A nonclosable unbounded operator on a Hilbert space.

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Let H be a separable Hilbert space with the standard orthonormal basis (ξ_n). Define T on H by $T\xi_n = n\xi_1$ and extend T to the dense linear subspace $D(T)$ of finite linear combinations of basis elements ξ_n (we denote the extension of T by the same T). Then T is a densely defined unbounded operator on H (since $\lim\limits_{n\to 0}\frac{\| T\xi_n \|}{\| \xi_n \|} = \lim\limits_{n\to 0} n = \infty$). Moreover T is not closable, for $\lim\limits_{n\to 0}\frac{\xi_n}{n} = 0$ but $\lim\limits_{n\to 0} T(\frac{\xi_n}{n}) = \xi_1$.