A Banach space which is not an inner product space.

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The supremum norm on $C[a,b]$ can not be obtained from an inner product. Since if $f(t) = 1$ and $g(t) = \frac{t-a}{b-a}$, then $\| f \| = \| g \| = 1, \| f - g \| = \sup\{|1 - \frac{t-a}{b-a}|; t \in [a,b]\} = 1$ and $\| f + g \| = \sup\{|1 + \frac{t-a}{b-a}|; t \in [a,b]\} = 2$ and so the parallelogram equality $\| f + g \|^2 + \| f - g \|^2 = 2 \| f \|^2 + 2 \| g \|^2$ (which is satisfied in every inner product space) isn't held.

Comment. Indeed this Banach space is not an inner product space in any equivalent norm.