A commutative Banach algebra A without any minimal ideals.

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Let $A = \mathcal{A}(\Delta)$[1] , J be a minimal ideal and, for $n \geq 0$, $I_n = \{f \in A\ ;\ f(0) = f'(0) = \ldots = f^{(n)}(0) = 0\}$ (recall $f^{(0)} = f$). Then $(I_n)_{n\geq 0}$ is a strictly decreasing sequense of (primary) ideals. Assuming $0 \neq f \in J$, then $0 \neq z^{n+1} f \in I_n \cap J$. So $I_n \cap J = J$. Hence $(\bigcap_{n=1}^{\infty} I_n) \cap J = J$ and so $J = 0$, since $\bigcap_{n=1}^{\infty} I_n = \{0\}$. Thus $\mathcal{A}$ has no minimal ideal.

[1]Let Δ denote the closed unit disc $\{z \in \mathcal{C}, |z| \leq 1\}$. Suppose that $A(\Delta)$ denoted the set of all elements of $C(\Delta)$ which are analytic on the interior of Δ. $A(\Delta)$ is a closed subalgebra of $C(\Delta)$