A closed ideal $I$ of a commutative $C^*$-algebra $A$ and a closed ideal $J$ of $I$ such that $J$ isn’t an ideal of $A$.

Let $A = C([0, 1])$, $I = Af$ and $J = Cf + Af^2$, where $f(t) = t; 0 \leq t \leq 1$. Then $J$ is an ideal of $I$ and $I$ is an ideal of $A$; but $f \in J$ and $f, f^\frac{1}{2} \notin J$ (otherwise, there exist $\lambda \in C$ and $g \in A$ such that $f.f^\frac{1}{2} = \lambda f + gf^2$. So $\lim_{t \to 0} t^\frac{1}{2} = \lambda + \lim_{t \to 0} tg(t)$. Therefore $\lambda = 0$ and $t^\frac{1}{2} = tg$ contradicting the continuity of $g$. Thus $J$ isn’t an ideal of $A$. 