A semisimple commutative Banach algebra with a closed two-sided ideal $I$ such that $\frac{A}{I}$ isn’t semisimple.

Suppose that $A$ is the algebra $C^m([0, 1])$ of all $m$ times continuously differentiable complex-valued functions on $[0, 1]$ with the norm $\| f \| = \sum_{k=0}^{m} \frac{1}{k!} \sup_{x \in [0, 1]} |f^{(k)}(x)|$.
Let $I = \{ f \in A; f(0) = f'(0) = 0 \}$. Then $\frac{A}{I}$ is not semisimple, since assuming $f_0$ to be $f_0(x) = x$, then $f_0^2 \in I$ and so $(f_0 + I)^2 = f_0^2 + I = 0$, hence $r(x) = \lim_n \| (f_0 + I)^n \|^\frac{1}{n} = 0$. Therefore $f_0 + I \in \text{Rad} \left( \frac{A}{I} \right)$. But $f_0 + I \neq 0$. So that $\frac{A}{I}$ is not semisimple.