A topologically nilpotent Banach algebra. (A Banach algebra A is called topologically nilpotent if the quantity $N_A(n) = \sup\{\|x_1...x_n\|^{\frac{1}{n}}$; $x_i \in A$; $\|x_i\| \leq 1$, $1 \leq i \leq n\}$ tends to zero as $n \to \infty$).

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The Banach algebra $C[0,1]$ with the supremum norm $\|.\|$ and convolution multiplication is topologically nilpotent:

Defining $u \in C([0,1])$ by $u(t) = 1$ $(0 \leq t \leq 1)$, we have $u^n(t) = \frac{t^{n-1}}{(n-1)!}$ $(n = 1, 2, \ldots)$ and so $\|u^n\| = \frac{1}{(n-1)!}$. For arbitrary $f_1, \ldots, f_n \in C([0,1])$, $|f_1 * f_2 * \ldots * f_n)(t)| \leq \|f_1\| \ldots \|f_n\| u^n(t)$. Hence $(\frac{\|f_1 * \ldots * f_n\|}{\|f_1\| \ldots \|f_n\|})^{\frac{1}{n}} \leq \frac{1}{((n-1)!)^{\frac{1}{n}}}$.

Now note that $\lim_n \frac{1}{((n-1)!)^{\frac{1}{n}}} = 0$.

Ref.

P.G. Dixon, G. A. Willis, Approximate identities in extensions of topological nilpotent Banach algebras., Proc. Royal of Edin., 122A, 45-52, 1992.