A Banach algebra having no bounded approximate identity.

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$\{xy\ ;\ x, y \in l^2\}$ is a proper subset of Banach algebra l^2 equipped with the coordinatewise operations. In fact $(\frac{1}{n}) \in l^2$ and if $x_n y_n = \frac{1}{n}$, then there exist an integer N such that for all $n > N$, $|x_n| \geq \dfrac{1}{\sqrt{n}}$ or for all $n > N$, $|y_n| \geq \dfrac{1}{\sqrt{n}}$, and hence $(x_n) \notin l^2$ or $(y_n) \notin l^2$. Now Cohen's factorization theorem [B&D,§11. Corollary 11] implies that l^2 has no bounded approximate identity.

Comment. Using BA37, we conclude that the Banach algebra l^2 has neither bounded aproximate identity nor unbounded one.

Ref.

[B&D] F.F. Bonsall and J. Duncan, Complete normed algebras, Springer-Verlag, 1973.

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