A commutative Banach algebra whose unit ball isn't norm compact.

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The unit ball of $C([0,1])$ is not compact with respect to the supremum norm, since if $p_n(x) = x^n$, then $\| p_n \| = 1$ and (p_n) has no convergent subsequence.
It's well-known that a normed space Y is finite dimensional iff $\{y \in Y; \| y \| \leq 1\}$ is compact (cf. [Ker, Theorem 2.5-5]). $C([0,1])$ is infinite dimensional, hence its unit ball is not compact.

Ref.

[Ker] E. Kreyszig, Introductory functional analysis with applications,John Wiley & Sons, 1978.