Two element x, y in a C^*-algebra A such that $sp(xy) \neq sp(yx)$.

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Let $A = B(l^2)$, x be the unilateral shift operator on l^2, defined by $T(\alpha_1, \alpha_2, ...) = (0, \alpha_1, \alpha_2, ...)$, and $y = T^*$. Then $TT^*(\alpha_1, \alpha_2, \ldots) = T(\alpha_2, \alpha_3, \ldots) = (0, \alpha_2, \alpha_3, \ldots)$ and $T^*T(\alpha_1, \alpha_2, \ldots) = T^*(0, \alpha_1, \alpha_2, \ldots) = (\alpha_1, \alpha_2, \ldots)$. Hence $sp(T^*T) = \{1\}$ but $0 \in sp(TT^*)$ (since $(TT^*)(1, 0, 0, \ldots) = (0, 0, \ldots)$).