A non-topologically nilpotent Banach algebra.

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1. The algebra $\mathcal{C}$ of complex numbers.

2.The Volterra algebra $L^1[0,1]$[1] isn't topologically nilpotent ; For establishing this, consider $x_i(t) = \begin{cases} 2^i & 0 \leq t \leq 2^{-i} \\ \\ 0 & 2^{-i} \leq t \leq 1 \end{cases}$.
Then $\|x_i\| = 1$ $(i = 1, 2, \ldots)$ and for all n, $\|x_1 \ldots x_n\| = 1$.

[1]The Banach space $L^1([0,1])$ with the product $(fg)(x) = \int_0^x f(x-u)g(u)du$ is a non-unital commutative Banach algebra and called Volterra algebra.