A Banach algebra with an unbounded approximate identity.

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Consider l^p as a Banach algebra with coordinatewise operations. Let $e_n = (\underbrace{1,1,1,\ldots,1}_{n},0,0,\ldots)$. Then $\sup_n ||e_n|| = \sup\{\sqrt[p]{n}\ ;\ n \in N\} = \infty$, and for every $x = (\alpha_n) \in l^2$, $\lim_n ||xe_n - x|| = \lim_n (\sum_{k=n+1}^{\infty} |\alpha_k|^p)^{\frac{1}{p}} = 0$. Thus (e_n) is required approximate identity.