A Hilbert space whose Hamel dimension and Hilbert dimension are different.

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The Hilbert space l^2 has the orthonormal basis (e_n) with $e_n(m) = \delta_{mn}$; $m, n \in \mathbf{N}$. Hence its Hilbert dimension is $\aleph_0$. But the set of all sequences $x_\alpha =< 1, \alpha, \alpha^2, \alpha^3, \cdots >, 0 < \alpha < 1$ is a linearly independent uncountable subset of l^2. Thus the Hamel dimension of l^2 isn't $\aleph_0$.

Comment. This Hilbert dimension is probably the only one which this can happen.