

A C^* -algebra that isn't a von Neumann algebra.

$K(H)$, where H is a separable infinite dimensional Hilbert space is a C^* -algebra but not a von Neumann algebra. In fact if $(e_n)_{n \in \mathcal{N}}$ is an orthonormal basis for H and $P_n = \sum_{i=1}^n e_i \overline{\otimes} e_i$, then P_n is a finite-rank projection converging strongly to the identity operator I (since for each $x \in H$, $I(x) = x = \sum_{i=1}^{\infty} \langle x, e_i \rangle e_i = \lim_n P_n(x)$). If $K(H)$ were a von-Neumann algebra, it should be $I \in K(H)$, a contradiction.