(a) A separable Banach algebra.

(b) A non-separable Banach algebra.

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(a) $\{r_1 + ir_2; r_1, r_2 \in \mathcal{Q}\}$ is a countable dense subset of $\mathcal{C}$. Hence $\mathcal{C}$ is a separable Banach algebra.

(b) l^∞ isn't separable. In fact if $S = \{a_1, a_2, \cdots, \}$ is a countable set in $l^\infty, a_n = (a_n^k)_{k\in\mathcal{N}}$, and $b_n = \begin{cases} 0 & |a_n^n| \geq 1 \\ 2 & |a_n^n| < 1 \end{cases}$, then $b = (b_n) \in l^\infty$ and for all n, $\| b - a_n \|_\infty \geq |b_n - a_n^n| \geq 1$. So that the neighborhood of b with the radius 1 doesn't intersect S. Thus S isn't dense in l^∞.

For another proof see [A&B, Problem 25.7].

Ref.

[A&B] C.D. Aliprantis and O. Burkinshaw, Problems in real analysis, Acad. Press, 1990.