A C^*-algebra A where every unitary element is of the form $exp(ih)$ for a self-adjoint $h \in A$.

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Suppose that $A = C([0,1])$. For each unitary $u \in A$, the mapping $t \longmapsto u_t$ from $[0,1]$ to the unitary group of G of A with $u_t(x) = u((1-t)x)$ connects u to $u(0)1$. If $u(0) = exp(i\theta)$ for some real number θ, $\{exp(it\theta)1; 0 \leq t \leq 1\}$ in G connects 1 to $u(0)1$. Therefore u is connected to 1. Now by [K&R3, Exercise 4.6.7], $u = exp(ih)$ for some $h \in A_h$.

Comment. By [K&R1, Theorem 5.2.1], A isn't W^*-algebra. **Ref.**

[K&R1] R.V. Kadilon, J.R. Ringrase, Fundamentals of the theory of operator algebras (I), Acad. Press, 1983.

[K&R3] R.V. Kadilon, J.R. Ringrase, Fundamentals of the theory of operator algebras (III), Acad. Press, 1991.