Two closed densely defined operators T and S on a Hilbert space such that $T+S$ isn't closable.

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Consider a separable infinite dimensional Hilbert space H with an orthonormal basis (ξ_n). Let $D=\{\eta\in H;\sum_{n=1}^{\infty}n^4|<\eta,\xi_n>|^2<\infty\},\zeta=\sum_{n=2}^{\infty}n^{-1}\xi_n$, and define the operators S and T with the domain D, which is dense in H, by

$$S\eta=\sum_{n=2}^{\infty}n^2<\eta,\xi_n>\xi_n \quad , \quad T\eta=S\eta+<S\eta,\zeta>\xi_1 \quad (\eta\in D).$$

Then $-S$ and T are closed densely defined and $T+(-S)$ isn't closable. (cf. Problem 2.8.43 of [K&R1])

Ref

[K&R1] R.V. Kadilon and J.R. Ringrase, Fundamentals of the theory of operator algebras (I), Acad. Press, 1983.