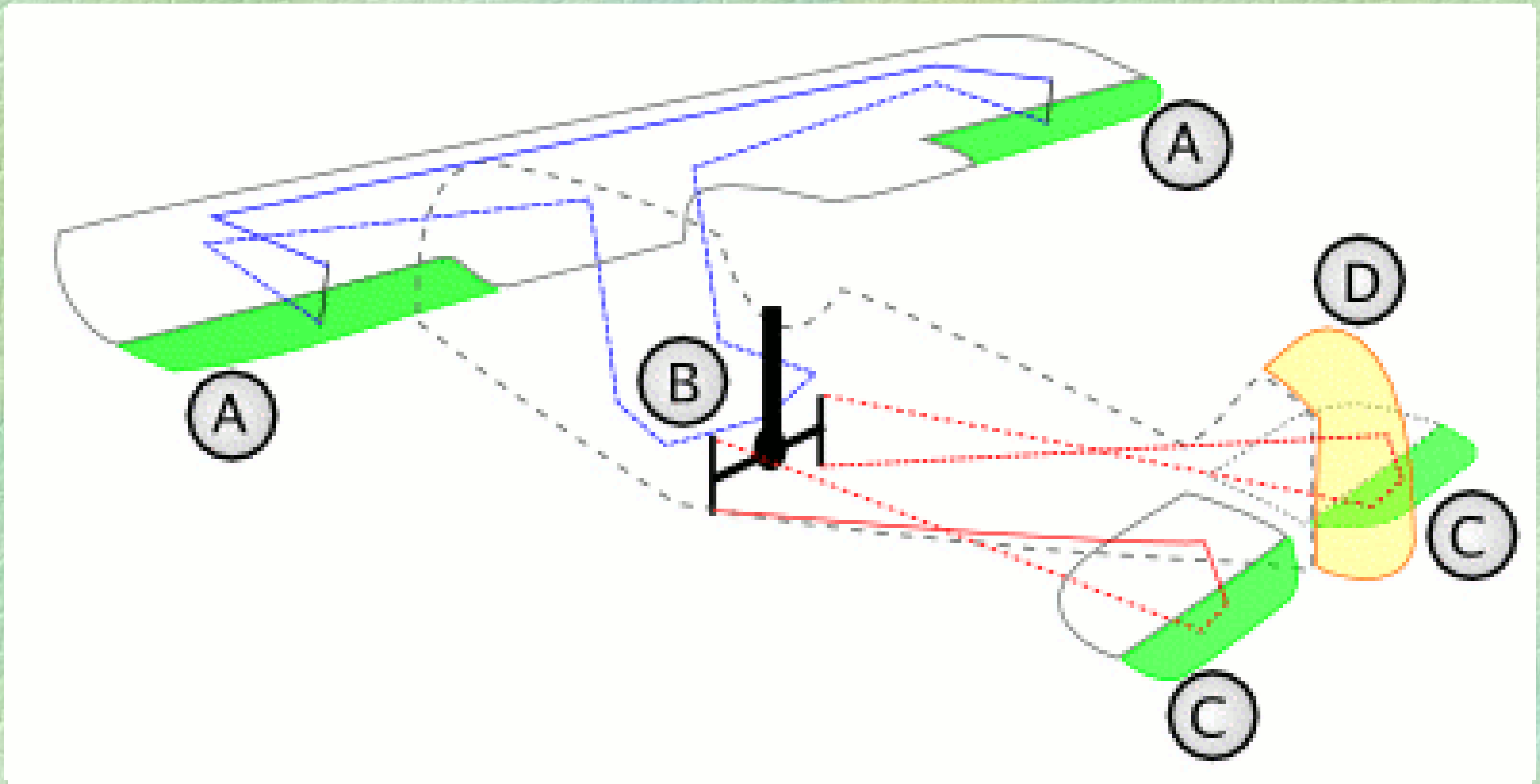
یک هواپیمای ساده شده



Position control system

A simplified aeroplane

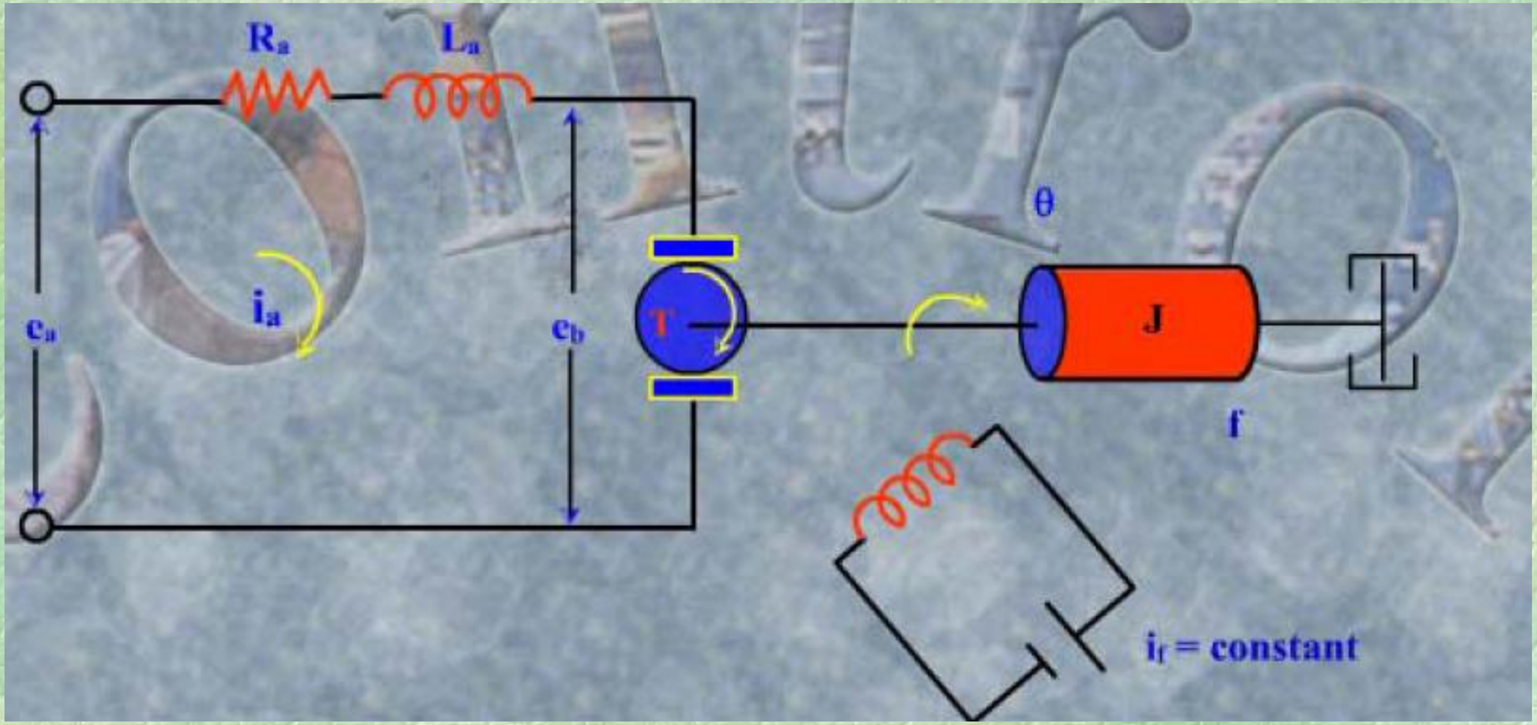
یک هواپیمای ساده شده



Example 3:Dynamics of a electromechanical system

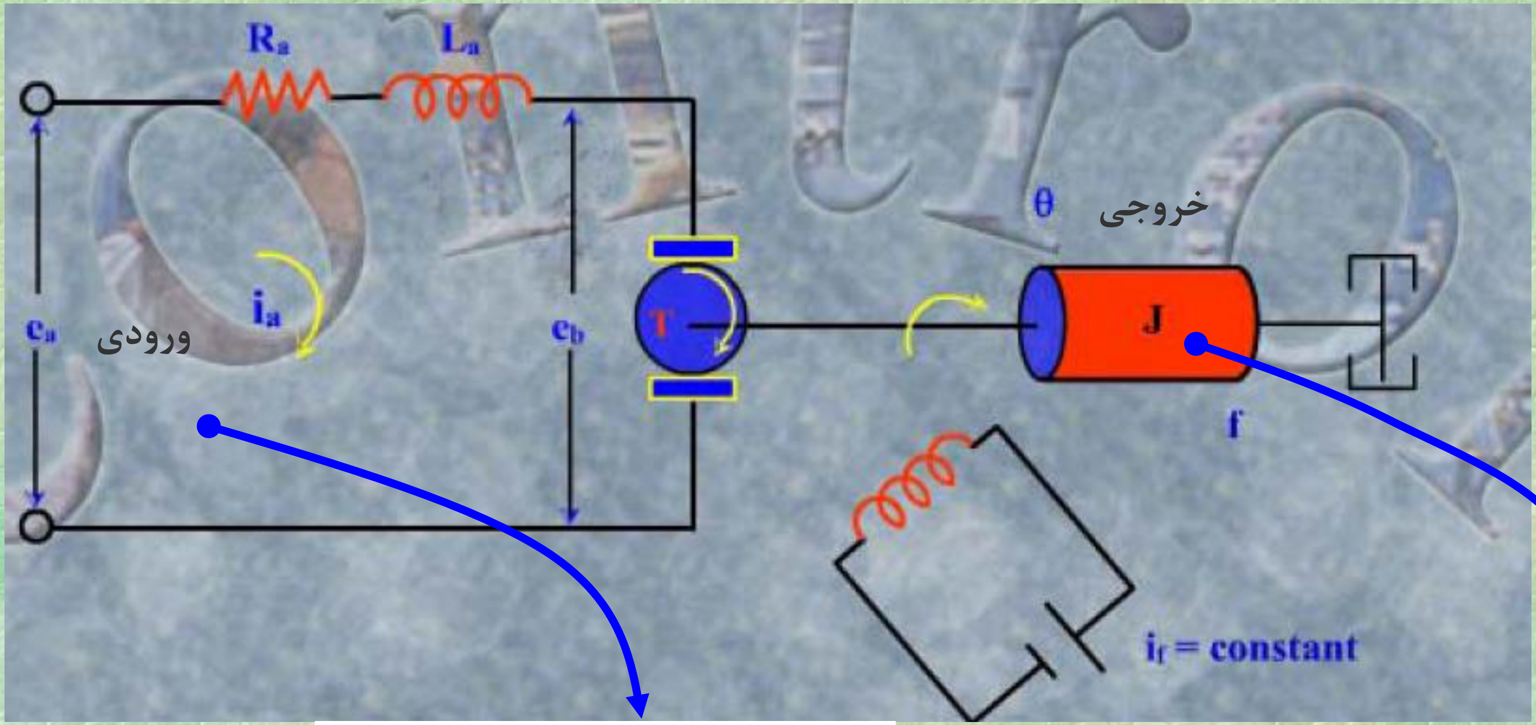
مثال ۳: دینامیک یک سیستم الکترومکانیکی

Position control system



Example 3: Continue

مثال ۳: ادامه



$$L_a \frac{di_a}{dt} + R_a i_a + e_b = e_a$$

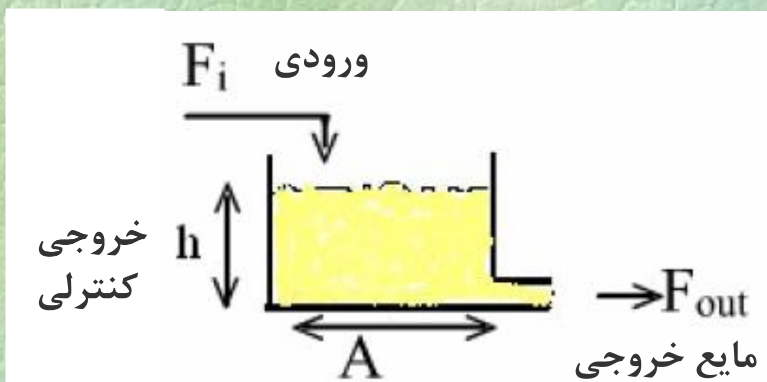
$$e_b = K_b \frac{d\theta}{dt}$$

$$J\frac{d^2\theta}{dt^2} + f\frac{d\theta}{dt} = T$$

$$T = Ki_a$$

Example 4: Dynamics of a liquid level system

مثال ۴: دینامیک یک سیستم سطح مایعات



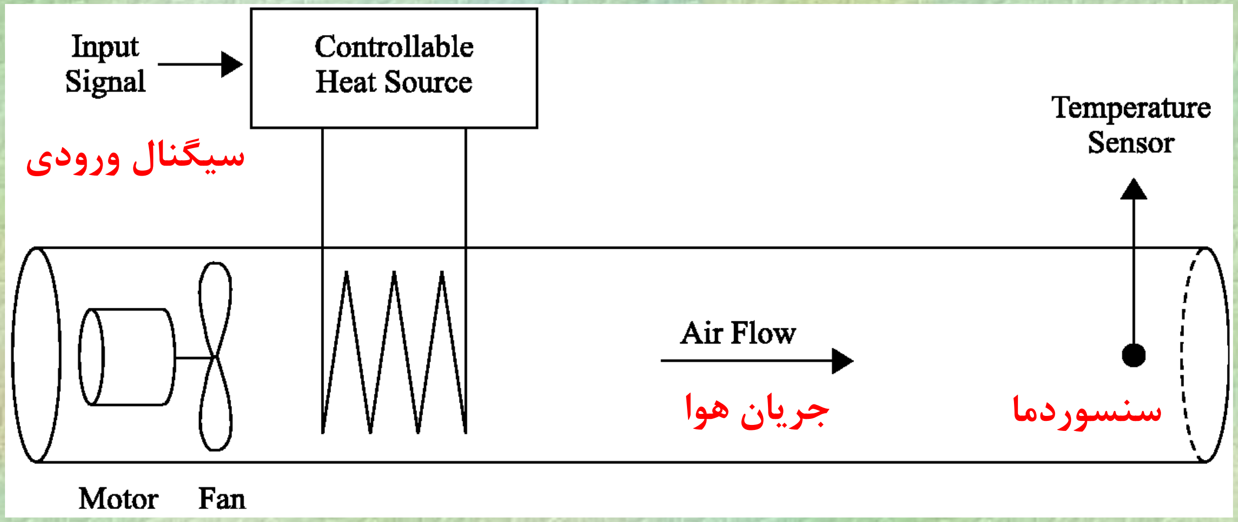
F_i = Inlet volumetric flow rate
F_{out} = Outlet volumetric flow rate
A = Cross sectional area of the tank
ρ = Density of water
h = Height of water

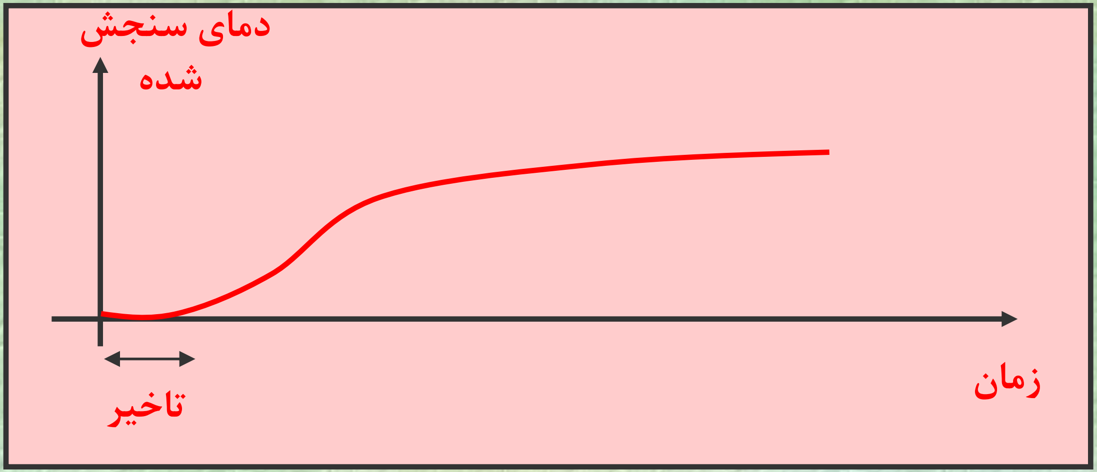
$$\frac{d(\rho A h)}{dt} = \rho F_i - \rho F_{out}$$

$$F_{out} = \alpha \sqrt{h}$$

Example 5: A system with pure time delay

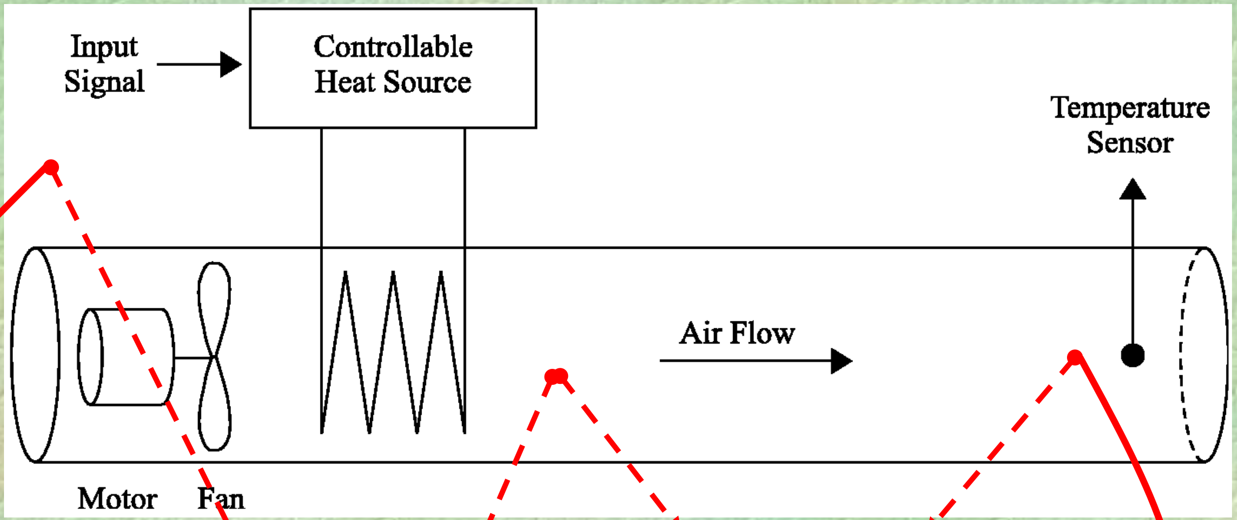
مثال ۵: سیستمی با تاخیر ثابت





Example 5: A system with pure time delay

مثال ۵: سیستمی با تاخیر ثابت



$$\frac{K}{(\tau s+1)}$$

$$H(s) = e^{-sT_d}$$

$$H(s) = \frac{Ke^{-sT_d}}{(\tau s+1)}$$

Exercises

2-1 Derive the output of example 2 excited by following input

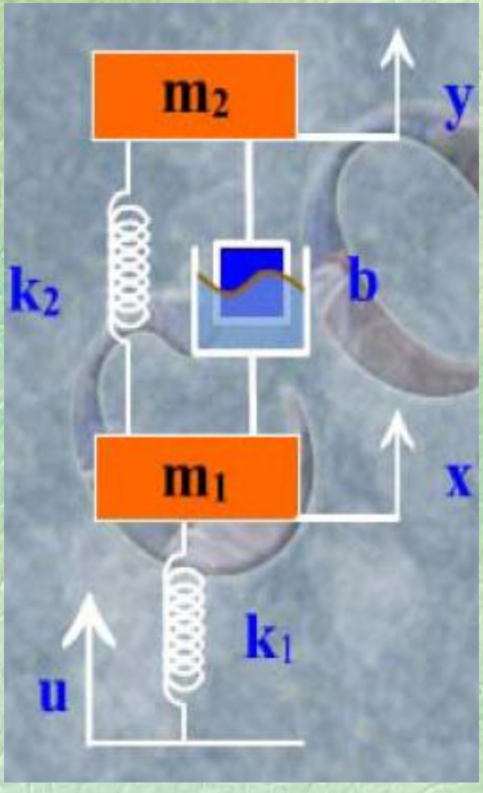
$u=sint$

2-2 Repeat example 3, but neglect the inductance of the motor.

2-3 Repeat example 3, but consider the motor as a series motor.

Exercises (Continue)

2-4 Find the mathematical model of the following system.



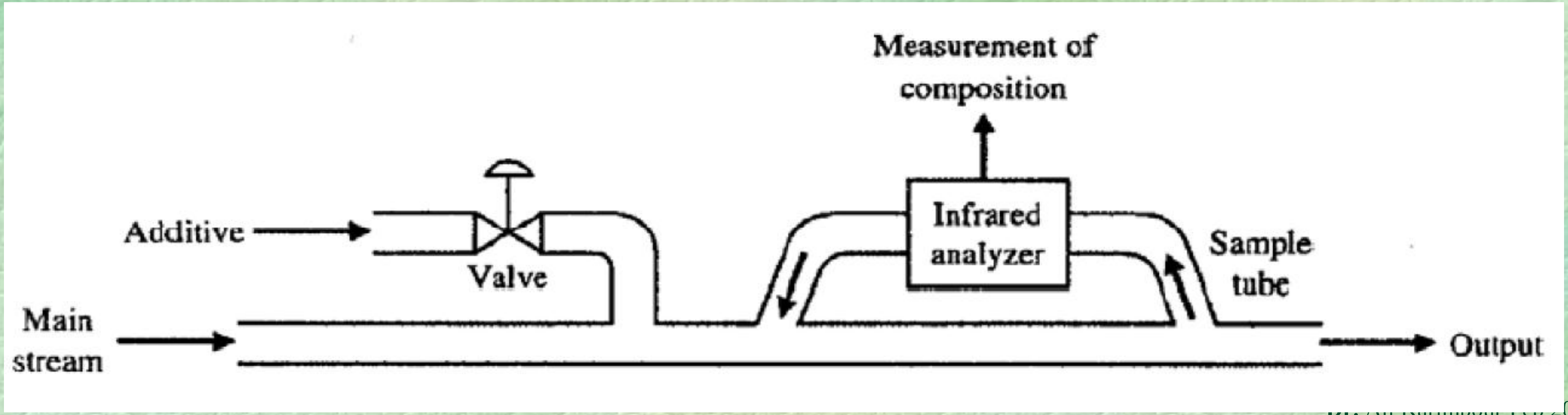
Final answer is:

$$m_2\ddot{y} + b\dot{y} + k_2 y = b\dot{x} + k_2 x$$

$$m_1\ddot{x} + b\dot{x} + (k_1 + k_2)x = b\dot{y} + k_2 y + k_1 u$$

Exercises (Continue)

2-5 In a control system of an industrial composition process , it is very important to control the chemical composition of the output. There is an infrared analyzer for measurement and the valve of additive stream is controllable. Complete the feedback loop and find the block diagram of the process.



Exercises (Continue)

2-6 The figure shows a system for extracting water for irrigation by use of solar energy. Draw the block diagram of the system operation.