A primitive C^*-algebra with a unique nontrivial closed bi-ideal (and so that it is not simple).

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Let H be a separable infinite dimensional Hilbert space and $A = B(H)$. Then $K(H)$ is a nontrivial closed bi-ideal of $B(H)$, and if I is a nontrivial closed bi-ideal of $B(H)$, we have $F(H) \subseteq I$ (cf. [Mur, Th. 2.4.7]). Hence $K(H) \subseteq I$. If $I \not\subseteq K(H)$, then I has an infinite-rank projection p (cf. [Mur, Cor. 4.1.14]). For each infinite-rank projection q, there exist $u \in B(H)$ such that $p = u^*u$ and $q = uu^*$ (if (e_n) and (f_n) are orthonormal basis for $p(H)$ and $q(H)$ resp., define $u(e_n) = f_n$ and $u = 0$ on $p(H)^\perp$) so $q = upu^* \in I$. Hence $I = B(H)$, a contradiction.

Since $B(H)' = \mathcal{C}1$ (For $(\mathcal{C}1)' = B(H)$ and this is because of $(\mathcal{C}1)'' = \mathcal{C}1$), the identity representation $B(H) \longrightarrow B(H)$ is a faithful irreducible representation. Hence $B(H)$ is primitive.

Ref.

[Mur] G.J. Murphy, C^*-algebras and operator theory, Academic Press, 1990.