A C^*-algebra A and a closed bi-ideal J of A such that $\frac{A}{J}$ and J are liminal, but A is not liminal.

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Let H be an infinite dimensional Hilbert space and I_H be the identity operator on H. Then $A = K(H) + \mathcal{C}I_H$ isn't liminal (otherwise, since identity representation $K(H) + \mathcal{C}I_H \longrightarrow B(H)$ is nonzero irreducible ($(K(H) + \mathcal{C}I_H)' = K(H)' = \mathcal{C}I_H$ (see $CW17$)) we should have $I_H \in K(H)$, a contradiction). But $K(H)$ is liminal, since each nonzero irreducible representation of $K(H)$ is unitarily equivalent to identity representation $K(H) \longrightarrow B(H)$ (see [Mur, page 146]). Also $\frac{A}{J} \simeq \mathcal{C}I_H$ which is finite dimensional and so is liminal (Every finite dimensional C^*-algebra B is liminal, since if (H_1, Ψ) is a nonzero irreducible representation of B, then for $x \neq 0$ in H_1, $\Psi(B)x$ is finite dimensional (for $\Psi(B)$ is finite dimensional). If $(u_\lambda)_\lambda$ be any approximate unit for B, then $(\Psi(u_\lambda))_\lambda$ strongly converges to I_H so $x \in [\Psi(B)x] = \Psi(B)x$. Hence $\Psi(B)x$ is a nonzero (closed) subspace of H_1 invariant for $\Psi(B)$, so by irreducibility, $\Psi(B)x = H_1$. Therefore H_1 is finite dimensional. Thus $\Psi(B) \subseteq B(H_1) = K(H_1)$).

Ref.

[**Mur**] G.J. Murphy, C^*-algebras and operator theory, Academic Press, 1990.