A commutative radical Banach algebra.

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1. A Banach space with all products taken to be zero. Then every element is quasi invertible.

2.The Banach space $L^1([0,1])$ with the product $(fg)(x) = \int_0^x f(x-y)g(y)dy$ has $f_\circ(t) = t, 0 \leq t \leq 1$ as a generator since $f_\circ^n(t) = \frac{t^{n-1}}{(n-1)!}$, the set of polynomials in one variable is L^p-dense in $C([0,1])$ and $C([0,1])$ is L^p-dense in $L^1([0,1])$ (cf. [Rud2, Theorem 2.14]).
Moreover $\| f_\circ^n \| = \int_0^1 |f_\circ^n(t)|dt = \frac{1}{n!}$, so $r(f_\circ) = \lim_n \| f_\circ^n \|^{\frac{1}{n}} = \lim_n (n!)^{\frac{-1}{n}} = 0$. Therefore this algebra doesn't have any character. Thus it is radical algebra.

Ref.

[**Rud2**] W. Rudin, Real and complex analysis, McGraw-Hill, 1986.