A $C^*$-algebra $A$ where every unitary element is of the form $\exp(\mathrm{i}h)$ for a self-adjoint $h \in A$.

Suppose that $A = C([0,1])$. For each unitary $u \in A$, the mapping $t \mapsto u_t$ from $[0,1]$ to the unitary group of $G$ of $A$ with $u_t(x) = u((1-t)x)$ connects $u$ to $u(0)1$. If $u(0) = \exp(\mathrm{i}\theta)$ for some real number $\theta$, $\{\exp(\mathrm{i}t\theta)1; 0 \leq t \leq 1\}$ in $G$ connects $1$ to $u(0)1$. Therefore $u$ is connected to $1$. Now by [K&R3, Exercise 4.6.7], $u = \exp(\mathrm{i}h)$ for some $h \in A_h$.

**Comment.** By [K&R1, Theorem 5.2.1], $A$ isn’t $W^*$-algebra. **Ref.**
