A nonclosed ideal that is not self-adjoint in a commutative C^*-algebra.

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Consider C^*-algebra $A = C(\Delta)$ and the ideal $I = fA = \{fg\ ;\ g \in A\}$, where $f(z) = z$. $f^*(z) = \bar{z}$ and if $f^* \in I$, then there exists an element $g \in A$ such that $f^* = fg$. So $g(0) = \lim_{z\to 0} g(z) = \lim_{z\to 0} \frac{\bar{z}}{z}$, a contradiction. Thus I isn't self-adjoint.