An (algebrically) simple Banach algebra.

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In the case commutative, consider the familiar Banach algebra $\mathcal{C}$. In the non-commutative case, consider the algebra $M_n(\mathcal{C})$ of all $n \times n$ matrices with entries in $\mathcal{C}$. Identifying $M_n(\mathcal{C})$ with $B(\mathcal{C}^n) = K(\mathcal{C}^n)$ we may regard $M_n(\mathcal{C})$ as a noncommutative C^*-algebra.

Suppose that I_{ij} is the matrix with the ij-entry 1 and 0 elswhere. Then $I_{ij}I_{\alpha\beta} = \delta_{j\alpha}I_{i\beta}$, where δ denotes Kronecker's δ. Let Δ be a nontrivial two-sided ideal in $M_n(\mathcal{C})$. There is a nonzero element $A = \sum_{i,j=1}^{n} a_{ij}I_{ij}$ in Δ, hence $a_{rs} \neq 0$ for some $1 \leq r, s \leq n$. But $I_{rs}AI_{sr} = (\sum_{j=1}^{n} a_{rj}I_{rj})I_{sr} = a_{rs}I_{rr} \in \Delta$. Hence $I_{ij} = I_{is}I_{sr}I_{rj} \in \Delta$ for all $1 \leq i, j \leq n$. Therefore $\Delta = M_n(\mathcal{C})$, a contradiction.