

## A commutative radical Banach algebra.

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1. A Banach space with all products taken to be zero. Then every element is quasi invertible.

2. The Banach space  $L^1([0, 1])$  with the product  $(fg)(x) = \int_0^x f(x-y)g(y)dy$  has  $f_{\circ}(t) = t, 0 \leq t \leq 1$  as a generator since  $f_{\circ}^n(t) = \frac{t^{n-1}}{(n-1)!}$ , the set of polynomials in one variable is  $L^p$ -dense in  $C([0, 1])$  and  $C([0, 1])$  is  $L^p$ -dense in  $L^1([0, 1])$  (cf. [Rud2, Theorem 2.14]).

Moreover  $\|f_{\circ}^n\| = \int_0^1 |f_{\circ}^n(t)|dt = \frac{1}{n!}$ , so  $r(f_{\circ}) = \lim_n \|f_{\circ}^n\|^{\frac{1}{n}} = \lim_n (n!)^{-\frac{1}{n}} = 0$ . Therefore this algebra doesn't have any character. Thus it is radical algebra.

## Ref.

[Rud2] W. Rudin, Real and complex analysis, McGraw-Hill, 1986.