**Two non-isomorphic Banach algebras with homeomorphically isomorphic invertible groups.**

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1. Let $A_1 = C([-1, \frac{-1}{2}] \cup [\frac{1}{2}, 1])$ and $A_2 = C([0,1] \cup \{2\})$. Since $[0,1] \cup \{2\}$ isn't homeomorphic to $[-1, \frac{-1}{2}] \cup [\frac{1}{2}, 1]$, $A_1$ isn't isomorphic to $A_2$. Also the function which sends $x \in Inv(A_1)$ to $y \in G_2$ defined by

$$y(t) = \begin{cases} x(t-1) & t \in [0, \frac{1}{2}] \\ \left[x(\frac{-1}{2})/x(\frac{1}{2})\right] x(t) & t \in [\frac{1}{2}, 1] \\ x(\frac{1}{2})/x(\frac{-1}{2}) & t = 2 \end{cases}$$

is the desired isomorphism.

**Ref.**

[**Zel**] W. Zelazko, Banach algebras, Elsevier Publishing Company, 1973.