A non-maximal primary ideal in a unital commutative Banach algebra A.

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Suppose that A is the algebra $C^m([0,1])$ of the complex valued m times continously differentiable functions on $[0,1]$ with the norm $\| f \| = \sum_{k=0}^{m} \frac{1}{k!} \sup_{x\in[0,1]} |f^{(k)}(x)|$. Let $x_0 \in [0,1]$ and $I = \{f \in A; f(x_0) = f'(x_0) = 0\}$. Then I is a closed two-sided ideal contained in only one maximal ideal; i.e. $\{f \in A : f(x_0) = 0\}$. Note that the maximal ideals of A are of the form $I_x = \{f \in A; f(x) = 0\}, x \in [0,1]$.

A conclusion is that $C^m([0,1])$ is not spectral synthesis, i.e. it has a closed two-sided ideal which is not the intersection of maximal ideals containing this ideal.

Comment. The disk algebra contains a nonmaximal prime ideal, namely $\{0\}$.