

A non-unital C^* -algebra with compact primitive ideal space.

If H is an infinite dimensional Hilbert space, then the non-unital C^* -algebra $K(H)$ is simple. By (CW17), $(K(H))' = \mathcal{C}1$, so the identity representation

$$K(H) \longrightarrow B(H)$$

$$T \longmapsto T$$

is a faithful irreducible representation, hence $\{0\}$ is a primitive ideal of $K(H)$. By (CW19), $K(H)$ is simple, so primitive ideal space of $K(H)$ is $\{\{0\}\}$, a compact space.