A closed ideal I of a commutative C^*-algebra A and a closed ideal J of I such that J isn't an ideal of A.

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Let $A = C([0,1])$, $I = Af$ and $J = \mathcal{C}f + Af^2$,where $f(t) = t; 0 \leq t \leq 1$. Then J is an ideal of I and I is an ideal of A; but $f \in J$ and $f.f^{\frac{1}{2}} \notin J$ (otherwise, there exist $\lambda \in \mathcal{C}$ and $g \in A$ such that $f.f^{\frac{1}{2}} = \lambda f + gf^2$. So $\lim_{t\to 0} t^{\frac{1}{2}} = \lambda + \lim_{t\to 0} tg(t)$. Therefore $\lambda = 0$ and $t^{\frac{1}{2}} = tg$ contradicting the continuity of g. Thus J isn't an ideal of A.