Two closed densely defined operators $T$ and $S$ on a Hilbert space such that $T+S$ isn’t closable.

Consider a separable infinite dimensional Hilbert space $H$ with an orthonormal basis $(\xi_n)$. Let $D = \{ \eta \in H; \sum_{n=1}^{\infty} n^4 | \langle \eta, \xi_n \rangle |^2 < \infty \}$, $\zeta = \sum_{n=2}^{\infty} n^{-1} \xi_n$, and define the operators $S$ and $T$ with the domain $D$, which is dense in $H$, by

$$S\eta = \sum_{n=2}^{\infty} n^2 < \eta, \xi_n > \xi_n , \quad T\eta = S\eta^+ < S\eta, \zeta > \xi_1 \quad (\eta \in D).$$

Then $-S$ and $T$ are closed densely defined and $T + (-S)$ isn’t closable. (cf. Problem 2.8.43 of [K&R1])

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