(a) A unital Banach algebra, except the algebra of complex numbers, without nontrivial idempotent.
(b) A unital Banach algebra with a nontrivial idempotent.
(Recall that 0 and 1 are called trivial idempotents.)

We show that the Banach algebra $C(X)$ has no nontrivial idempotent iff $X$ is connected:
Let $0 \neq f \neq 1$ be an idempotent. Then $X = f^{-1}(\{0\}) \cup f^{-1}(\{1\})$ implies that $X$ is not connected. Conversely if $X$ is disconnected and $X = G_1 \cup G_2$ with open disjoint sets $G_1$ and $G_2$, then $f(x) = \begin{cases} 
1 & x \in G_1 \\
0 & x \in G_2 
\end{cases}$ is a trivial idempotent of $C(X)$.

Comment. If $A$ is a (not necessarily commutative) Banach algebra with an element $a \in A$ such that $sp(a)$ is not connected, then $A$ has a nontrivial idempotent. (cf. [B&D, Remarks of Prop. 7.9])

Ref.