Two non-isometrically isomorphic spaces with the same duals. So that a such dual space could not be a $W^*$-algebra under any multiplication and involution.

$\mathcal{C}_0$ and $\mathcal{C}$ are both closed subspaces of $l^\infty$. In addition for each $x = (x_n) \in l^1$, $\rho_x : c_0 \rightarrow \mathcal{C}$ given by $(y_n) \mapsto \sum_{n=1}^{\infty} x_n y_n$ is a bounded linear functional on $c_0$ with the norm $\| \rho_x \| = \| x \|$. Clearly $c_0^\#$ is isometrically isomorphic to $l^1$. Also for each $x = (x_n) \in l^1$, $\eta_x : c \rightarrow \mathcal{C}$ given by $(y_n) \mapsto x_1 \lim_{n \to \infty} x_n + \sum_{n=1}^{\infty} x_n y_n$ is a bounded linear functional on $c$ with the norm $\| \eta_x \| = \| x \|$. Obviously $c^\#$ is isometrically isomorphic to $l^1$. But by BA25.DVI the closed unit ball of $c_0$ has no extreme point while the closed unit ball $c$ contains at least $(1, 1, 1, \ldots)$ as an extreme point (since if $1 = tx_n + (1 - t)y_n$ with $|x_n| \leq 1$ and $|y_n| \leq 1$, then $1 = tRx_n + (1 - t)Ry_n$ for all $n$, so that $Rx_n = Ry_n = 1$ and hence $x_n = y_n = 1$ for each $n$). Thus $c_0$ and $c_1$ are not isometrically isomorphic.

Now by [Sak1, Corollary 1.13.3], $l^1$ can not be a $W^*$-algebra.

Re.