A Banach algebra $A$ such that $\text{Rad}(A)$ is a proper subset of the set 
$\{x; r(x) = 0\}$ of all quasi-nilpotent elements.

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1. Suppose that $H$ be a Hilbert space with $\dim H \geq 2$. Let $x, y \in H - \{0\}$ 
and $<x, y> = 0$. The norm of rank one operator $(x \overline{y})(z) = <z, y > x$ 
is $||x|| ||y|| 
eq 0$. So $x \overline{y} \neq 0$. Also $(x \overline{y})^2(z) = (x \overline{y})(<z, y > x) = <z, y > x = 0$ so $(x \overline{y})^2 = 0$. Hence it is quasi-nilpotent. But $B(H)$ 
is semi-simple. Therefore $x \overline{y} \notin \text{Rad}(B(H)) = \{0\}$.

2. Let $A = M_2(\mathbb{C}) \simeq B(\mathbb{C}^2)$. $A$ is a $C^*$-algebra so $\text{Rad}(A) = \{0\}$. 
The element $\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$ has the spectrum $\{0\}$ and so $r(\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}) = 0$. Hence 
$\text{Rad}(A)$ is not equal to $\{x; r(x) = 0\}$.