A Hilbert space whose Hamel dimension and Hilbert dimension are different.

The Hilbert space $l^2$ has the orthonormal basis $(e_n)$ with $e_n(m) = \delta_{mn}$; $m, n \in \mathbb{N}$. Hence its Hilbert dimension is $\aleph_0$. But the set of all sequences $x_\alpha = \langle 1, \alpha, \alpha^2, \alpha^3, \cdots \rangle$, $0 < \alpha < 1$ is a linearly independent uncountable subset of $l^2$. Thus the Hamel dimension of $l^2$ isn’t $\aleph_0$.

**Comment.** This Hilbert dimension is probably the only one which this can happen.