A semisimple commutative Banach algebra with a closed two-sided ideal I such that $\frac{A}{I}$ isn't semisimple.

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Suppose that A is the algebra $C^m([0,1])$ of all m times continously differentiable complex-valued functions on $[0,1]$ with the norm $\| f \| = \sum_{k=0}^{m} \frac{1}{k!} \sup_{x\in[0,1]} |f^{(k)}(x)|$. Let $I = \{f \in A; f(0) = f'(0) = 0\}$. Then $\frac{A}{I}$ is not semisimple, since assuming $f_\circ$ to be $f_\circ(x) = x$, then $f_\circ^2 \in I$ and so $(f_\circ + I)^2 = f_\circ^2 + I = 0$, hence $r(x) = \lim_n \| (f_\circ + I)^n \|^{\frac{1}{n}} = 0$. Therefore $f_\circ + I \in Rad(\frac{A}{I})$. But $f_\circ + I \neq 0$. So that $\frac{A}{I}$ is not semisimple.