

Two element x, y in a C^* -algebra A such that $sp(xy) \neq sp(yx)$.

Let $A = B(l^2)$, x be the unilateral shift operator on l^2 , defined by $T(\alpha_1, \alpha_2, \dots) = (0, \alpha_1, \alpha_2, \dots)$, and $y = T^*$. Then $TT^*(\alpha_1, \alpha_2, \dots) = T(\alpha_2, \alpha_3, \dots) = (0, \alpha_2, \alpha_3, \dots)$ and $T^*T(\alpha_1, \alpha_2, \dots) = T^*(0, \alpha_1, \alpha_2, \dots) = (\alpha_1, \alpha_2, \dots)$. Hence $sp(T^*T) = \{1\}$ but $0 \in sp(TT^*)$ (since $(TT^*)(1, 0, 0, \dots) = (0, 0, \dots)$).