A Banach algebra A such that $Rad(A)$ is a proper subset of the set $\{x; r(x) = 0\}$ of all quasi-nilpotent elements.

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1.Suppose that H be a Hilbert space with $dimH \geq 2$. Let $x, y \in H - \{0\}$ and $< x, y >= 0$. The norm of rank one operator $(x\overline{\otimes}y)(z) =< z, y > x$ is $\| x \| \| y \| \neq 0$. So $x\overline{\otimes}y \neq 0$. Also $(x\overline{\otimes}y)^2(z) = (x\overline{\otimes}y)(< z, y > x) =< z, y >< x, y > x = 0$ so $(x\overline{\otimes}y)^2 = 0$. Hence it is quasi-nilpotent. But $B(H)$ is semi-simple. Therefore $x\overline{\otimes}y \notin Rad(B(H)) = \{0\}$.

2.Let $A = M_2(\mathcal{C}) \simeq B(\mathcal{C}^2)$. A is a C^*-algebra so $Rad(A) = \{0\}$. The element $\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$ has the spectrum $\{0\}$ and so $r(\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}) = 0$. Hence $Rad(A)$ is not equal to $\{x; r(x) = 0\}$.