

A nonclosed ideal that is not self-adjoint in a commutative C^* -algebra.

Consider C^* -algebra $A = C(\Delta)$ and the ideal $I = fA = \{fg ; g \in A\}$, where $f(z) = z$. $f^*(z) = \bar{z}$ and if $f^* \in I$, then there exists an element $g \in A$ such that $f^* = fg$. So $g(0) = \lim_{z \rightarrow 0} g(z) = \lim_{z \rightarrow 0} \frac{\bar{z}}{z}$, a contradiction. Thus I isn't self-adjoint.