A primitive $C^*$-algebra with a unique nontrivial closed bi-ideal (and so that it is not simple).

Let $H$ be a separable infinite dimensional Hilbert space and $A = B(H)$. Then $K(H)$ is a nontrivial closed bi-ideal of $B(H)$, and if $I$ is a nontrivial closed bi-ideal of $B(H)$, we have $F(H) \subseteq I$ (cf. [Mur, Th. 2.4.7]). Hence $K(H) \subseteq I$. If $I \not\subseteq K(H)$, then $I$ has an infinite-rank projection $p$ (cf. [Mur, Cor. 4.1.14]). For each infinite-rank projection $q$, there exist $u \in B(H)$ such that $p = u^*u$ and $q = uu^*$ (if $(e_n)$ and $(f_n)$ are orthonormal basis for $p(H)$ and $q(H)$ resp., define $u(e_n) = f_n$ and $u = 0$ on $p(H)^\perp$) so $q = upu^* \in I$. Hence $I = B(H)$, a contradiction.

Since $B(H)' = C_1$ (For $(C_1)' = B(H)$ and this is because of $(C_1)' = C_1$), the identity representation $B(H) \twoheadrightarrow B(H)$ is a faithful irreducible representation. Hence $B(H)$ is primitive.

Ref.