A commutative Banach algebra whose unit ball isn’t norm compact.

The unit ball of $C([0,1])$ is not compact with respect to the supremum norm, since if $p_n(x) = x^n$, then $\| p_n \| = 1$ and $(p_n)$ has no convergent subsequence.

It’s well-known that a normed space $Y$ is finite dimensional iff $\{ y \in Y; \| y \| \leq 1 \}$ is compact (cf. [Ker, Theorem 2.5-5]). $C([0,1])$ is infinite dimensional, hence its unit ball is not compact.

Ref.