An incomplete inner product space.

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The linear space $C[a,b]$ of all continuous complex-valued functions on $[a,b]$ with the inner product $< f,g >= \int_a^b f(x)\overline{g(x)}dx$ is not complete with respect to the norm $\| f \| =< f,f >^{\frac{1}{2}} = (\int_a^b |f(x)|^2 dx)^{\frac{1}{2}}$. In fact the sequence (f_n) where

$$f_n(x) = \begin{cases} 0 & a \leq x < \frac{b+a}{2} \\ (n+n_0)(x - \frac{b+a}{2}) & \frac{b+a}{2} \leq x \leq \frac{b+a}{2} + \frac{1}{n+n_0} \\ 1 & \frac{b+a}{2} + \frac{1}{n+n_0} < x \leq b \end{cases}$$

(n_0 is a natural number greater than $\frac{2}{b-a}$)
is a Cauchy but not convergent.