A non-unital C^*-algebra with compact primitive ideal space.

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If H is an infinite dimensional Hilbert space, then the non-unital C^*-algebra $K(H)$ is simple. By (CW17), $(K(H))' = \mathcal{C}1$, so the identity representation

$$\begin{array}{ccc} K(H) & \longrightarrow & B(H) \\ T & \longmapsto & T \end{array}$$

is a faitful irreducible representation, hence $\{0\}$ is a primitive ideal of $K(H)$. By (CW19), $K(H)$ is simple, so primitive ideal space of $K(H)$ is $\{\{0\}\}$, a compact space.