A construction of a bounded approximate identity for a commutative C^*-algebra A.

****

Let $A = C_0(X)$ be a commutative C^*-algebra. Consider the set Λ consisting of all compact subsets of X. $(\Lambda, \subseteq)$ is a directed set. For each compact subset K of X, by Urysohn's lemma, there exists a function $f_K \in C_0(X)$ equal to 1 on K satisfying $0 \leq f \leq 1$. For each $g \in C_0(X)$ and given $\varepsilon > 0$, $K_0 = \{x \in X; |g(x)| \geq \varepsilon\}$ is compact. Hence for all $K \supseteq K_0$, $\|f_K g - g\|_\infty = \sup_{x \in X} |f_K(x)g(x) - g(x)| < \varepsilon$. Therefore $\lim_{K \in \Lambda} f_K g = g$, Thus $(f_K)_{K \in \Lambda}$ is a bounded aproximate identity for A.