A $C^*$-algebra that isn’t a von Neumann algebra.

$K(H)$, where $H$ is a separable infinite dimensional Hilbert space is a $C^*$-algebra but not a von Neumann algebra. In fact if $(e_n)_{n \in \mathbb{N}}$ is an orthonormal basis for $H$ and $P_n = \sum_{i=1}^n e_i \otimes e_i$, then $P_n$ is a finite-rank projection converging strongly to the identity operator $I$ (since for each $x \in H$, $I(x) = x = \sum_{i=1}^{\infty} < x, e_i > e_i = \lim_n P_n(x)$). If $K(H)$ were a von-Neumann algebra, it should be $I \in K(H)$, a contradiction.