An algebrically semisimple non-commutative Banach algebra.

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We show that $B(X)$, the algebra of bounded linear mappings from normed space X into X is semi-simple:

Suppose that $x_0 \neq 0$ is fixed in X. Then $I_{x_0} = \{T \in B(X); Tx_0 = 0\}$ is a left ideal in $B(X)$.

We shall show that it is maximal. Let J be a left ideal properly containing I_{x_0}. Then $Jx_0 = \{Tx_0; T \in J\}$ is a nonzero linear subspace of X which is invariant under each $S \in B(X)$. If $Jx_0 \neq X$, then there exists a nonzero $y \in Jx_0$ and an element $z \in X$ such that $z \notin Jx_0$. If $S \in B(X)$ such that $Sy = z$, then $z \in Jx_0$ for Jx_0 is invariant under all elements of $B(X)$. Thus $Jx_0 = X$. So that there exists $U \in J$ such that $Ux_0 = x_0$. For each $T \in B(X), TU - UT \in I_{x_0}$. Hence $T \in J + I_{x_0} \subseteq J$. Therefore $B(X) = J$. Thus $Rad(B(X)) \subseteq \cap_{0 \neq x \in X} I_x = \{0\}$. Therefore $B(X)$ is algebrically semisimple.