A normed algebra with non-open group of invertibles (and so the algebra is not Banach).

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Let $A = \mathcal{C}[z]$[1], then $Inv(\mathcal{C}[z]) = \mathcal{C} - \{0\}$, hence the elements $p_n(z) = 1 + \frac{z}{n} (n \in \mathcal{N})$ aren't invertible. But $\lim_n p_n(z) = 1 \in Inv(\mathcal{C}[z])$. Therefore $A - Inv(A)$ isn't closed.

[1]The set $\mathcal{C}[z]$ of all polynomials in an indeterminate z with complex coefficients under usual operations on polynomials and with the norm $\|p\| = \sup_{|\lambda| \leq 1} |p(\lambda)|$ is a normed algebra.