A non-maximal primary ideal in a unital commutative Banach algebra $A$.

Suppose that $A$ is the algebra $C^m([0,1])$ of the complex valued $m$ times continuously differentiable functions on $[0,1]$ with the norm $\|f\| = \sum_{k=0}^{m} \frac{1}{k!} \sup_{x \in [0,1]} |f^{(k)}(x)|$. Let $x_0 \in [0,1]$ and $I = \{ f \in A; f(x_0) = f'(x_0) = 0 \}$. Then $I$ is a closed two-sided ideal contained in only one maximal ideal; i.e. $\{ f \in A : f(x_0) = 0 \}$. Note that the maximal ideals of $A$ are of the form $I_x = \{ f \in A; f(x) = 0 \}, x \in [0,1]$.

A conclusion is that $C^m([0,1])$ is not spectral synthesis, i.e. it has a closed two-sided ideal which is not the intersection of maximal ideals containing this ideal.

**Comment.** The disk algebra contains a nonmaximal prime ideal, namely $\{0\}$.